On the Fluctuation Resolution Limit of An Optical Modulation Interferometer

S. M. KOZEL
The Moscow Institute of Physics and Technology
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A theoretical model of a modulation optical interferometer designed to measure the angular dimensions of a source is considered. Expressions have been obtained for the fluctuation limits of sensitivity and precision. Experiments are described which indicate that the theoretical sensitivity and precision limits can be obtained in practice. The influence of atmospheric perturbations in a stellar modulation interferometer is estimated.

1. STATEMENT OF THE PROBLEM

WE DESIGNATE as an optical mudulation interferometer an interferometer arrangement where: (a) one or more of the parameters of the system (for example, the position of one of the mirrors) change periodically with time and thereby give rise to periodic changes in the interference pattern, which depends on some physical quantity to be investigated, and (b) these changes generate through a photoelectric process an electrical signal which is then separated by suitable electrical filter.

The sensitivity (resolving power) of this arrangement is determined by its optical parameters and the smallest detectable electrical signal. It is well known that in an expedient design the second of the factors is determined solely by the fluctuations and not by the technical imperfections of the system (as for example, the amplifier drift). The smallest detectable electrical signal (the fluctuation limit of sensitivity) can be made as small as desired if a filter with a high time constant is used. This means that in principle the sensitivity (resolving power) of a modulation optical interferometer with given optical parameters can be made as high as desired[1].

A similar statement can be made also with respect to the precision with which an optical quantity surpassing the threshold value can be measured with the modulation optical interferometer. With proper design, the measurement error of the instrument is determined by the fluctuation error and can be made as small as desired by increasing the observation time. The modulation optical interferometer can also be used to measure small periodic (or transformed into periodic) changes in the relative motion of two light beams[1,2] (cf. also survey, Ref. 3). The fluctuation limit of sensitivity of these measurements has been calculated theoretically and attained experimentally by Bershtein[4]. Under relatively simple realizable conditions this limit is on the order of 10^{-2} or even 10^{-3} A. It should also be pointed out that in a similar problem Tolansky[5] was able to obtain the maximum possible sensitivity on the order of tens of Angstrom units by means of multiple beam interferometry in conjunction with a visual analysis of the interference pattern.

A number of applications of the modulation optical interferometer and related arrangements is described in the literature (see for example, Refs. 6 and 7). Equally as important as interference measurements where it is required to determine changes in path differences are measurements where it is required to determine changes in the intensity contrast of an interference pattern caused by changing one of the system parameters. Measurements of this type are used to obtain information on the size of a light source (Michelson's stellar interferometer) or of the degree of its monochromaticity. A modulation optical interferometer properly designed can also be used for measurements which will be discussed in this paper.

It is of interest to estimate for a certain model of the modulation optical interferometer the theoretical fluctuation limit of detectable changes in intensity of the interference pattern, and thereby obtain an estimate of the fluctuation limit of resolution and of the fluctuation error in measuring the intensity of the interference pattern. It is also of interest to investigate experimentally the attainability of the theoretical values of the fluctuation limits of sensitivity and precision.

2. THEORETICAL MODEL OF THE MODULATION INTERFEROMETER

We will consider a sufficiently simple theoretical model of a modulation interferometer designed to obtain information on the size of a light source. In the choice of the theoretical model consideration was given to the fact that it was contemplated to use this model as a basis for an experimental setup (as was indeed done). Partly because of this a readily realizable practical arrangement of the Rayleigh type interferometer was chosen.

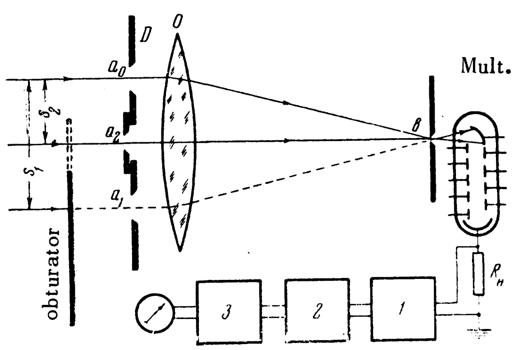

FIG. 1. Theoretical model of the modulation optical interferometer designed to measure the angular spread of a light source.

