Solving the system (15), and taking Eqs. (11)-(13) into account, we get for the amplitude of the excitation

$$q_1 = \frac{M_{10}k_1^{-1}\,\mathrm{Im}\,M_{11} - (1+M_{11})\,k_1^{-1}\,\mathrm{Im}\,M_{10}}{(1+M_{00})(1+M_{11}) - M_{01}M_{10}}. \quad (16)$$

Taking it into consideration that $\sin k_\alpha r/k_\alpha$ is an analytic function of k_α^2, we can represent $x_{\alpha\beta}$ in the form of a series in k_1^2. Substituting this series in $M_{1\gamma}$, and expanding $\exp ik_1 s$ in the radicand in powers of k_1, we get

$$M_{1\gamma} = M_{1\gamma}^{(0)} + ik_1 M_{1\gamma}^{(1)} + k_1^2 M_{1\gamma}^{(2)} + ik_1^3 M_{1\gamma}^{(3)} + \ldots \quad (\gamma = 0; 1). \quad (17)$$

The series (17) converges, generally speaking, only over some region of variation of k_1. The radius of convergence is determined by the form of $V_{\alpha\beta}(r)$. So far as $M_{0\gamma}$ is concerned, we can, by making use of (5), transform Eq. (13) to the form

$$M_{0\gamma} = \sum_\beta \int_0^\infty e^{i\varkappa_1 s + ik_{01}s}\, V_{0\beta}(s) x_{\beta\gamma}(s)\,ds. \quad (18)$$

Expanding $\exp i\kappa_1 s$ in powers of κ_1, we get

$$M_{0\gamma} = M_{0\gamma}^{(0)} + i\varkappa_1 M_{0\gamma}^{(1)} + \varkappa_1^2 M_{0\gamma}^{(2)} + k_1^2 M_{0\gamma}^{(3)} + \ldots \quad (19)$$

Substituting (17) and (19) in Eq. (16), and then in Eq. (6), and limiting ourselves to terms of no higher order in k_1 and κ_1 than the second, we obtain

$$\sigma = 4\pi\frac{k_1}{k_0}\,a_1\,\frac{1 + a_2 k_1^2}{1 + a_3 k_1 + a_4\varkappa_1 + a_5\varkappa_1^2 + a_6\varkappa_1 k_1 + a_7 k_1^2}. \quad (20)$$

The coefficients $a_1 \ldots a_7$ are expressed in terms of $M_{\alpha\gamma}^{(i)}$. In the case in which the threshold of excitation is sufficiently high, we can expand κ_1 near the threshold in powers of k_1^2. We then have, with accuracy up to terms in k_1^2,

$$\sigma = 4\pi\,(k_1/k_0)a_1\,(1 + a_2 k_1^2)\,/\,(1 + a_3 k_1 + a_4 k_1^2). \quad (21)$$

In a number of cases, we can neglect the term $V_{10}F_0$ in (7). This is the so-called method of distorted waves.[1,2] In such an approximation, $M_{01} = 0$, whence, as is easy to show, $a_3 = 0$. Thus, in the approximate method of distorted waves, the linear term of Eqs. (20) and (21) is absent.

[1] N. Mott and G. Massey, *Theory of atomic collisions*, IIL, Ch. VIII (Russian translation).

[2] H. S. W. Massey, Rev. Mod. Phys. **28**, 3 (1956).

Translated by R. T. Beyer
133

Statistical Retardation in Dielectric Breakdown of Solid Dielectrics

E. A. KONOROVA
P. N. Lebedev Physical Institute, Academy of Sciences, USSR

(Submitted to JETP editor June 28, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 603-604 (March, 1957)

SEITZ[1] made the assumption that, in computing the dielectric strength of crystals on the basis of Hippel's criterion, it is necessary to account for the fluctuations in the behaviour of the electrons in the energy exchange with the lattice. The possibility of such fluctuations is described by a certain probability that the electron will be accelerated to the ionization potential without collisions with the lattice. This assumption leads to the possibility of statistical retardation in the breakdown of thin layers of solid dielectrics, and also to the dependence of the dielectric strength of crystals on thickness (for the thickness range 10^{-3} to 10^{-4} cm).[2,3] In 1952-1954, Japanese investigators[2,5] obtained data on the dependence of dielectric strength on the time of application of the voltage and on the thickness for samples of mica 3 to 5×10^{-4} cm thick (dotted lines in Figs. 1, 2). These authors concluded that Seitz' assumptions were confirmed experimentally. The size of electron avalanches and also the number of initial electrons formed per second were computed. The quantities computed in this manner were in accordance with the approximate computations of Seitz.

