These formulas become much simpler in the extreme relativistic limit. After integrating over angles, the relative differential probability becomes identical for magnetic and electric transitions, namely

$$d\gamma_j = \frac{(2j+1)\,\alpha^2}{2\,(2\pi)^2}$$

$$\times \left(1 - \frac{m^2(\Delta E - k)}{\Delta E \varepsilon_+ \varepsilon_-}\right)^j \frac{(\varepsilon_+^2 + \varepsilon_-^2)\,k)}{(\Delta E - k)^2 \Delta E^3}\, dk\, d\varepsilon_+.$$

The ratio of the differential probability for internal Compton effect to the probability for ordinary internal pair-conversion is roughly given by

$$d\gamma_j / d\beta_j = (\alpha/2\pi)\, k dk / (\Delta E - k)^2,$$

provided that $\Delta E\text{-}K >> m$.

In conclusion the author thanks I. S. Shapiro for valuable advice and assistance.

1. E. G. Melikian, J. Exptl, Theoret, Phys. (U.S.S.R.) 31, 1088 (1956); Soviet Phys. JETP

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The Momentum Distribution of Interacting Fermi Particles

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WE consider a system composed of a large number of interacting Fermi-particles. It is to be expected that among the excited states of the system there will exist states whose energy can be expressed as a sum of energies of quasi-particles. The energy of a quasi-particle of momentum p is

$$\varepsilon_p = v_0 (p - p_0),$$

where p_0 is the momentum at the top of the Fermi sea of the quasi-particles, $v_0 = v(p_0)$ is the velocity of a quasi-particle at the Fermi surface, $p > p_0$ for quasi-particles and $p < p_0$ for holes. The momentum p_0 need not coincide with the limiting momentum p_0^0 determined by the density,

$$p_0^0 = (3\pi^2 n)^{1/3}, \qquad (\hbar = 1).$$

It is easy to see that the quasi-particles have an attenuation proportional to $(p - p_0)^2$. This means that for p_0 not close to p_0 an excited state of a system with strong interactions cannot be described in terms of quasi-particles. As $p \to p_0$ the state becomes describable in terms of quasi-particles even when the interaction is strong. We shall prove that the momentum distribution of the particles in the ground state has a discontinuity at $p = p_0$, for any kind of interaction. This result refers to the distribution of particles and not of quasi-particles.

The one-particle Green's function is defined by

$$G(\mathbf{r}_1, t_1, \mathbf{r}_2, t_2) = i\,\langle T e^{iHt_1}\Psi(\mathbf{r}_1)\, e^{-iH(t_1 - t_2)}\,\Psi^+(\mathbf{r}_2)\, e^{iHt_2}\rangle, \tag{1}$$

where the expectation value is taken in the ground state of the system $\Psi(\mathbf{r}) = \Sigma a_\mathbf{p} e^{i\mathbf{pr}}$, and $a_\mathbf{p}$ is the annihilation operator for a particle of momentum p. If there is no external field, G is a function only of $r = |\mathbf{r}_1 - \mathbf{r}_2|$ and $\tau = t_1 - t_2$. Expressing G as a Fourier series in coordinate space, we find

$$G(r, \tau) = \sum G(p, \tau)\, e^{i\mathbf{pr}}; \tag{2}$$

$$G(p, \tau) = \begin{cases} ie^{iE_0\tau} < a_p e^{-iH\tau} a_p^+ >, & \tau > 0, \\ -ie^{-iE_0\tau} < a_p^+ e^{iH\tau} a_\mathbf{p} >, & \tau < 0. \end{cases}$$

This equation connects the function $G(p, \tau)$ with the momentum distribution of particles in the ground state, which is

$$n(p) = < a_p^+ a_\mathbf{p} > = iG(p, \tau)\,|_{\tau \to -0}.$$

Writing

$$G(p, \tau) = \int G(p, \varepsilon)\, e^{-i\varepsilon\tau}\, d\varepsilon / 2\pi,$$

we obtain

$$n(p) = i\int G(p, \varepsilon)\, e^{-i\varepsilon\tau}\, d\varepsilon / 2\pi,$$
$$\tau \to -0.$$