Consider a distant light source of angular dimension Ψ observed through an (optical) objective. If the objective is covered by a non-transparent screen with two parallel slits, separated from each other by a distance s, a system of interference fringes will be observed in the focal plane of the objective. The intensity distribution of the interference fringes is uniquely given by the separation of the slits, i.e. the base s and the angular dimension of the source. Consider now that in one of the interfering light beams a section is created with a periodically varying optical path length. We will refer to this operation as the modulation of the interference fringes. It can be realized, for example, by an oscillating plano-parallel glass plate covering one of the slits. The modulation of the interference fringes leads to intermittent time variations of the interference pattern. If now a slit which is parallel to the interference fringes and is narrow compared with their width is placed in the focal plane of the objective and the light beam passing this slit directed to the cathode of a photo multiplier, the photo-multiplier current would reproduce as a time function the spatial intensity distribution of the interference pattern.

Let us now assume that the screen covering the objective has three slits a_0, a_1, and a_2 (as shown in Fig. 1), and that a mechanical closing structure (obturator) covers alternately slits a_1 and a_2 with frequency F_0. In this case the length of the base s will be the variable quantity. We will refer to this operation as base-length modulation. We will assume that in the light beam common to both bases modulation of the interference fringes is accomplished as before. At a finite angular dimension of the light source, i.e. $\Psi > 0$, the modulation of the base length s leads to periodical changes in the intensity contrast of the interference fringes and consequently to modulation of the photomultiplier current with a modulation frequency F_0. The depth of this modulation depends on the angular dimension of the source and the geometry of the arrangement. For a point source ($\Psi = 0$) the modulation is absent, since the intensity contrast is equal to unity in either base.

Suitable electronic apparatus permits determination of the site of the light source from the presence of periodic amplitude modulation in the variable component of the photocurrent. The electronic apparatus (see Fig. 1) may, in particular, consist of an amplifier *1* tuned to the frequency with which the interference slits are passing by the slit of the photomultiplier, an amplitude detector *2*, and a narrow-band filter *3* to separate the frequency F_0 of the baselength modulation. In this arrangement the smallest detectable electrical signal of frequency F_0 at the detector output determines the threshold value of the angular dimension of the light source and, consequently, the limit of the resolving power of the setup.

This raises an additional problem. Suppose the angular dimension of the source exceeds the minimum detectable value; with what accuracy can it be measured? The unknown accuracy is obviously determined by the smallest detectable changes in the modulation depth of the electrical signal. The smallest detectable change will be a minimum if a null method is used to measure the electrical signal. This will exclude the effect of slow drifts in the amplification of the instrument (see, for example, Ref. 8). The precision of the measurements is then determined by the same factors as the threshold value of sensitivity—by the amplifier and filter fluctuations and bandwidths.

In our arrangement the null method can be accomplished by varying the width of the slit a_2. The variable portion of the photocurrent which corresponds to base s_2 is proportional to the width of this slit (we will denote the width of any slit with the same symbol as the slit itself). It is obviously possible to choose such a slit width $a_2 < a_1$ which will compensate for the increase in the variable component of the photocurrent due to shifting from bases s_1 to s_2. The instrument at the filter output will then read zero. This will also eliminate at sufficiently high modulation frequencies F_0 (in practice, 20 – 30 cps) the effect of slow drifts in the amplification of the system. The width of the slit a_2 can be callibrated directly in values of the angular dimension of the source.

A close analysis of Fig. 1 makes apparent one intrinsic shortcoming of the system, which fortunately can be easily removed. The point is, that when $a_1 \neq a_2$ the "dc" component of the photocurrent will also be modulated and with it, consequently, the shot noise of the photo-multiplier. Such modulation of the internal noise is extremely undesirable in modulation systems. In our system this modulation can be eliminated, for example, in the following way. A new slit, a_3, is cut through the screen not far from slit a_2. This slit is covered with a thick plano-parallel glass plate in order that the oscillations of the light waves passing through this slit are incoherent with the oscillations of the waves passing through the other slits. With a proper choice of a_3, the modulation of the "dc" component of the photocurrent is eliminated. The slit a_3 is not shown in Fig. 1.

To conclude the theoretical investigation it is necessary to express the fluctuation limit of sensitivity (the smallest detectible angular dimension of the source) Ψ_{fl} and the fluctuation error $|\Delta\Psi/\Psi|_{fl}$ (of the measurement) in terms of the optical and electrical parameters of our model. It would be of interest to consider this problem from the general statistical point of view which is at present extensively used in problems of radio-signal detection (see, for example, Refs. 9 and 10). However, we will restrict ourselves in this paper to a simpler approach using the signal to noise ratio. We shall assume arbitrarily, as is customary in problems of this kind, that the limiting value of the signal which can be detected is approached when the signal to noise ratio at the output of the system is unity. The smallest detectable angular dimension Ψ_{fl} of the source will thus be determined as the value which results in a signal to noise ratio of one at the output of the system (provided that the system is balanced for $\Psi = 0$). In a similar way we will determine the relative fluctuation error $|\Delta\Psi/\Psi|_{fl}$.

