Production of Proton-Antiproton Pairs by High Energy γ-Quanta

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The production of proton–antiproton pairs by high energy γ-quanta on nuclei is investigated by a semi-phenomenological method. A knowledge of the asymptotic expression of the function of each nucleon is sufficient for the computation of the effective cross section. On the assumption that the nucleus is "black" with respect to the nucleons, the asymptotic expression has been found as a superposition of a plane and diffracted waves. The calculations are extended to the case of an arbitrary interaction law between the nucleons and nucleus. The differential cross section is then expressed in terms of the amplitude of elastic scattering of nucleons on the nucleus.

IN view of the latest reports on the discovery of the antiproton[1], it is of interest to compute the effective cross section for formation of proton-antiproton pairs, without recourse to the weak coupling approximation. In the present paper, a semiphenomenological theory is developed which describes the formation of proton-antiproton pairs by high energy γ-quanta on nuclei; the theory is analogous to the one developed by Pomeranchuk[2] for π-mesons pair production.

We will assume that the nucleus is, with respect to the nucleon, an "absolutely black" sphere of radius R (below, we will generalize the results to an arbitrary interaction law between nucleons and nucleus). The wave function can then be determined outside the nucleus as a superposition of plane and diffracted waves. The probability of pair formation is, at small angles between the momenta of the nucleons and of the γ-quanta, totally determined by this wave function; this is so because the main role is played by distances which are large as compared to the nuclear radius. Let us note that in this ultrarelativistic region only small angles are practically important, because the cross section falls rapidly with increasing angles.

We will assume that the nucleons are described by the Dirac equation. The anomalous magnetic moment is irrelevant at such large energies of the γ-quanta.

The matrix element for pair production will be written in the form:

$$M = ie\sqrt{\frac{2\pi}{\omega}}\int\bar{\psi}_1(\gamma\mathbf{j})\,\psi_2\,e^{i\mathbf{k}_\gamma\mathbf{r}}\,d\mathbf{r}; \qquad (1)$$

$$\gamma_k = -i\beta\alpha_k\ (k=1,2,3),\ \gamma_4=\beta,\ \bar{\psi}=\psi^*\beta.$$

Here γ_i are the Dirac matrices, $\mathbf{k}_\gamma$ is the wave vector of the γ-quantum of frequency ω and of polarization $\mathbf{j}$ (we put $\hbar = c = 1$ everywhere), ψ_1, ψ_2 are the proton-antiproton wave functions, respectively. Each one of them can be taken to be a superposition of plane and incoming spinor waves[3]. The expression for such a ψ-function can be obtained from the expression for a plane and outgoing spinor waves written down by Akhiezer[4]. Indeed, if we write the asymptotic expression for the solution of the Dirac equation in the form of a superposition of plane and outgoing waves:

$$\psi = ue^{i\mathbf{pr}} + f(p\theta)\frac{e^{ipr}}{r}, \quad u = \begin{pmatrix} v \\ \sigma_p v/(E+m)\end{pmatrix}, \qquad (2)$$

θ is the angle between the vectors $\mathbf{p}$ and $\mathbf{r}$; θ is a two-dimensional angular vector. The solution of the same Dirac equation with the asymptotic expression in the form of a plane and incoming waves, is easily obtained in the form:

$$\psi' = ue^{i\mathbf{pr}} + f'(p\theta)\frac{e^{-ipr}}{r},$$

$$f'(p\theta) = i\,\Sigma_y f^*(-p\theta), \qquad i\,\Sigma_y = \begin{pmatrix} i\sigma_y & 0 \\ 0 & i\sigma_y\end{pmatrix}, \qquad (3)$$

where σ_y is the Pauli matrix. The spinor v has to be substituted by the spinor $-i\sigma_y v$.

Using expression (3) and the value of the amplitude f, we obtain:

$$\psi_1 = u_1e^{i\mathbf{p}_1\mathbf{r}} - \frac{1}{4\pi p_1}\int\left(\gamma\frac{\partial}{\partial\mathbf{r}} - \gamma_4E_1 - m\right)(\mathbf{p}_1\gamma)\,u_1 \times \frac{\exp\{-ip_1|\mathbf{r}-\mathbf{s}_1|\}}{|\mathbf{r}-\mathbf{s}_1|}d\mathbf{s}_1 = u_1e^{i\mathbf{p}_1\mathbf{r}} - \Phi_1,$$

