$$n(p) = i \int G(p, \varepsilon)\, e^{-i\varepsilon\tau}\, d\varepsilon / 2\pi,$$
$$\tau \to -0.$$

In the last integral we must not take the limit $\tau = 0$ before integration, since the integral $\int G(p, \epsilon)\, d\epsilon$ diverges along the real axis. For finite negative τ, we can replace the integral along the real axis by an integral round a closed contour C consisting of the real axis together with a semi-circle at infinity in the upper half-plane. After this we can set $\tau = 0$. Thus we have

$$n(p) = i \int_C G(p, \varepsilon)\, d\varepsilon / 2\pi. \quad (3)$$

The Green's function must have poles corresponding to quasi-particles. This follows from the representation of the Green's function in terms of the eigenstates of the whole system, according to the procedure of Lehmann.[1] Therefore, for p close to p_0,

$$G(p, \varepsilon) = Z / (\varepsilon_p - \varepsilon - i\gamma(p)) + f(p, \varepsilon)$$

where $f(p, \epsilon)$ is a function regular at $\epsilon = \epsilon_p - i\gamma$, and γ defines the attenuation of a quasi-particle and changes sign at $p = p_0$ as is required in order to give the correct sign for the attenuation of holes. The constant Z may be called the renormalization constant of the Green's function. When $p < p_0$, $\gamma < 0$ and G has a pole in the upper half-plane near to the real axis. When $p > p_0$, $\gamma > 0$ and this pole crosses to the lower half-plane where it is outside the contour C. Therefore,

$$n(p_0 - 0) - n(p_0 + 0) = Z, \quad (4)$$

and since $0 \leq n(p) \leq 1$, the renormalization constant satisfies the inequality $|Z| \leq 1$.

1 H. Lehman, Nuovo Cimento 11, 342 (1954); reproduced in "Problems of Modern Physics," 3, 1955.

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The μ–decay of K–particles and Hyperons

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(Submitted to JETP editor November 22, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 400–402 (February, 1957)

RECENTLY Schwinger[1] suggested that the weak interactions of μ-mesons and neutrinos with pions and K-particles are primary, and that the interactions of μ-mesons with hyperons and nucleons are secondary effects of the weak primary boson-fermion interaction. We here consider some elementary consequences of this hypothesis and discuss possible experimental tests of it.*

We suppose that the decays

$$\pi^{\pm} \to \mu^{\pm} + \nu \text{ and } K^{\pm} \to \mu^{\pm} + \nu, \quad (1)$$

are primary, and that all other interactions of μ-meson and neutrino with baryons and heavy mesons are results of a chain of interactions of which the process (1) constitutes one link. Such a chain of interactions can describe in particular the μ-decay of hyperons (e. g., $\Lambda^\circ \to p + K^- \to p + \mu^- + \nu$) and the so-called $K_{\mu 3}$ -decay of K-particles (e. g., $K^+ \to \pi^\circ + K^+ \to \pi^\circ + \mu^+ + \nu$). The other links in the chain must be strong interactions. Thus the other links cannot be processes in which strangeness is not conserved, such as $K^+ \to \pi^+ + \pi^\circ$, $\Lambda^\circ \to p + \pi^-$, etc.

The last remark implies that every μ-decay of particles with strangeness +1 (the K^+ and K°-particle and the anti-hyperons $\bar{\Lambda}$ and $\bar{\Sigma}$) must go via the μ-decay of the K^+ ($K^+ \to \mu^+ + \nu$) while the μ-decay of particles with strangness −1 (K^- and $\bar{K}^\circ$ and the hyperons Λ and Σ must go via the K^- decay ($K^- \to \mu^- + \nu$). So for the K°-particle, the decay

$$K^0 \to \mu^+ + \nu + \pi^- \quad (2)$$

is allowed, while

$$K^0 \to \mu^- + \nu + \pi^+; \quad (2')$$

is forbidden, and for the $\bar{K}^\circ$, the decay

$$\overline{K^0} \to \mu^- + \nu + \pi^+ \quad (3)$$

is allowed while

$$\overline{K_0} \to \mu^+ + \nu + \pi^-. \quad (3')$$

is forbidden. Also, in order to construct the two-step chain for the $K_{\mu 3}$ decay, we must have two types of K-particle, a scalar θ and a pseudoscalar τ, if the K-particle spin is zero. Otherwise, since the pion is pseudoscalar, parity would not be conserved in the process $K \to K + \pi$, and this is a strong interaction which must conserve parity. If we assume that the processes $\theta \rightleftarrows \tau + \pi$ and $\tau \rightleftarrows \theta + \pi$ conserve isotopic spin, it is easy to derive a relation between the probabilities of the processes

$$a)\ K^0 \rightarrow \mu^+ + \nu + \pi^-, \quad b)\ K^+ \rightarrow \mu^+ + \nu + \pi^0.$$

We find* that $w_a = 2w_b$.