In the analysis of the system it is convenient to introduce instead of the angular dimension Ψ of the light source the normalized dimension ψ, defined as Ψ/Ψ_0, where $\Psi_0 = \lambda/s_1$, and s_1 is the length of the larger base (i.e. the distance between the slits a_0 and a_1). We will, as before, refer to ψ_{fl} as the fluctuation limit of sensitivity and to $|\Delta\psi/\psi|_{fl} = |\Delta\Psi/\Psi|_{fl}$ as the relative fluctuation error of the measurement. In the analysis we shall assume that the source is monochromatic. More detailed calculations show, however, that the obtained results can be adapted also for sources with rather broad spectral bands (practically up to 500 – 700 Å).

3. THE BALANCE CONDITION

If the shot noise of the photomultiplier is neglected the current i of the photocathode is the sum of the dark current i_d and a component proportional to the light flux striking the photo-cathode through slit b (see Fig. 1). Slit b is assumed to be narrow compared with the width of the interference fringes and even more so compared with the dimensions of the Fraunhofer diffraction pattern produced by slits a, and is located near the common center of these images. The light flux striking slit b through each slit a is therefore proportional to the square of the width of the respective slit.

Applying the general theory of interferometry, we obtain for the base s_1, i.e. when the slits a_0 and a_1 are open,

$$i = I\{a_0^2 + a_1^2 + 2a_0a_1V_1U_1\cos\varphi(t)\} + i_d, \quad (1)$$

and for the base s_2, when the slits a_0, a_2 and a_3 are open,

$$i = I\{a_0^2 + a_2^2 + a_3^2 + 2a_0a_2V_2U_2\cos\varphi(t)\} + i_d. \quad (2)$$

Here I is the value of the photocurrent when only one of the slits a is open and its width is unity; V_1 and V_2 are the values of the intensity contrast of the interference pattern for $s = s_1$ and $s = s_2$ respectively, given by

$$V = \sin[\pi\psi(S/S_1)]/\pi\psi(S/S_1); \quad (3)$$

U_1 and U_2 are the values of the function

$$U = \sin[\pi\gamma(S/S_1)]/\pi\gamma(S/S_1), \quad (4)$$

for $s = s_1$ and $s = s_2$ respectively, which describes the leveling off of the variable component of the photocurrent because of the finite width of slit b. The parameter γ is the ratio of the width b to the width of the interference fringes at base s_1. The function $\varphi(t)$ describes the modulation of the interference fringes.

It is obvious that when

$$a_2^2 + a_3^2 = a_1^2, \tag{5}$$

the modulation of the quasi-constant component of the photocurrent will be absent. If in addition, $a_1 = a_0$, then

$$i = i_0 \{2 + 2m(t) \cos \varphi(t)\} + i_d, \tag{6}$$

where

$$m(t) = \begin{cases} V_1 U_1 \text{ if } s = s_1, \\ \xi V_2 U_2 \text{ if } s = s_2, \end{cases} \tag{7}$$

Here $i_0 = I a_0^2$, and $\xi = a_2 / a_0$.

We will assume that $\varphi(t)$ is a linear sawtooth function of time with an amplitude $\varphi_0 = 2\pi n$, where n is a sufficiently large number. In this case we may write approximately

$$\cos \varphi(t) = \cos \omega_0 t, \tag{8}$$

where $\omega_0 = 8\pi n/T$.

The system will obviously be balanced when $m(t)$ is independent of time. Denoting by ξ_0 the value of ξ at balance, we obtain

$$V_1 U_1 = \xi_0 V_2 U_2, \tag{9}$$

or, using equations (3) and (4),

$$\xi_0 = (s_2 / s_1)^2 \frac{\sin \pi\gamma}{\sin \pi\gamma (s_2 / s_1)} \frac{\sin \pi\psi}{\sin \pi\psi (s_2 / s_1)}. \tag{10}$$

4. DETERMINATION OF FLUCTUATION LIMITS OF SENSITIVITY AND PRECISION

We will now consider with what accuracy it is possible to satisfy condition (10) in the presence of shot noise in the photomultiplier. (The shot noise of the photomultiplier is under usual conditions of the experiment several orders of magnitude higher than the noise in other part of the system). We will therefore trace the useful signal and the noise through all sections of the system shown in Fig. 1.

(a). *Photocathode Current.* Taking the shot noise into consideration and taking equation (8) into account, equation (6) becomes

$$i = i_0 \{2 + 2m(t) \cos \omega_0 t\} + i_d + i_n \tag{11}$$

where i_n is the noise component of the photocurrent.

