

Concerning the Lifetime of the Two Forms of the π^0 Meson.

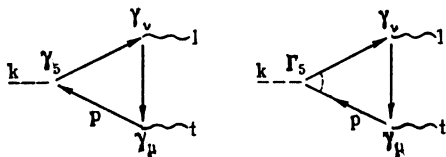
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ACCORDING to the theory of Fermi and Jang¹, it is assumed that π mesons are formed by nucleons in contact interaction (potential well of width $\hbar/Mc \sim 10^{-14}$ and depth 25×10^9 ev. According to this theory there exist two forms of π^0 mesons; $\pi^0(P + \bar{P})$ and $\pi^0(N + \bar{N})$; where P denotes proton, N neutron, \bar{P} antiproton, \bar{N} antineutron. In this note are presented results of calculations of the lifetime for the two form of the π^0 meson.

Computations were made by the general method of the invariant perturbation theory for a pseudoscalar meson with pseudoscalar coupling in the approximation $(\mu/M)^2 \ll 1$, where μ denotes the mass of the meson and M the mass of the nucleon, under the assumption that the expansion in the power of $g^2/\hbar c$ is valid. It was considered that the disintegration of the two forms of π^0 mesons occurs according to two mutually exclusive Feynman diagrams (Figs. 1a and b),



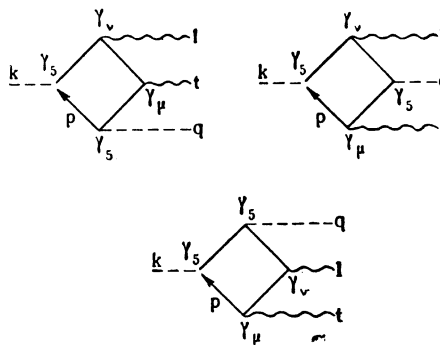
where k is the 4-momentum of the meson, t, l are 4-momentum quanta, p is the 4-momentum of the virtual nucleon, $\gamma_i = -i\beta\alpha_i$ ($i = 1, 2, 3$), $\gamma_4 = \beta$, $\gamma_5 = \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5^2 = 1$, $\hbar = c = 1$, the upper portion which includes the charged meson, computed in Refs. 2 and 3.

The life time obtained was:

$$\frac{1}{\tau} \approx \left[1; \frac{3}{\pi^2} \left(\frac{g^2}{\hbar c} \right)^2 \right] \frac{1}{16\pi^2} \left(\frac{g^2}{\hbar c} \right) \left(\frac{e^2}{\hbar c} \right)^2 \left(\frac{\mu}{M} \right)^2 \frac{\mu c^2}{\hbar},$$

where the first term in the square parenthesis corresponds to the $\pi^0(P + \bar{P})$ meson, the second term to the $\pi^0(N + \bar{N})$ meson. Within the limits of the method used, the life times of the two π^0 mesons, are generally speaking, different.

By the same method and using the same approximations the conversion probability of one π^0 meson into the other was evaluated by the simpler diagram (Fig. 2)



where q denotes the 4-momentum of the end meson. The logarithmic deviation was removed by subtraction. For the life time in this process we obtained the value:

$$1/\tau \approx (N^4/192\pi) (g^2/\hbar c)^2 (e^2/\hbar c)^2 (\mu/M)^4 (m/\mu)^3 mc^2/\hbar,$$

where m denotes the mass of the electron, $N = (\mu_1 - \mu_2)/m$ the mass difference of the mesons.

¹ E. Fermi and C. Jang, Phys. Rev. **76**, 1739 (1949).

² A. D. Galanin, J. Exptl. Theoret. Phys. (U.S.S.R.) **26**, 417 (1954).

³ A. D. Galanin and V. G. Solovev, J. Exptl. Theoret. Phys. (U.S.S.R.) **27**, 112 (1954).

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On the Derivation of the Low Equation in the Theory of Meson Scattering

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THERE is shown in this note a new method of deriving Low's equations, the analysis of which is very important in discussions pertaining to the theory of scattering of mesons by nucleons. It is shown that they may be written in a form, where the concrete type of interaction enters in a simple way through a nonhomogeneous term.* In the analysis of

*It should be noted that a series of causality conditions and renormalization, assumed in the derivation of the dispersion relations, are thereby imposed on the interaction and also the condition of sufficiently rapid decay of certain expressions under the integral in Refs. 3, 4.

the scattering of π -mesons by nucleons, let us introduce the amplitude (T -matrix) of scattering connected with the corresponding matrix element of the S -matrix in the following manner.

$$\begin{aligned} (p'\sigma', k'j | S | p\sigma, ki) &= \delta_{\sigma\sigma'} \delta_{ij} \delta(\mathbf{p}' - \mathbf{p}) \delta(\mathbf{k} - \mathbf{k}') \\ &- (2\pi i) [(2\pi)^3 2k'_0 2k_0]^{-1/2} \delta(p' + k' - p - k) \\ &\times (p'\sigma', k'j | T | p\sigma, ki). \end{aligned} \quad (1)$$

Here $p, k(p', k')$ denote the momenta at $-\infty (+\infty)$ of the incident (scattered) nucleon and meson, satisfying the relations

$$\begin{aligned} p + k &= p' + k', \quad \gamma p = \gamma p' = -m^2, \\ k^2 &= k'^2 = -\mu^2, \end{aligned} \quad (2)$$

where m and μ are experimental masses; all other magnitudes are considered also renormalized.