$$\Phi_2 = u_2 e^{-i\mathbf{p}_2\mathbf{r}} - \frac{1}{4\pi p_2}\int\left(\gamma\frac{\partial}{\partial\mathbf{r}} + \gamma_4 E_2 - m\right)(\mathbf{p}_2\gamma)\, u_2 \times \frac{\exp\{ip_2|\mathbf{r}-\mathbf{s}_2|\}}{|\mathbf{r}-\mathbf{s}_2|}\, d\mathbf{s}_2 = u_2 e^{-i\mathbf{p}_2\mathbf{r}} - \Phi_2, \quad (4)$$

where $\mathbf{p}_1$, E_1 and $\mathbf{p}_2$, E_2 are the proton and antiproton momenta and energies, respectively. The integration over $\mathbf{s}_1(\mathbf{s}_2)$ is along a circle of radius R, perpendicular to $\mathbf{p}_1(\mathbf{p}_2)$ and passing through the center of the nucleus.

Let us transform the expressions for the functions Φ_1 and Φ_2 writing them in the form of a Fourier integral

$$\Phi_1 = \frac{1}{8\pi^3 p_1}\int\left(\gamma\frac{\partial}{\partial\mathbf{r}} - \gamma_4 E_1 - m\right) \times (\mathbf{p}_1\gamma)\, u_1 \frac{\exp\{-i\mathbf{q},\, \mathbf{r}-\mathbf{s}_1\}}{q^2 - p_1^2 + i\varepsilon}\, d\mathbf{q}\, d\mathbf{s}_1; \quad \varepsilon \to 0. \quad (5)$$

The fuction Φ_2 will be written in an analogous form.

Let us introduce the angular vectors $\mathbf{k}_1$, $\mathbf{k}_2$.

$$\mathbf{p}_1 = p_1\frac{\mathbf{k}_\gamma}{k_\gamma}\left(1 - \frac{k_1^2}{2p_1}\right) + \mathbf{k}_1, \quad (6)$$

$$\mathbf{p}_2 = p_2\frac{\mathbf{k}_\gamma}{k_\gamma}\left(1 - \frac{k_2^2}{2p_2}\right) + \mathbf{k}_2, \quad \mathbf{k}_\gamma\mathbf{k}_1 = \mathbf{k}_\gamma\mathbf{k}_2 = 0,$$

$$\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}, \quad (\mathbf{k}_1 - \mathbf{k}_2)/2 = \mathbf{b}.$$

Using the condition E_1, $E_2 \gg m$, where m is the nucleon rest mass, we get

$$M = e\,(32\pi^3/\omega^3)^{1/2} E_1 E_2 k^{-1} R J_1(kR)\, \bar{u}_1 T u_2,$$

$$T = \frac{1}{m^2 + k_1^2}\left(\gamma\mathbf{j} - \frac{\gamma_3(\gamma\mathbf{j})(\gamma\mathbf{k})}{2p_2}\right) + \frac{1}{m^2 + k_2^2}\left(\gamma\mathbf{j} - \frac{\gamma_3(\gamma\mathbf{k})(\gamma\mathbf{j})}{2p_1}\right) + \frac{1}{m^2 + b^2}\left(-\gamma\mathbf{j} + \frac{1}{4p_1}\gamma_3(\gamma\mathbf{k})(\gamma\mathbf{j}) + \frac{1}{4p_2}\gamma_3(\gamma\mathbf{j})(\gamma\mathbf{k})\right), \quad (\gamma\mathbf{k}_\gamma) = \gamma_3\omega. \quad (7)$$

The main contribution to the integrals (1) comes from distances to the nucleus $p_1 p_2/m^2\,\omega \gg R$, if $k_1 \sim k_2 \leqslant m$ (see also Ref. 2). The use of the asymptotic expressions for the γ-function is therefore legitimate.

Let us make a summation of $|M|^2$ over the nucleon spins in the final state. Let us also note that, in the integration over k and b, the main role is played by the values $k \sim 1/R$ and $b \sim m$; wherever possible, we shall drop terms $\sim k$ with respect to terms $\sim b$.