Since the signal component of the current is a function of time, the noise i_n is a nonstationary random process. However it can be shown that in our case the noise at the amplifier output is the same as if the photocurrent were a stationary random process with a mean square spectral density $G_i(f)$ corresponding to the case when $m = 0$. Moreover (see, for example, Ref. 11),

$$G_i(f) = 2e(1 + B)(2i_0 + i_d), \tag{12}$$

where e is the electron charge and B is a constant which, according to Chichik[11], can be assumed to be equal to 1.5.

(b). *Detector Input Voltage.* The regular portion of the photocurrent establishes a voltage at the detector input which, apart from its phase, can be represented by

$$E_c \cos \omega_0 t, \quad E_c = 2 i_0 K_1 m(t). \tag{13}$$

In this equation $K_1(f)$ is the frequency response of of the instrument portion up to the detector. For simplicity we will assume that $K_1(f)$ is a rectangle of width Δf. In writing equation (13) the assumption was made that $\Delta f \gg F_0$.

The spectral density $G_u(f)$ of the noise voltage at the detector input within the band Δf is given by

$$G_u(f) = 2eK_1^2(1 + B)(2i_0 + i_d). \tag{14}$$

Outside of this band $G_u(f) = 0$.

(c). *Detector Output Current.* Assuming specifically that the detector is linear and denoting the slope of its characteristic by S we obtain, using the result obtained by Bunimovich[12]:

$$I_{lf}(t) = (S\sigma / \sqrt{2\pi})\,[\alpha q(t) + e^{-\alpha q(t)}], \tag{15}$$

$$G_{lf}(F_0) \approx G_{lf}(0) = (S^2\sigma^2 / 4\pi\Delta f)(\tilde{b}_1 + \tilde{b}_2). \tag{16}$$

In these equations $I_{lf}(t)$ is the regular component of the low frequency detector current; $G_{lf}(F)$ is the spectral density of the low frequency current fluctuations (the condition $\Delta f \gg F_0$ is assumed); $G_{lf}(F_0)$ is practically equal to $G_{lf}(0)$; σ is the norm of the noise voltage at the detector output, given by

$$\sigma = [G_u(f)\,\Delta f]^{1/2}; \tag{17}$$

$q(t)$ is a shorthand notation for

$$q(t) = E_c(t) / \sigma\sqrt{2}; \tag{18}$$

α is a constant coefficient equal to $2/\sqrt{\pi}$, and $\tilde{b}_1$ and $\tilde{b}_2$ are time averages of the functions

$$b_1(q) = (8/\pi)[1-1.5\, e^{-\alpha q(t)} + 0.5\, e^{-3\alpha q(t)}],$$
$$b_2(q) = 1.5\, e^{-\alpha q(t)} - 0.5\, e^{-3\alpha q(t)}. \tag{19}$$

Expanding $I_{lf}(t)$ into a Fourier series and taking into account that $\Delta q(t) \ll 1$, the following approximate expression is readily obtained for the amplitude I_1 of the first harmonic (of frequency F_0):

$$I_1 = 2\pi^{-3/2}\, i_0 K_1 S a(1-e^{-\alpha q_0})\,[\xi V_2 U_2 - V_1 U_1], \tag{20}$$

where q_0 is the time average of $q(t)$:

$$q(t) = q_0 + \Delta q(t). \tag{21}$$

In the sense of the problem we assumed $\Delta q \ll 1$, since otherwise it would be possible to detect the amplitude modulation without any filter. Considering that $\Delta q(t)$ is small we may also in calculating $G_{lf}(F_0)$ replace $q(t)$ in (19) by its time average q_0. This gives

$$G_{lf}(F_0) = (S^2\sigma^2 / 4\pi\Delta f) \times (2{,}5 - 2{,}3\, e^{-\alpha q_0} + 0{,}8\, e^{-3\alpha q_0}). \tag{22}$$

In the determination of q_0 it is natural to assume that the balance condition (10) is satisfied. Then,

$$q_0 = i_0\, [e(1+B)(2i_0 + i_d)\Delta f]^{-1/2}\, \xi_{00}, \tag{23}$$

where ξ_{00} is a shorthand notation for $V_1 U_1$. It should be noted that by its definition ξ_{00} is equal to ξ_0 for $s_2 = 0$.

