

errors in measurement of the quantities in the table are as follows: for σ_{1-1} , $\pm 30\%$; for N^-/N^+ , $+18\%$; and for σ_{-11} , $\pm 15\%$. As is seen from this Table, the equality (3) is not satisfied in a significant part of the energy interval investigated.

Thus the relation (1) cannot be of universal applicability to processes of energy exchange between an ionic cluster and a material.

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³ Ia. M. Fogel' and R. V. Mitin, J. Exptl. Theoret. Phys. **30**, 450 (1956); Soviet Phys. JETP **3**, 334 (1956).

⁴ Fogel', Ankudinov and Slabospitskii, J. Exptl. Theoret. Phys. **32**, 453 (1957).

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On the Possibility of Observing the Variation of the Chemical Potential of Electrons of a Metal in a Magnetic Field

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It is known, the quantization of the energy levels of electrons in a magnetic field leads to a number of phenomena (diamagnetism of the electron gas, De Haas-van Alfen effect, Shubnikov-de Hass effect, *et al.*) As a rule, the variation of the chemical potential of the electron gas in the magnetic field is insignificant in all these phenomena.

The dependence of the chemical potential ζ on the magnetic field can be detected directly by measuring the contact potential difference $\delta\varphi$ between two samples of the same metal, if one of these samples is placed in a strong magnetic field.

The contact potential difference which arises is determined by the difference of the chemical potentials

$$e\delta\varphi = \zeta(H) - \zeta_0; \quad \zeta_0 = \zeta(0).$$

to the accuracy of a constant component related to the different states of the sample surfaces.

According to Ref. 1, this difference is:

$$e\delta\varphi = \frac{2\sqrt{2\pi}}{dU/d\zeta} \left(\frac{e\hbar H}{c} \right)^{1/2} \frac{G(\zeta_0, H)}{|\partial^2 S(\zeta_0, p_z)/\partial p_z^2|^{1/2}}, \quad (1)$$

where

$$G(\zeta_0, H) = \sum_{k=1}^{\infty} \frac{1}{k^{1/2}} \Psi'(k\lambda) \sin \left[\frac{kc}{e\hbar H} S_m(\zeta_0) \mp \frac{\pi}{4} - 2\pi k\gamma \right] \\ \times \cos \left[\frac{k}{2m_0} \frac{dS_m}{d\zeta} \right];$$

$$\Psi'(z) = z/\sinh z; \quad \lambda = (\pi c\theta/e\hbar H) dS_m/d\zeta$$

(the notation is the same as in Ref. 1). If the expression for the oscillating part of the magnetic moment M_{osc} [Ref. 1, Eq. (3.2)] is used, then there can be obtained:

$$e\delta\varphi = - \frac{\alpha M H_{osc}}{N}, \quad \alpha = U \frac{dS_m}{d\zeta} / S_m \frac{dU}{d\zeta}; \quad (2)$$

α is a nondimensional function of the angles between the field and the crystallographic directions; $\alpha = \frac{2}{3}$ in case of an isotropic quadratic dependence; N is the number of electrons.

We took into account only that variation of the chemical potential in (1), which is related to the oscillating component of the thermodynamic potential since that variation of the chemical potential which is caused by the presence of monotonic components (diamagnetic and paramagnetic) is proportional to

$$\delta\zeta \sim \mu H (\mu H/\zeta),$$

and, as we will see below, is considerably less than the oscillatory part (certainly, for low temperatures).

In order to estimate the order of magnitude of the effect expected, it can be considered that

$$|\partial^2 S(\zeta_0, p_z)/\partial p_z^2|_m^{-1/2} G(\zeta_0, H) \sim 1, \\ e\delta\varphi \sim (\mu H/\zeta_0)^{1/2}; \quad \mu = e\hbar/m^*c.$$

For

$$H \sim 10^4 \text{G}, \quad \zeta_0 \sim 10^{-12} \text{эрг}, \quad \mu \sim 10^{-20}, \quad \delta\varphi \sim 10^{-6} \text{V}.$$

This estimate is lowered considerably since it is not taken into account that the effective mass of the majority of metals for which the De Haas-van

Alfen effect is observed is considerably less than the mass of the free electron (for which $\mu \sim 10^{-20}$).

If $m^* = 10^{-2}m_0$ is taken, then it is shown that $\delta\varphi \sim 10^{-3}$ V. Moreover, it should be taken into account that $\zeta_0 \sim 10^{-12}$ erg corresponds to one electron per atom ($n \sim 1$) which, as a rule, also does not hold ($n < 1$).

If there are several electron groups then the potential difference arising is determined as

$$e \delta\varphi = 2V \sqrt{2\pi} \left(\frac{e\hbar H}{c} \right)^{3/2} \sum \left| \frac{\partial^2 S}{\partial p_z^2} \right|_m^{-1/2} G / \sum dU_0/d\zeta. \quad (3)$$

Here, the summation in the numerator and denominator is made over all the electron groups. However, it should be taken into account that even the weak domain structure, which always exists in single crystals, leads to the diffusion or the total smoothing of the oscillations in components corresponding to the bands with a large number of electrons for small fields. Actually, it is easy to show¹ that it is simply necessary to average the right side of (3) over the various domains to take the domain structure into account. Here, naturally, the components containing the significant random variations of the sine argument drop out,* *i.e.*, precisely those components which correspond to bands with a large number of electrons.

2. The variation of the chemical potential of an electron gas in a magnetic field leads to a dependence of the emission current (both the cold and the thermionic emission) on the magnetic field². Hence, the oscillations in the magnetic field must be observed only in the cold electron emission case**. Portions of the electrons located on the "tail" of the Fermi distribution where they behave substantially as classical particles participate in the thermionic emission. Consequently, a periodic dependence on the magnetic field naturally does not arise.

Direct calculation of the thermionic current under the assumption that the magnetic field is perpendicular to the metal surface and that the dispersion law is quadratic and isotropic, yields

* We note that $S_m(\zeta)$ is a function of the angles between the magnetic field and the crystallographic axes.

** The variation of the total cold emission current in a magnetic field is extremely slight. In particular, a periodic dependence on the magnetic field is apparently not observable in practice. Consequently, we do not dwell on these questions here.

$$I(H) = \frac{\beta H \cosh(\beta_0 H / kT)}{kT \sinh(\beta H / kT)} I_0,$$

where I_0 is the thermionic current in the absence of the magnetic field, $\beta = e\hbar/2m^*c$ is the electron diamagnetic moment, m is its effective mass, $\beta_0 = e\hbar/2mc$ is the electron spin magnetic moment.

It is interesting to note that the thermionic current can either increase or decrease in strong magnetic fields depending on the relation between the effective mass m^* and the mass of the free electron m . If $m^* = m$, then

$$I(H) \approx \frac{\beta H}{kT \tanh(\beta H / kT)} I_0,$$

that is, the thermal current increases linearly with the magnetic field $\tanh x \sim 1$, $x \gg 1$ for $\beta H \gg kT$.

It should be noted that the experimental observation of thermionic current variation in a magnetic field is extremely difficult because of the low current intensity at low temperatures.

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Natural Line Widths of Microwaves

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WHEN molecules are contained in a space whose linear dimensions are much smaller than the emitted wavelength, as often occurs in the microwave range, the molecules radiate as a single quantum-mechanical system and the line width of the spontaneous emission depends on the number of molecules. Dicke and Romer^{1,2} showed that the line width is proportional to the number of molecules for "bound" states. The present note will show that this result is also valid for "unbound" states of a molecular system.