It is well known[2–4] that the ratio of μ^+ to μ^- decays in a beam of K°-particles will vary with time or with distance from the point of origin of the beam. This depends on the fact that the ratio of $\bar{K}^\circ$ to K° in the beam is variable. Consider for definiteness a beam of θ°-particles. At $t = 0$ there are no $\bar{\theta}^\circ$-particles present:

$$N(\theta^0,\ t=0) = N_0; \quad N(\bar{\theta}^0,\ t=0) = 0.$$

Then[3] after a time t,

$$N(\theta^0,\ t) = \frac{1}{4} N_0 \left[e^{-t/\tau_1} + e^{-t/\tau_2} + 2e^{-t/2\tau_1 - t/2\tau_2} \cos \Delta m t\right],$$

$$N(\bar{\theta}^0,\ t) = \frac{1}{4} N_0 \left[e^{-t/\tau_1} + e^{-t/\tau_2} - 2e^{-t/2\tau_1 - t/2\tau_2} \cos \Delta m t\right].$$

Here τ_1 and τ_2 are the θ_1° and θ_2° lifetimes, and Δm is the difference between their masses; we recall that the particles which have definite charge-parity[5] are θ_1° and θ_2°, and not θ° and $\bar{\theta}^\circ$. Since processes (2′) and (3′) are forbidden, while the probabilities of (2) and (3) are equal by virtue of charge-symmetry, the ratio of μ^+ to μ^- decays will vary in the beam according to the formula

$$\frac{n_{\mu^+}}{n_{\mu^-}} = \frac{e^{-t/\tau_1} + e^{-t/\tau_2} + 2e^{-t/2\tau_1 - t/2\tau_2} \cos \Delta m t}{e^{-t/\tau_1} + e^{-t/\tau_2} - 2e^{-t/2\tau_1 - t/2\tau_2} \cos \Delta m t}. \quad (4)$$

This is a special case of a more general formula found by Zel'dovich[4]. A similar law holds for the μ-decay of the τ°-particle.

If the hypothesis of the primary character of the $K\mu\nu$-interaction is correct, then in the $K_{\mu 4}$-decay

$$(K \rightarrow \mu + \nu + 2\pi)$$

the K^+ and K° must emit μ^+-mesons while the K^- and $\bar{K}^\circ$ must emit μ^--mesons. However, the probability of $K_{\mu 4}$ decay must be very small compared with that of $K_{\mu 3}$-decay. The decay of Λ and Σ hyperons, which have strangeness -1, must go via the μ-decay of a K^--particle. This means that hyperon decays can only produce μ^- and never μ^+-mesons. Thus the processes

$$\Lambda^0 \rightarrow p + \mu^- + \nu, \quad \Sigma^- \rightarrow n + \mu^- + \nu$$

are allowed, while the μ-decay of the Σ^+-hyperon is forbidden.

Finally we consider the μ-decay of the cascade Ξ-hyperon. Our hypothesis requires that in μ-decay the strangeness is either conserved (if the μ-decay goes via a pion) or is changed by one unit (if the μ-decay goes via a K-particle). Since the Ξ-hyperon has strangeness -2, it can decay by

$$\Xi^- \rightarrow \Lambda^0 (\Sigma^0) + \mu^- + \nu, \quad \Xi^0 \rightarrow \Sigma^+ + \mu^- + \nu$$

but not by

$$\Xi^- \rightarrow n + \mu^- + \nu, \quad \Xi^0 \rightarrow p + \mu^- + \nu$$

Decays with μ^+ emission are of course forbidden for Ξ-hyperons.

If these various consequences of Schwinger's hypothesis are not confirmed by experiment, it means that the hypthesis is incorrect. However, an experimental confirmation of the effects will not imply that the hypothesis is correct. In fact the same consequences can be derived from quite different assumptions. For example, they follow from a hypothesis of Sachs,[6] according to which μ-decays conserve "attribute" and the "attribute" of μ^- and neutrino together is equal to that of K^- or Λ or Σ. All the selection rules which we have found hold in the Sachs theory, independently of whether the primary interaction of μ, ν is with pions and K-particles or with nucleons and hyperons. In particular, the forbiddenness of (2′) and (3′) was deduced by Sachs,[6] and Eq. (3) by Sachs and Trieman,[7] starting from the "attribute" hypothesis.

The author expresses his thanks to Professor I. Ia. Pomeranchuk for much valuable criticism.

*After this letter was in print we received a typescript of the second part of Schwinger's work, including some of the results reported here.

*This result was found independently by S. G. Matinian (private communication).

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