(d) *Filter Ouput Voltage.* The amplitude of the sinusoidal voltage of frequency F_0 at the filter output is

$$A = K_2 I_1. \tag{24}$$

The dispersion of the filter output noise is

$$\sigma_f^2 = K_2^2 G_{lf}(F_0)\Delta F, \tag{25}$$

where ΔF is the filter bandwidth.

(e) *Fluctuation Error of the Measured Angular Dimension of the Source.* In accordance with the assumption made in Section 2 we will determine the fluctuation error $|\Delta\psi/\psi|_{fl}$ from the condition of equality of signal and noise at the filter output, i.e. from

$$A^2/2 = \sigma_f^2. \tag{26}$$

Setting $\xi = \xi_0 + \Delta\xi$ and taking into account equations (20, (22), (24), (25), and (26), we obtain after some simple transformations

$$|\Delta\xi/\xi_0|_{fl} = \frac{\pi}{2} X(\alpha q_0) \left[\frac{\Delta F}{\Delta f}\right]^{1/2}, \tag{27}$$

where

$$X(\alpha q_0) = \frac{(2{,}5 - 2{,}3\, e^{-\alpha q_0} + 0{,}8\, e^{-3\alpha q_0})^{1/2}}{\alpha q_0 (1 - e^{-\alpha q_0})}, \tag{28}$$

$|\Delta\xi/\xi_0|_{fl}$ represents the normalized fluctuation error of the measured quantity ξ_0 for a given angular spread of the source. $|\Delta\psi/\psi|_{fl}$ is readily obtained from this quantity.

From equation (10) it follows that

$$\Delta\xi/\xi_0 = \pi\psi[\cot\pi\psi - (S_2/S_1)\cot\pi\psi(S_2/S_1)](\Delta\psi/\psi), \tag{29}$$

from which

$$|\Delta\psi/\psi|_{fl} = \tfrac{1}{2} X(\alpha q_0)\, Y(\psi)\, [\Delta F/\Delta f]^{1/2}, \tag{30}$$

where,

$$Y(\psi) = \frac{1}{\psi[(S_2/S_1)\cot\pi\psi S_2/S_1 - \cot\pi\psi]} \tag{31}$$

It is obvious that equation (30) is correct for values of ψ only which are not too close to either zero or unity.

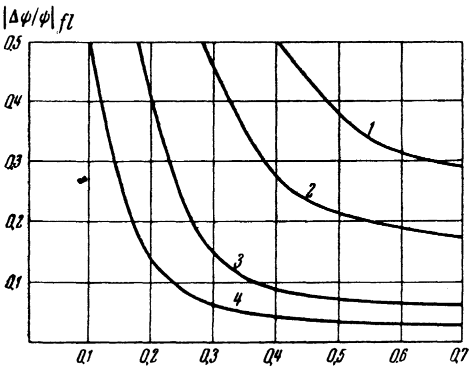

FIG. 2. The fluctuation error $|\Delta\psi/\psi|_{fl}$ as a function of the relative angular dimension ψ of the source with the photocurrent i_0 as a parameter. The values of i_0 are: $1 - 5\times10^{-16}$, curve $2 - 10^{-15}$, curve $3 - 10^{-14}$, and curve $4 - 10^{-13}$ amp.

(f) *Fluctuation Limit of Sensitivity.* To determine ψ_{fl} we will rewrite equation (20). We will first assume that the apparatus is balanced for a point source. Then, $\xi = \xi_0 = U_1/U_2$. Taking U_1 in equation (20) outside the parenthesis and setting it equal to unity, we obtain, approximately

$$I_1 = \frac{2}{\pi\sqrt{\pi}}\, i_0 K_1 S\alpha\,(1 - e^{-\alpha q_0})\,[V_2 - V_1]. \tag{32}$$

In view of the relatively small value of $\pi\psi$ in comparison with unity, we may expand V_1 and V_2 in a power series in terms of $\pi\psi$ and neglect all terms of order higher than 2. This gives,

$$V_2 - V_1 = (\tfrac{1}{6})\,\pi^2\psi^2[1-(s_2/s_1)^2]. \quad (33)$$

According to our previous assumption the signal is considered detected when the signal to noise ratio at the filter output is not less than unity. If this condition for signal detection is written down, we obtain after some simple transformations

$$\psi_{fl} = [1-(s_2/s_1)^2]^{-\frac{1}{2}}[X(aq_0)]^{\frac{1}{2}}[\Delta F/\Delta f]^{\frac{1}{4}}. \quad (34)$$

In Fig. 2 the fluctuation error $|\Delta\psi/\psi|_{fl}$ is shown as a function of ψ for several values of i_0. Fig. 3 shows the fluctuation limit of sensitivity ψ_{fl} as a function of the photocurrent i_0 (lower graph). The following values for the parameters are assumed for the plots: $B = 1.5$, $i_d = 2\times10^{-16}$ amp,, $\Delta f = 1$ cps, $s_2/s_1 = \frac{2}{3}$ and $\gamma = 0.35$. (This value of γ is an optimum from the point of view of signal to noise ratio.)