The effective cross section of the process is

$$d\sigma_{\mathbf{j}} = 2\pi\sum|M|^2(2\pi)^{-6}\, dp_{1z}\, d\mathbf{k}_1\, d\mathbf{k}_2\, |F_{+-}|^2. \quad (8)$$

The expression for the cross section contains an auxiliary factor F_{+-}, which is a form factor determined by the indefiniteness of the nucleons and which depends on the invariant frequency of the γ-quantum in the rest system of each of the nucleons. The form factor contains as well the interaction between nucleons[2,4]:

$$F_{+-} = F_{+-}\left(\frac{m^2 + b^2}{\mu m}\frac{\omega}{2E_1};\ \frac{m^2 + b^2}{\mu m}\frac{\omega}{2E_2}\right), \quad (9)$$

where μ is the rest mass of the π-meson (the dimensions of the nucleon are assumed to be $\sim 1/\mu$). We obtain the following expression for the differential cross section of the process:

$$d\sigma_{\mathbf{j}} = \frac{e^2R^2}{\pi^2}\frac{d\mathbf{k}\, J_1^2(kR)}{k^2}\frac{d\mathbf{b}\, dE_1}{2(m^2+b^2)^2\omega^3}\,[m^2\omega^2 + b^2(E_1^2 + E_2^2) + 2E_1E_2(\mathbf{j}\times\mathbf{b})^2 - 2E_1E_2(\mathbf{j}\mathbf{b})^2]\,|F_{+-}|^2, \quad (10)$$

$$E_1 + E_2 = \omega, \quad d\mathbf{k} = k\,dk\,d\varphi_k, \quad d\mathbf{b} = b\,db\,d\varphi_b.$$

Averaging (10) over the γ-quantum polarizations, we obtain

$$d\sigma = \frac{e^2R^2}{2\pi^2}\frac{d\mathbf{k}\, J_1^2(kR)}{k^2}\frac{d\mathbf{b}\, dE_1}{(m^2+b^2)^2\omega^3}\,[m^2\omega^2 + b^2E_1^2 + b^2(\omega - E_1)^2]\,|F_{+-}|^2. \quad (11)$$

Integrating over $\mathbf{k}$ and over the directions of the vector $\mathbf{b}$, we get

$$d\sigma = e^2R^2\frac{b\,db\,dE_1}{(m^2+b^2)^2\omega^3}\,[m^2\omega^2 + b^2E_1^2 + b^2(\omega - E_1)^2]\,|F_{+-}|^2. \quad (12)$$

Let us note that the differential cross section reaches large nuclear values for small angles between the momenta of the formed particles and the γ-quantum momenta $b \lesssim m$. The integration over E_1 and b can be performed only by letting $F_{+-} = 1$.

$$d\sigma(E_1) = \frac{e^2R^2}{2}\left\{\frac{b_{\max}^2}{m^2 + b_{\max}^2}\frac{1}{\omega} + \frac{E_1^2 + (\omega - E_1)^2}{\omega^3} \times \left(\ln\frac{m^2 + b_{\max}^2}{m^2} - \frac{b_{\max}^2}{m^2 + b_{\max}^2}\right)\right\} dE_1 \quad (13)$$

where $b_{max} \sim m$ and is limited by the condition that all the derivation is valid only for small angles. The total cross section of the process has the form:

$$\sigma = \frac{e^2R^2}{3}\left(\frac{1}{2}\frac{b^2_{max}}{m^2+b^2_{max}} + \ln\frac{m^2+b^2_{max}}{m^2}\right). \quad (14)$$

This integration can give erroneous results because, for large values of its arguments (as in our case), the form factor can decrease the cross section appreciably. Some properties of the form factor, important for the theory, can be obtained by comparing the differential cross section with the experimental data. If we let $F_+=1$, then it can be seen from (11) that the effective angles are $\theta \sim m/E$.

Formula (13) gives the energy distribution of the formed protons. It has a minimum for $E_1 = E_2 = \omega/2$. Let us note that the total cross section for pair formation turns out to depend neither on the energy of the inpinging quantum, nor on the rest mass of the formed particles.

The results obtained for a concrete model of the nucleus – the "absolutely black" nucleus – can be generalized to an arbitrary interaction law between nucleon and nucleus – in analogy to what has been done for the π-meson[5]. In this case the cross section is expressed through the spinor amplitudes for elastic scattering of nucleons on a nucleus.

The wave function will be written in the form

$$\psi_1 = u_1 e^{i\mathbf{p}_1\mathbf{r}} + f_1(p_1\theta_1)e^{-ip_1r}/r,$$
$$\psi_2 = u_2 e^{-i\mathbf{p}_2\mathbf{r}} + f_2(p_2\theta_2)e^{ip_2r}/r, \quad (15)$$

where θ^i is the angle between the vector $\mathbf{r}$ and $-\mathbf{p}_i = 1,2$. The amplitude f_1 is obtained from the usual spinor amplitude f for elastic scattering, in the transition to a converging asymptote[3].