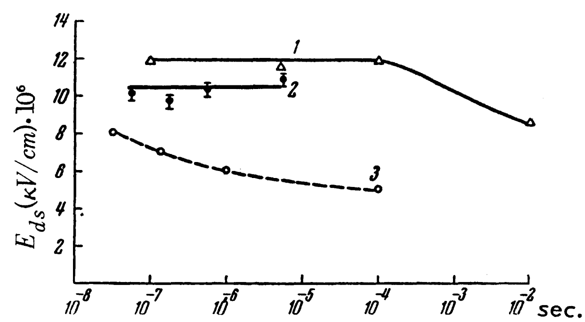

FIG. 1. Dependence of the dielectric strength of glass and mica on the duration of voltage application. 1 – glass, 2 – mica, 3 – mica according to the data, Ref. 5

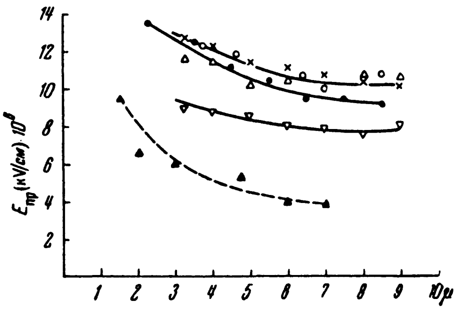

FIG. 2. Dependence of electric strength of glass and mica on the thickness. ● — mica; ▲ — mica, Ref. 2; △ — glass, 5 μ sec pulse; ○ — glass, 0.2 μ sec pulse; × — glass, 100 μ sec pulse; ▽ — glass 10^{-2} sec pulse.

We have also measured the dielectric strength and its dependence on the time of application of the voltage and on the thickness of mica-muscovite and glass of the following composition: SiO_2 – 68%, B_2O_3 – 20%, Al_2O_3 – 3%, Na_2O – 4%, K_2O – 5%, As_2O_3 – 0.25%. A linearly increasing voltage was applied to the samples. The time from the beginning of the voltage rise to the puncture of the sample could be varied between 10^{-2} sec and 5×10^{-8} sec. The dependence of the puncture voltage (E_p) on the sample thickness was obtained for pulse duration of 5×10^{-6} sec. Measurements of the puncture voltage for short pulses were made with the CRT oscillograph type KO-20 (experimental factory VEI), and for pulses of 10^{-2} sec duration with an electrostatic voltmeter across the kenotron. Errors in voltage measurements did not exceed 10% and, judging by the calibration oscillograms, were generally smaller.

Mica plates 2 to 10μ thick and glass films 3 to 10μ thick were used in the experiments. Silver electrodes were formed on the mica surfaces, after carefully cleaning of the latter with benzol, by evaporation in a vacuum in form of two discs 1.5 mm diameter on one side and 5 mm diameter on the other. The samples were placed between two electrodes, sphere and plane (sphere diameter 4mm) with a mixture of glycerine and alcohol poured over them. Since in most cases the samples were punctured at the center of of the smaller disc, it may be considered that puncture took place in a uniform field. Glass samples were punctured between sphere-plane electrodes; a 50% mixture of glycerine and alcohol was used as the medium. Thickness measurements of the samples were made with a ±3% accuracy.

Results of measurements are shown in Figs. 1 and 2. Each point on the graph represents the mean value of dielectric strength measurements on 10 to 16 samples. The mean square error does not exceed 6%. As seen from Fig. 2, the puncture strength of mica and glass increases noticeably with the decrease in thickness from 5 to 2.5μ. This is in agreement with the data obtained earlier (Joffe and Aleksandrov, 1932) for mica at constant voltage, although at constant voltage secondary phenomena could have taken place, such as heating of the sample and formation of space charges. The general form of this dependence agrees also with results of measurements obtained by Ryu and Kawamura[2] for mica-biotite (dotted curve, Fig. 2). However, no dependence of the puncture strength on the duration of voltage application was observed even in sufficiently thin samples, which is contrary to the results of the Japanese investigators (dotted curve, Fig. 1).

The dependence of dielectric strength on the duration of voltage application is very frequently determined by the non-uniformity of the field in the sample. Since the absolute values of dielectric strength for mica-muscovite obtained in our experiments are higher than those published in Ref. 3, the question arises whether the dependence obtained in Refs. 3 and 5 is not in fact related to a non-uniform field.

For glass, with sufficiently short pulses (from 10^{-4} and shorter), the dielectric strength also does not depend on time, and is noticeably lower for pulses of 10^{-2} sec duration. As shown in Refs. 6 to 8, this dependence can be satisfactorily explained as due to the heating of the sample by the prepuncture currents, increased in the ionic conductivity and, as a result of this, formation of ionic space charges which distort the field at the electrodes.

Thus, it may be considered that in the puncture of glass or mica, statistical retardation is generally either absent or less than 10^{-8} sec.