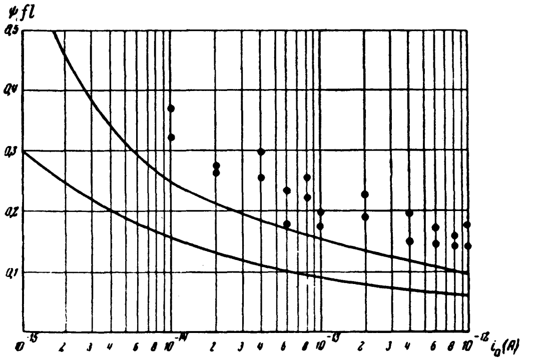

FIG. 3. The fluctuation limit of sensitivity ψ_{fl} as a function of the photocurrent i_0. For the upper curve $i_d = 4\times10^{-15}$ amp, $B = 6.5$; for the lower curve $i_d = 4\times10^{-16}$ amp, $B = 1.5$.

5. THE EXPERIMENT

The described thoretical model was adapted as a basis for developing the experimental setup. A more detailed arrangement of the setup is shown in Fig. 4.

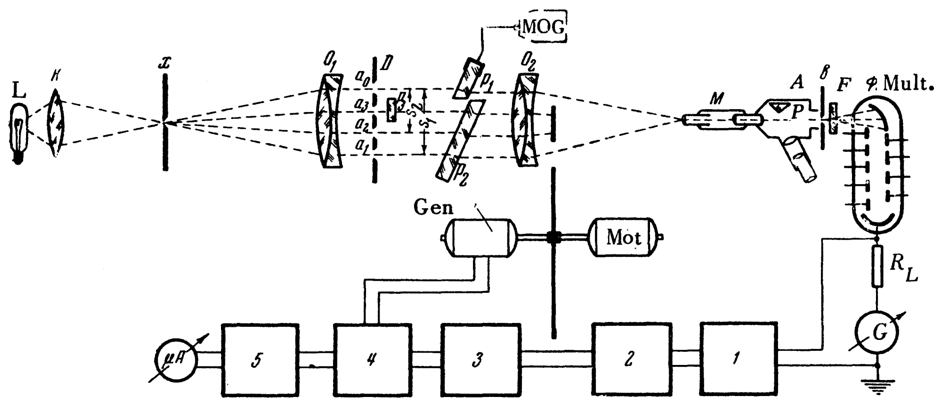

FIG. 4. Schematic diagram of the experimental set-up.

The light source used is a calibrated slit x illuminated with a very strong motion-picture projection lamp. The light beam from the collimator illuminates diaphragm D. Four parallel slits are cut through the diaphragm. The widths two slits, a_2 and a_3, can be varied by means of micrometer screws. The interference fringes are modulated by means of a plane parallel plate p_1 on which angular oscillations are imparted from a mechanical oscillation generator MOG. Plate p_2 is used for compensation of the constant difference of the (optical) path lengths. The modulation of the base length is produced by means of a obturator of frequency $F_0 = 23$ cps. The interference fringes formed in the focal plane of the objective O_2 are projected with large magnification on the slit of the photomultiplier. The width of this slit is only a fraction of the width of the interference fringes and is such to give the optimum condition for γ, i.e. $\gamma = 0.35$.

For visual control of the position of the interference fringes an adaptor A behind the microscope is used. By means of a tilting prism P this enables to observe the interference pattern through a viewing telescope. When the prism is tilted back the light beam passing through the slit b and a green filter F ($\Delta\lambda \approx 500$ A) strikes the photocathode of a FEU-19-M multiplier. The voltage of the photomultiplier load is applied to the input of amplifier *1*. This amplifier is tuned to the frequency $f_0 = 1100$ cps with which the interference fringes are passing (the slit b) and has a bandwidth $\Delta f = 100$ cps. The photomultiplier output voltage is applied to linear detector *2*. A sinusoidal signal of frequency $F_0 = 23$ cps appears at the output of the detector. The balance is accomplished at the output of a narrowband filter tuned to the frequency F_0, comprising a heterodyne filter which consists of a preliminary filter *3*, a balancing detector *4* and a dc filter *5*. A reference voltage is applied to the balancing detector from a generator, the rotor of which is rigidly coupled with the obturator. The phase of the reference voltage can be varied by rotating the stator of the generator. Its value was chosen to obtain maximum sensitivity for the filter. The bandwidth of the heterodyne filter, determined by the dc filter, was set equal to 1 cps.