Let us transform to the Fourier representation:

$$\bar{f}_1(p_1\theta_1)\frac{e^{ip_1r}}{r} = \int \bar{\chi}_1(\mathbf{q})e^{i\mathbf{qr}}\,d\mathbf{q}; \quad (16)$$

$$\bar{\chi}_1(\mathbf{q}) = (2\pi)^{-3}\int \bar{f}_1(p_1\theta_1)\frac{e^{ip_1r}}{r}e^{-i\mathbf{qr}}\,d\mathbf{r} \quad (17)$$

and analogously for f_2. The polar axis is chosen in the direction of the vector $-\mathbf{p}_1$. Making use of the smallness of all the angles, we get

$$\bar{\chi}_1(\mathbf{q}) = (2\pi)^{-3}\int \bar{f}_1(p_1\theta)\exp\left\{i(p_1-q)r + \frac{iqr}{2}(\theta-\theta_{1q})^2\right\}rdrd\theta$$

$$= (2\pi)^{-2}\bar{f}_1(p_1\theta_{1q})/(q^2-p_1^2-i\varepsilon); \quad \varepsilon \to 0. \quad (18)$$

Analogously, we have;

$$\chi_2(\mathbf{q}) = (2\pi)^{-2}f_2(p_2\theta_{2q})/(q^2-p_2^2-i\varepsilon); \quad \varepsilon \to 0. \quad (19)$$

Here θ_{iq} is the angle between the vectors $-\mathbf{p}_i$ and $\mathbf{q}$, $i=1,2$.

Let us investigate the integrals involved in the matrix element (1). The integrals containing either f_1 or f_2 are:

$$\int\left(\bar{f}_1(p_1\theta_1)\frac{e^{ip_1r}}{r}(\gamma\mathbf{j})u_2e^{-i\mathbf{p}_2\mathbf{r}} + \bar{u}_1e^{-i\mathbf{p}_1\mathbf{r}}(\gamma\mathbf{j})f_2(p_2\theta_2)\frac{e^{ip_2r}}{r}\right)e^{i\mathbf{k}_\gamma\mathbf{r}}\,d\mathbf{r}$$
$$= \int\left(\bar{\chi}_1(\mathbf{q})(\gamma\mathbf{j})u_2e^{i(\mathbf{q}+\mathbf{k}_\gamma-\mathbf{p}_2)\mathbf{r}} + \bar{u}_1(\gamma\mathbf{j})\chi_2(\mathbf{q})e^{i(\mathbf{q}+\mathbf{k}_\gamma-\mathbf{p}_1)\mathbf{r}}\right)d\mathbf{q}\,d\mathbf{r} \quad (20)$$
$$= \frac{4\pi}{\omega(m^2+b^2)}(p_2\bar{f}_1(\mathbf{k})(\gamma\mathbf{j})u_2 + p_1\bar{u}_1(\gamma\mathbf{j})f_2(\mathbf{k})).$$

The integral containing both f_1 and f_2 has the form

$$\int\bar{f}_1(p_1\theta_1)\frac{e^{ip_1r}}{r}(\gamma\mathbf{j})f_2(p_2\theta_2)\frac{e^{ip_2r}}{r}e^{i\mathbf{k}_\gamma\mathbf{r}}\,d\mathbf{r}$$
$$= \int\bar{\chi}_1(\mathbf{q})(\gamma\mathbf{j})\chi_2(\mathbf{q}')e^{i(\mathbf{q}+\mathbf{q}'+\mathbf{k}_\gamma)\mathbf{r}}\,d\mathbf{q}\,d\mathbf{q}'\,d\mathbf{r}$$
$$= (2\pi)^3\int\bar{\chi}_1(\mathbf{q})(\gamma\mathbf{j})\chi_2(-\mathbf{k}_\gamma-\mathbf{q})\,d\mathbf{q}. \quad (21)$$

We let $\mathbf{q} = \mathbf{q}_z + \mathbf{g}$ $(\mathbf{k}_\gamma \cdot \mathbf{g}) = 0$. (21) is then rewritten in the following form:

$$\frac{2}{\pi}\int d\mathbf{g}\bar{f}_1(\mathbf{k}_1+\mathbf{g})(\gamma\mathbf{j})f_2(\mathbf{k}_2-\mathbf{g})$$
$$\int_{-\infty}^{\infty} dq_z/(q_z^2+g^2-p_1^2-i\varepsilon)(|\omega+q_z|^2+g^2-p_2^2-i\varepsilon). \quad (22)$$

Let us evaluate the integral over q_z. Introducing the new variable $\boldsymbol{\eta} = \mathbf{k}_1 + \mathbf{g}$, we get

$$\frac{2i}{\omega(m^2+b^2)}\int\bar{f}_1(\boldsymbol{\eta})(\gamma\mathbf{j})f_2(\mathbf{k}-\boldsymbol{\eta})\,d\boldsymbol{\eta}. \quad (23)$$

We have made use of the fact that the effective $k \sim 1/R \ll b \sim m$. The integration over $\boldsymbol{\eta}$ in (23) is performed over the whole surface orthogonal to the vector $\mathbf{k}_\gamma$. In the evaluation of the integrals (20) and (23), the main role is played by distances $p_1p_2/\mu^2\omega \gg R$, therefore the use of the asymptotic expressions for the ψ-function is legitimate.