From the definition of the scattering amplitude and the unitarity condition of the S -matrix it follows directly that

$$\begin{aligned} &(p'\sigma', k'j | T | p\sigma, ki) \\ &= -\bar{u}(p'\sigma') \left\{ (1 + \text{Int}) \sum_{\lambda_i=1,0} (\gamma k)^{\lambda_1} (\tau_j \tau_i)^{\lambda_2} \rho_{\lambda_1 \lambda_2}((p+k)^2, (p-p')^2) \right\} u(p, \sigma), \\ &\text{Int} \equiv k \leftrightarrow -k', \quad i \leftrightarrow j; \quad p' + k' = p + k. \end{aligned} \quad (4)$$

Here $\rho_{\lambda_1 \lambda_2}((p+k)^2, (p-p')^2)$ denote certain functions which can be represented, as analysis of the Feynman diagrams⁶ and consideration of the causality condition will show, in the form:

$$\begin{aligned} \rho_{\lambda_1 \lambda_2}((p+k)^2, (p-p')^2) &= \frac{\delta(\lambda_1 \lambda_2)}{(p+k)^2 + m^2 - i\epsilon} \\ &+ \int_{(m+\mu)^2}^{\infty} dM_1^2 \int_{(2\mu)^2}^{\infty} dM_2^2 \frac{\omega_{\lambda_1 \lambda_2}(M_1^2, M_2^2)}{[(p+k)^2 + M_1^2 - i\epsilon][(p-p')^2 + M_2^2 - i\epsilon]}, \end{aligned} \quad (5)$$

where $\delta(\lambda_1 \lambda_2)$ is a constant depending on the method of renormalization of the charge; dependence of $\delta(\lambda_1 \lambda_2)$ on λ_1 is determined by the form of interaction and can be readily established by comparing $P_{\lambda_1 \lambda_2}$ with the first term of the perturbation theory expansion. Further analysis will be conducted in the laboratory system of coordinates [$p = (m, 0)$].

$$\begin{aligned} (T - T^+) &\equiv [(p'\sigma', k'j | T | p\sigma, ki) - (p'\sigma', k'j | T^+ | p\sigma, ki)] \\ &= -2\pi i \sum_n \int dp_n(n | T | p'\sigma', k'j)^* (n | T | p\sigma, ki) \\ &\quad \times \delta(p_n - p - k), \end{aligned} \quad (3)$$

where it is necessary to take $p + k = p^1 + k^1$ on the right side of the equation; (n) is the complete specification of the state as an example of which we choose the system of eigenfunctions of the energy vector—the momentum P at infinity with eigenvalues $p^2 = p_n^2$. Note that for coherent forward scattering the left side of equation (3) is the imaginary part of the scattering amplitude, and on the right side represents the complete scattering cross section with an accuracy up to a coefficient proportional to the flow of the incident particles. Thus, in this case, (3) is equivalent to the well-known “optical” theorem.⁵

Let us further make use of the scatter amplitude written in a general form which follows⁶⁻⁸ from the investigation of the property of invariance and symmetry of the given amplitude*

The following inequalities which follow from (2) are valid for the subsequent portion of the analysis; $(p - k')^2 + M_1^2 = -(m + \mu)^2 + 2m(k'_0 + \mu) + M_1^2 > 0$, $(p + k)^2 + m^2 = -2mk_0 - \mu^2 < 0$, $(p - p')^2 + M_2^2 = -2m(m - p'_0) + M_2^2 > 0$. (6) Considering (4-6) and making use of (2), the difference $(T - T^+)$ [Eq. (3)] can be written in form**

$$\begin{aligned} (T - T^+) &\equiv -2i\bar{u}(p'\sigma') \left\{ (1 + \text{Int}) \sum_{\lambda_i=1,0} (\tau_j \tau_i)^{\lambda_2} (\gamma k)^{\lambda_1} \text{Im} [\rho_{\lambda_1 \lambda_2}((p+k)^2, (p-p')^2)] \right\} u(p, \sigma) \\ &= -2\pi i\bar{u}(p'\sigma') \left[\sum_{\lambda_i=1,0} (\tau_j \tau_i)^{\lambda_2} (\gamma k')^{\lambda_1} P \int_{(2\mu)^2}^{\infty} \frac{\omega_{\lambda_1 \lambda_2}(-(p+k)^2, M_2^2)}{(p-p')^2 + M_2^2} dM_2^2 \right] u(p, \sigma) \end{aligned} \quad (7)$$

$(p + k) = p^1 + k^1$. It follows from the comparison of (3) and (7) that in the laboratory system of computation,

* The symbols of Ref. 6 are generally used herein.