We made experimental estimates of sensitivity and precision with which the angular dimension of a source can be measured. The result of these measurements is shown in Fig. 3, where the smallest detectable values of ψ are shown as a function of the photocurrent i_0. A comparison with the lower graph of the same figure, which gives the theoretical values of the fluctuation limit of sensitivity calculated for the parameter values assumed in Section 4, shows a wide discrepancy in the results. This discrepancy can be explained by the fact that the photomultiplier used in the experiment had parameters inferior than assumed in the theoretical calculations. Measuring the spectral density of the shot noise of the photomultiplier used in the experiment at 1100 cps it was found that $i_d = 4 \times 10^{-15}$ amp and $B = 6.5$. The theoretical dependence of ψ_{fl} on i_0 for the corrected values of i_0 and B is shown in the upper graph of Fig. 3.

The result of experimental estimates of the error in measuring the angular dimension of the source is shown in Fig. 5. Also shown in the same figure are the theoretical curves of the fluctuation error calculated for the parameters of the experimental set-up. The agreement between the experimental and theoretical results can be regarded as satisfactory. The theoretically found fluctuation limits of sensitivity and precision can thus be attained also in practice.

An essential advantage of the modulation interferometer is its noise rejection with respect to shifts of the interference fringes due to changes in path differences of the interfering beams. Such shifts may result, in particular, from mechanical disturbances of the interferometer. Based on theoretical considerations it may be expected that the modulation interferometer is insensitive to random shifts of the interference fringes provided the following conditions are fulfilled: (1) the mean square value of the shifts is less than or equal to the width of the fringes and (2) the correlation time of the shifts is much larger than $\tau = 1/\Delta f$.

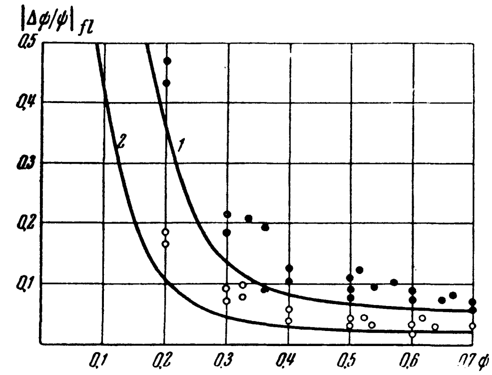

FIG. 5. The fluctuation error $|\Delta\psi/\psi|_{fl}$ as a function of the relative angular dimension ψ of the source. $B = 6.5$; $i_d = 4 \times 10^{-15}$ amp. Curve *1* and points marked ● are for $i_0 = 4 \times 10^{-14}$ amp; curve *2* and points marked ○ – for $i_0 = 4 \times 10^{-13}$ amp.

These theoretical considerations have also been confirmed experimentally. In a number of investigated cases where the presence of mechanical disturbances made the interference pattern completely indistinguishable in visual observations the modulation interferometer was still practically able to utilize completely its theoretical possibilities.

6. SOME REMARKS ON THE POSSIBILITY OF USING THE OPTICAL MODULATION INTERFEROMETER TO MEASURE ANGULAR DIAMETERS OF STARS

It is well known that the idea of using the interference principle for measuring angular diameters of stars was first expressed by Fizeau in 1868. Later this idea was developed further by Michelson, who proposed the so-called stellar interferometer, in which the folded optical paths made it possible in principle to increase indefinitely the dimensions of the base and, consequently, to increase indefinitely the resolving power of the interferometer (see, for example, Ref. 13). In 1920 Michelson was able to measure the angular diameters of some of the bigger stars. Equipment was used with base lengths of 6 and 18 meters. Michelson considered that a further increase in the base length is prevented by the atmospheric and mechanical disturbances which impose an upper limit on the practically realizable base values and at the same time establish the lower limit of the measurable angular diameters. This consideration is in accordance with the capabilities of visual observations.

Some years ago Bershtein and Gorelik pointed out in their notes[14] that the application of radio-physical methods to analysis of the interference pattern of an optical steller interferometer may substantially widen its potentialities. One of the steps in this direction may be the application of the balanced modulation interferometer described above. Of special importance here is the previously mentioned stability of the modulation interferometer with respect to phase fluctuations of the interference fringes.

It is of interest to estimate the potentialities of the modulation method in its application to the stellar interferometer when atmospheric disturbances are considered. Below we will give a brief report on the results of these estimates.