In the last integral we must not take the limit $\tau = 0$ before integration, since the integral $\int G(p, \epsilon)\, d\epsilon$ diverges along the real axis. For finite negative τ, we can replace the integral along the real axis by an integral round a closed contour C consisting of the real axis together with a semi-circle at infinity in the upper half-plane. After this we can set $\tau = 0$. Thus we have

$$n(p) = i\int_C G(p, \varepsilon)\, d\varepsilon / 2\pi. \qquad (3)$$

The Green's function must have poles corresponding to quasi-particles. This follows from the representation of the Green's function in terms of the eigenstates of the whole system, according to the procedure of Lehmann.[1] Therefore, for p close to p_0,

$$G(p, \varepsilon) = Z / (\varepsilon_p - \varepsilon - i\gamma(p)) + f(p, \varepsilon)$$

where $f(p, \epsilon)$ is a function regular at $\epsilon = \epsilon_p - i\gamma$, and γ defines the attenuation of a quasi-particle and changes sign at $p = p_0$ as is required in order to give the correct sign for the attenuation of holes. The constant Z may be called the renormalization constant of the Green's function. When $p < p_0$, $\gamma < 0$ and G has a pole in the upper half-plane near to the real axis. When $p > p_0$, $\gamma > 0$ and this pole crosses to the lower half-plane where it is outside the contour C. Therefore,

$$n(p_0 - 0) - n(p_0 + 0) = Z, \qquad (4)$$

and since $0 \le n(p) \le 1$, the renormalization constant satisfies the inequality $|Z| \le 1$.

1 H. Lehman, Nuovo Cimento 11, 342 (1954); reproduced in "Problems of Modern Physics," 3, 1955.

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The μ–decay of K–particles and Hyperons

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RECENTLY Schwinger[1] suggested that the weak interactions of μ-mesons and neutrinos with pions and K-particles are primary, and that the interactions of μ-mesons with hyperons and nucleons are secondary effects of the weak primary boson-fermion interaction. We here consider some elementary consequences of this hypothesis and discuss possible experimental tests of it.*

We suppose that the decays

$$\pi^{\pm} \to \mu^{\pm} + \nu \text{ and } K^{\pm} \to \mu^{\pm} + \nu, \qquad (1)$$

are primary, and that all other interactions of μ-meson and neutrino with baryons and heavy mesons are results of a chain of interactions of which the process (1) constitutes one link. Such a chain of interactions can describe in particular the μ-decay of hyperons (e. g., $\Lambda^{\circ} \to p + K^- \to p + \mu^- + \nu$) and the so-called $K_{\mu 3}$-decay of K-particles (e. g., $K^+ \to \pi^{\circ} + K^+ \to \pi^{\circ} + \mu^+ + \nu$). The other links in the chain must be strong interactions. Thus the other links cannot be processes in which strangeness is not conserved, such as $K^+ \to \pi^+ + \pi^{\circ}$, $\Lambda^{\circ} \to p + \pi^-$, etc.

The last remark implies that every μ-decay of particles with strangeness $+1$ (the K^+ and K°-particle and the anti-hyperons $\bar{\Lambda}$ and $\bar{\Sigma}$) must go via the μ-decay of the K^+ ($K^+ \to \mu^+ + \nu$) while the μ-decay of particles with strangness -1 (K^- and $\bar{K}^{\circ}$ and the hyperons Λ and Σ must go via the K^- decay ($K^- \to \mu^- + \nu$). So for the K°-particle, the decay

$$K^0 \to \mu^+ + \nu + \pi^- \qquad (2)$$

is allowed, while

$$K^0 \to \mu^- + \nu + \pi^+; \qquad (2')$$

is forbidden, and for the $\bar{K}^{\circ}$, the decay

$$\overline{K^0} \to \mu^- + \nu + \pi^+ \qquad (3)$$

is allowed while

$$\overline{K_0} \to \mu^+ + \nu + \pi^-. \qquad (3')$$

is forbidden. Also, in order to construct the two-step chain for the $K_{\mu 3}$ decay, we must have two types of K-particle, a scalar θ and a pseudoscalar τ, if the K-particle spin is zero. Otherwise, since the pion is pseudoscalar, parity would not be conserved in the process $K \to K + \pi$, and this is a strong interaction which must conserve parity. If