As far as the dependence of the dielectric strength on thickness, in this range of thicknesses, it can be explained not only on the basis of Seitz' assumptions but also on the basis of other theories of electric puncture, for example the theory of Chuchenkov[9].

It appears to us, on the basis of these investigations, that there are not sufficient grounds so far for the conclusions made in Refs. 2 and 5.

The author takes this opportunity to thank Prof. G. I. Skanavi for his interest and attention to this work.

[1] F. Seitz, Phys. Rev. **76**, 1376 (1949)

[2] G. Ryu and H. Kawamura, J. Phys. Soc. Japan **9**, 438 (1954)

[3]Kawamura, Onuki and Ohkura, J. Phys. Soc. Japan 7, 528 (1952)

[4]H. Kawamura, J. Phys. Soc. Japan 8, 731 (1953)

[5]Kawamura, Ohkura and Kikuchi, J. Phys. Soc. Japan 9, 541 (1954)

[6]K. G. Keller, Physica, 14, 475 (1948)

[7]K. G. Keller, Physica, 17, 511 (1951)

[8]I. Wermeer, Physica 20, 313 (1954)

[9]V. A. Chuchenkov, Usp. Fiz. Nauk 54, 185 (1954)

Translated by J. L. Herson
134

The Applicability of the Relation of Detailed Balance for a Cluster of Ions of Stationary Composition

IA. M. FOGEL'
Physical-Technical Institute,
Academy of Sciences, U.S.S.R.

(Submitted to JETP editor July 15, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 604-605
(March, 1957)

IN a recently published work of Korsunskii, Leviant, and Pivovar[1] on the basis for the treatment of the experimental data presented in another work of these authors[2], a confirmation is made of the fact that to processes of charge exchange between ions of a cluster and molecules of a substance, occurring upon passage of the ionic cluster through the substance, one can apply the relation

$$\sigma_{ik} / \sigma_{ki} = N_k / N_i, \tag{1}$$

where σ_{ik} and σ_{ki} are effective cross sections for an ionic transition from the charge-state i to the charge-state k and vice versa; N_k/N_i is the ratio of the number of ions with charge k to the number of ions with charge i in a cluster of stationary composition. The composition of a cluster passing through a layer of material is determined by the differential equations

$$dN_k / dn = \sum_{i=1}^{m} \sigma_{ik} N_i - N_k \sum_{i=1}^{m} \sigma_{kl}, \tag{2}$$

where n is the thickness of the layer of material in atoms per cm^2, and 1, 2, ... m are the charge-states of the ions of the cluster. The cluster attains a stationary state for the condition $dN_k/dn = 0$. The relation (1) is obtained from this condition for the charge-states fulfilling the condition $|i - k| = 1$, if for $|i - k| > 1$ all $\sigma_{ik} = \sigma_{ki} = 0$ (processes of exchange of more than one electron do not occur). Since the conditions indicated in Ref. 2 are not satisfied, its fulfillment in this work proves to be purely by chance, inasmuch as the relation (1) is not derived from any sort of general considerations.

The applicability of Eq. (1) can be subjected to a direct check in processes of capture and loss of two electrons in single collisions of protons and negatively charged hydrogen ions with molecules of hydrogen. For these processes, according to Eq. (1), the equality

$$\sigma_{1-1} / \sigma_{-11} = N^- / N^+. \tag{3}$$

must hold. The effective cross-sections σ_{1-1} for capture of two electrons by protons upon collision with hydrogen molecules, and the ratios N^-/N^+ in a hydrogen cluster of stationary composition, formed in the passage of protons through hydrogen, were determined in the work of Fogel' and Mitin[3]. On the other hand, we determined the effective cross sections of σ_{-11} for the loss of two electrons in collisions of negative hydrogen ions with molecules of hydrogen[4]. In the Table, values of the quantities $\sigma_{1-1}/\sigma_{-11}$ and N^-/N^+ are presented for various energies of the ions of the cluster. The

Energy of Ions, kev.	$\sigma_{1-1}\cdot 10^{18}$ cm^2	$\sigma_{-11}\cdot 10^{17}$ cm^2	$\frac{\sigma_{1-1}}{\sigma_{-11}}\cdot 10^2$	$\frac{N^-}{N^+}\cdot 10^2$	$\frac{\sigma_{1-1}/\sigma_{-11}}{N^-/N^+}$
10,3	7,2	3,8	18,7	17,5	1,1
15,2	11,4	4,1	28,1	13,1	2,1
20,7	12,3	3,8	32,3	9,1	3,6
24,9	8,9	3,9	25,7	6,3	4,1
29,6	7,2	3,7	19,5	4,3	4,5
31,8	4,2	3,1	13,1	3,5	3,7