** For the derivation of (6) it is necessary to remember that, according to the accepted symbols, $u = u^* \gamma_0$, $(\gamma_0 \gamma_u)^+ = \gamma_0 \gamma_u$

$$\begin{aligned}
 P & \int_{(m+\mu^2)}^{\infty} dM_1^2 \int_{(2\mu)^2}^{\infty} dM_2^2 \frac{\omega_{\lambda_1\lambda_2}(M_1^2, M_2^2)}{[-(\mu^2 + m^2 + 2mk_0) + M_1^2][(p-p')^2 + M_2^2]} \\
 & = \frac{1}{\pi i} \int_{\mu}^{\infty} d\tilde{k}_0 \frac{\text{Im } \rho_{\lambda_1\lambda_2}((\mu^2 + m^2 + 2m\tilde{k}_0), (p-p')^2)}{\tilde{k}_0 - k_0} \Bigg|_{p+k=p'+k'}
 \end{aligned} \tag{8}$$

Now, separating in (3) the obvious dependence on $p_0 + k_0$, and examining separately the terms of the expression of (7) within the square parentheses,

Eq. (4) can from the present viewpoint, be finally rewritten in a form equivalent to the first of Low's infinite system of equations:

$$\begin{aligned}
 & (p'\sigma', k'j | T | p\sigma, ki) \\
 = & -\bar{u}(p'\sigma') \left\{ (1 + \text{Int}) \sum_{\lambda_i=1,0} (\gamma_k)^{\lambda_1} (\tau_j \tau_i)^{\lambda_2} \frac{\delta(\lambda_1\lambda_2)}{(p+k)^2 - m^2 - i\epsilon} \right\} u(p, \sigma) \\
 + & (1 + \text{Int}) \sum'_n \int dp_n \left\{ \frac{(n | T | p'\sigma', k'j)^* (n | T | p\sigma, ki)}{p_{0n} - p_0 - k_0 - i\epsilon} \delta(p_n - p - k) \right\},
 \end{aligned} \tag{9}$$

where prime at the summation symbol indicates that the summation is carried out over all n , except the case $p^2 n = -m^2$ and the momenta satisfy conditions (2). Thereby also, the physical state of the nucleon is considered as the lowest stable state and the difference between the proton and neutron masses is neglected. We note that this reasoning could have been simplified by the introduction of Gilbert's concept of the transformation operator, since the hermitian and the skew hermitian operators $T + T^+$ and $T - T^+$ are, in view of the causality principle, mutual forms in the sense of Hilbert's transformation, with an accuracy up to the addition of terms which differ by a permutation of the integrations.

In conclusion the author expresses his deep gratitude to Prof. D. D. Ivanenko, whose remark with reference to Low's equation and dispersion relations, made at the Moscow Conference on High Energy in May 1956, served as the starting point for this work.

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The Hall Effect in Bismuth Under Pressures up to 30,000 Kg/cm²

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IN the course of our work on the investigation of the Hall effect in bismuth¹, we built an apparatus for measurements at pressures up to 30,000 kg/cm². A pressure up to 30,000 kg/cm² is created inside a microlite matrix, which is surrounded on all sides by a lead casing and receives support on almost all sides in a steel conical mounting. The microlite matrix was prepared for us in the Glass Laboratory the Mendeleev Moscow Chemicotechnical Institute.* The means for transmitting the pressure is silver chloride, which transmits hydrostatic pressure sufficiently well for a pressure chamber of di-

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³M. L. Goldberger, Phys. Rev. **99**, 979, (1955).

⁴M. Oehme, Phys. Rev. **101**, 1503 (1956).

⁵M. Lax, Phys. Rev. **78**, 306 (1950).

⁶A. M. Brodskii, Dokl. Akad. Nauk. SSSR **111**, 787 (1956).

⁷Y. Nambu, Phys. Rev. **100**, 394 (1955).

**For the derivation of (6) it is necessary to remember that, according to the accepted symbols, $u = u^* \gamma_0$, $(\gamma_0 \gamma_\mu)^+ = \gamma_0 \gamma_\mu$.

*We consider it our pleasant duty to express our thanks to Prof. I. I. Kitaigorodskii and scientific technician Ts. M. Gurevich for the great labor furnished by them in working out the technology and preparing these matrices.