$$P \int_{(m+\mu^2)}^{\infty} dM_1^2 \int_{(2\mu)^2}^{\infty} dM_2^2 \frac{\omega_{\lambda_1\lambda_2}(M_1^2, M_2^2)}{[-(\mu^2+m^2+2mk_0)+M_1^2]\,[(p-p')^2+M_2^2]}$$
$$= \frac{1}{\pi i}\int_{\mu}^{\infty} d\widetilde{k}_0 \frac{\operatorname{Im}\rho_{\lambda_1\lambda_2}((\mu^2+m^2+2m\widetilde{k}_0),(p-p')^2)}{\widetilde{k}_0-k_0}\Bigg|_{p+k=p'+k'} \quad (8)$$

Now, separating in (3) the obvious dependence on p_0+k_0, and examining separately the terms of the expression of (7) within the square parentheses, Eq. (4) can from the present viewpoint, be finally rewritten in a form equivalent to the first of Low's infinite system of equations:

$$(p'\sigma', k'j \mid T \mid p\sigma, ki)$$
$$= -\bar{u}(p'\sigma')\Big\{(1+\mathrm{Int})\sum_{\lambda_i=1,0}(\gamma_k)^{\lambda_1}(\tau_j\tau_i)^{\lambda_2}\frac{\delta(\lambda_1\lambda_2)}{(p+k)^2-m^2-i\varepsilon}\Big\}u(p,\sigma)$$
$$+(1+\mathrm{Int})\sum_n{}'\int dp_n\Big\{\frac{(n\mid T\mid p'\sigma',k'j)^*(n\mid T\mid p\sigma, ki)}{p_{0n}-p_0-k_0-i\varepsilon}\,\delta(\mathbf{p}_n-\mathbf{p}-\mathbf{k})\Big\}, \quad (9)$$

where prime at the summation symbol indicates that the summation is carried out over all n, except the case $p^2 n=-m^2$ and the momenta satisfy conditions (2). Thereby also, the physical state of the nucleon is considered as the lowest stable state and the difference between the proton and neuteron masses is neglected. We note that this reasoning could have been simplified by the introduction of Gilbert's concept of the transformation operator, since the hermitian and the skew hermitian operators $T+T^+$ and $T-T^+$ are, in view of the causality principle, mutual forms in the sense of Hilbert's transformation, with an accuracy up to the addition of terms which differ by a permutation of the integrations.

In conclusion the author expresses his deep gratitude to Prof. D. D. Ivanenko, whose remark with reference to Low's equation and dispersion relations, made at the Moscow Conference on High Energy in May 1956, served as the starting point for this work.

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**For the derivation of (6) it is necessary to remember that, according to the accepted symbols, $u=u^*\gamma_0$, $(\gamma_0\gamma_\mu)^+=\gamma_0\gamma_\mu$.

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The Hall Effect in Bismuth Under Pressures up to 30,000 Kg/cm²

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IN the course of our work on the investigation of the Hall effect in bismuth[1], we built an apparatus for measurements at pressures up to 30,000 kg/cm^2. A pressure up to 30,000 kg/cm^2 is created inside a microlite matrix, which is surrounded on all sides by a lead casing and receives support on almost all sides in a steel conical mounting. The microlite matrix was prepared for us in the Glass Laboratory the Mendeleev Moscow Chemico-Technical Institute.* The means for transmitting the pressure is silver chloride, which transmits hydrostatic pressure sufficiently well for a pressure chamber of di-

*We consider it our pleasant duty to express our thanks to Prof. I. I. Kitaigorodskii and scientific technician Ts. M. Gurevich for the great labor furnished by them in working out the technology and preparing these matrices.

ameter 11-12 mm and height 4 mm. A thin plate of bismuth is embedded in the silver chloride, with five contacts which make it possible to measure the resistance, the dependence of resistance on magnetic field, and the Hall effect, as functions of pressure. The magnetic field is introduced into the matrix through the plungers exerting the pressure. In one of the plungers are found four miniature electrical conductors. The measurements were made on bismuth of 99.99% purity.

The occurrence of the I-II and II-III transitions in bismuth was clearly seen by jumps in the resistance. With increasing pressure the Hall effect in bismuth progressivley decreases, and in modification III, it becomes at least three orders of magnitude smaller than for bismuth under normal pressure. The dependence of resistance on magnetic field in modification III also falls below the limits of sensitivity of the measuring system. Noting that in modification III, according to the measurements of Bridgman[2] and also of Butuzov and Gonikberg[3], the melting point increases with pressure; and that, according to the measurements of Chester and Jones[4], superconductivity occurs in bismuth III, it is obviously possible to assume that bismuth III is a true metal, in contrast to bismuth I, which is characterized as a "semi-metal". Since details of the change of Hall effect with pressure in modification I obviously depend on the orientation of the plate, the necessity arose of carrying out the experiments with monocrystalline plates, after which we shall publish a detailed article.

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Hydrodynamic Fluctuations

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A GENERAL theory of hydrodynamic fluctuations can be constructed by introducing "outside" terms into the equation of motion of the liquid, as was done by Rytov[1] for the fluctuations of an electromagnetic field in continuous media; he introduced corresponding "outside" fields in Maxwell's equations.

The introduction of such additional terms can be accomplished in different equivalent ways. The most advantageous is the form in which the fluctuations of the "outside quantities" at the various points of the liquid are not correlated with one another. This is accomplished by the introduction of "outside stress tensor" s_{ik} in the Navier-Stokes equation and the "outside heat flow" vector g in the heat conduction equation (the equation of continuity remains unchanged). The system of hydrodynamic equations then takes the form

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \mathbf{v}) = 0, \tag{1}$$

$$\rho \frac{\partial v_i}{\partial t} + \rho (\mathbf{v}\nabla) v_i = -\frac{\partial p}{\partial x_i} + \frac{\partial \sigma'_{ik}}{\partial x_k}, \tag{2}$$

$$\rho T \left(\frac{\partial s}{\partial t} + \mathbf{v}\nabla s\right) = \frac{1}{2}\sigma'_{ik}\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i}\right) - \operatorname{div}\mathbf{q}, \tag{3}$$

$$\sigma'_{ik} = \eta\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i} - \frac{2}{3}\delta_{ik}\frac{\partial v_l}{\partial x_l}\right) + \zeta \frac{\partial v_l}{\partial x_l}\delta_{ik} + s_{ik}, \tag{4}$$

$$\mathbf{q} = -\varkappa \Delta T + \mathbf{g} \tag{5}$$

(all the notation agrees with that used in our book[2]). To these equations should be added the relations which define the mean values of the products of components s_{ik} and g_i. We do this by first assuming the fluctuations to be classical (*i.e.*, their frequencies $\omega \ll kT/\hbar$), while the viscosity and the thermal conductivity of the liquid are non-dispersive.

The rate of change of the total entropy of the liquid S is given by the expression [see Ref. 2, Sec. 49]

$$\dot{S} = \int\left\{\frac{\sigma'_{ik}}{2T}\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i}\right) - \frac{\mathbf{q}\nabla T}{T^2}\right\} dV. \tag{6}$$

Following the general rules of fluctuation theory laid down in Ref. 3, Secs, 117, 120, we select as