The differential cross section for the process is

$$d\sigma_j = \frac{e^2}{\pi^2}\frac{E_1^2E_2^2}{\omega^3(m^2+b^2)^2}|F_{+-}|^2\sum\Big|\frac{1}{p_2}\bar{u}_1(\gamma\mathbf{j})f_2(\mathbf{k}) + \frac{1}{p_1}\bar{f}_1(\mathbf{k})(\gamma\mathbf{j})u_2 \quad (24)$$
$$+\frac{i}{2\pi p_1p_2}\int\bar{f}_1(\boldsymbol{\eta})(\gamma\mathbf{j})f_2(\mathbf{k}-\boldsymbol{\eta})d\boldsymbol{\eta}\Big|^2 dE_1\,d\mathbf{k}\,d\mathbf{b}$$

($E_1+E_2=\omega$). The summation over spins can be performed in a general form if the functions f_1 and f_2 have the form (we change the notation $u_1f_1(\mathbf{k})$, where f_1 and f_2 are scalar quantities. Then, averaging (24) over j, we get:

$$d\sigma' = \frac{e^2}{2\pi^2}\frac{m^2\omega^2+b^2(E_1^2+E_2^2)}{(m^2+b^2)^2\omega^3}\Big|\frac{1}{p_1}f_1^*(\mathbf{k})+\frac{1}{p_2}f_2(\mathbf{k}) + \frac{i}{2\pi p_1p_2}\int f_1^*(\boldsymbol{\eta})f_2(\mathbf{k}-\boldsymbol{\eta})d\boldsymbol{\eta}\Big|^2|F_{+-}|^2 d\mathbf{k}\,d\mathbf{b}\,dE_1. \quad (25)$$

The distribution in k, i. e., in momenta transferred to the nucleus, is therefor model-dependent.

In the semi-transparent model of the nucleus[6], using $k\sim 1/R \ll k_1,\ k_2\sim m$:

$$f_1(k) = -ip_1u_1\int_0^R(1-\exp\{-\alpha_1^*\sqrt{R^2-s_1^2}\})\times J_0(ks_1)s_1ds_1;$$

$$f_2(k) = ip_2u_2\int_0^R(1-\exp\{-\alpha_2\sqrt{R^2-s_2^2}\})\times J_0(ks_2)s_2ds_2; \quad (26)$$

$$\alpha_1 = \varkappa_1 - 2ip_1(n_1-1), \quad \alpha_2 = \varkappa_2 - 2ip_2(n_2-1).$$

where $\varkappa$ and n are the absorption coefficient and the index of refraction of the nucleonic wave in the nucleus. Using these amplitudes, and integrating over k and φ_b, we get

$$d\sigma(b_1E_1) = \frac{e^2}{\pi}\frac{m^2\omega^2+b^2E_1^2+b^2(\omega-E_1)^2}{(m^2+b^2)^2\omega^3}\sigma_s|F_{+-}|^2bdbdE_1. \quad (27)$$

Here σ_s is the total elastic diffraction scattering cross section for a nucleon on a semi-transparent nucleus, with a total absorption coefficient $\varkappa=\varkappa_1+\varkappa_2$ and, correspondingly, with a total phase shift $2p_1(n_1-1)+2p_2(n_2-1)$, where the coefficients 1 and 2 refer the proton and antiproton.

As a conclusion, I wish to express my thankfulness to I. Ia. Pomeranchuk for the position of the problem and helpful advices.

Note added in proof. In the case of the "black" nucleus, the cross section is obtained in the assumption that the nucleus has the same radius with respect to protons as well as antiprotons. If one rejects this assumption and assumes that the radius is R_1 with respect to protons and R_2 with respect to antiprotons, the expressions(10) to (14) obtained above will be made valid by replacing R by R_1 (if $R_1>R_2$), or by R_2 (if $R_2>R_1$). In the case of a semi transparent nucleus, we obtain cumbersome results which we do not report here.

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124