In our calculations we used a theoretical model of the modulation stellar interferometer which was in principle very close to the model shown in Fig. 1. Since stars can be replaced as uniformly luminous disks we have, instead of equation (3),

$$V = 2J_1(1.22\pi\psi s/s_1)/1.22\pi\psi s/s_1, \qquad (35)$$

where $J_1(X)$ is the Bessel function of first order, and ψ now denotes the quantity $\Psi_{s_1}/1.22\lambda$. In addition, making use of a folded path of the rays it is possible to obtain in the modulation stellar interferometer the same width of the interference fringes for both bases s_1 and s_2. The indicated distinguishing characteristics are not essential. Their effect is that the values of ψ_{fl} and $|\Delta\psi/\psi|_{fl}$ obtained for theoretical model of the stellar interferometer in the absence of atmospheric disturbances differ by no more than 5 to 10 percent from the corresponding results for the investigated model. These can therefore be used also for the stellar interferometer.

The photocathode current i_0 can be calculated from the following approximate equation:

$$i_0 = 3\cdot 10^{-14}\cdot 2.5^{-m}\ \mathrm{A}, \qquad (36)$$

where m is the visual star magnitude. This equation was obtained under the assumption that the investigated model has geometrical dimensions of the same order as the latest version of Michelson's stellar interferometer. The sensitivity of the photocathode was assumed to be 3×10^{-5} amperes per lumen.

The following was found concerning the effect of atmospheric disturbances:

(a) There is no sense in using modulation interferometer unless the average turbulence angle t_0 is much smaller than λ/l, where l is the linear dimension of the interferometer mirror (the notation of Danjon and Kude[15] is used). It should be pointed out that the corresponding condition is apparently even more stringent for the common stellar interferometer.

(b) If the condition $t_0 \ll \lambda/l$ is fulfilled it is always possible, at least in principle, to choose such parameters (for example, the modulation frequency F_0) that the phase fluctuations of the interference fringes, which are caused by the fluctuations in the optical path lengths in the passage of the light through the turbulent medium, have a negligible effect on the results of the measurement.

(c) As to the intensity fluctuations, the calculations show that they increase the spectral density of the noise component of the photocurrent from the value given by Eq. (12) to

$$i_0^2 g(|f-f_0|), \qquad (37)$$

where $g(f)$ is the spectral density of the relative intensity fluctuations. This result is obtained under the assumption that the intensity fluctuations in the various paths of the interferometer are independent of each other.

A number of experiments on the investigation of $g(f)$ is described in the literature (see, for example, Ref. 16 and 17). On the basis of these studies it was possible to estimate $g(|f-f_0|)$ for frequencies close to f_0, i.e., for frequencies in the pass band of the amplifier. For good atmospheric conditions it may be assumed that in the region of the amplifier pass band $g(|f-f_0|)$ is 3×10^{-4} cps.

It is convenient to introduce the ratio of the spectral density of the photocurrent fluctuations brought about by the atmospheric disturbances to the spectral density of the shot noise in the amplifier pass band. Denoting this ratio by C and assuming for B the value 1.5 we obtain

$$C = 2\cdot 10^{14}\, i_0 = 6\cdot 2.5^{-m}. \qquad (38)$$

It can be shown that under the atmospheric conditions discussed above for very bright stars the fluctuation limit of sensitivity ψ_{fl} and the fluctuation error in measuring the angular quantity $|\Delta\,\psi/\psi|_{fl}$ are given approximately by

$$\psi_{fl} = (\psi_{fl})^{0}\,(1 + C)^{1/4}, \tag{39}$$

$$|\Delta\,\psi/\psi|_{fl} = |\Delta\,\psi/\psi|_{fl}^{0}\,(1 + C)^{1/2}. \tag{40}$$

In the above equations the index 0 refers to the ideal case of quiet (undisturbed) atmosphere. The effect of "atmospheric" noise is, as should be expected, stronger for the bright stars than for the weak stars. For $m = 0$, for example, "atmospheric" noise increases the fluctuation error in measuring the angular quantity by 2.65 times, and the smallest detectable angular dimension by 1.63 times. At a very quiet state of the atmosphere the effect of atmospheric disturbances is even smaller.

Comparisons with visual observations show that the use of the modulation optical interferometer to measure the angular dimension of a light source does not under readily realizable experimental conditions result in a large increase of the resolving power. However, the theoretical possibilities of the modulation interferometer may be fully realized under conditions of strong disturbances (noise rejection). This, in part, implies that the modulation stellar interferometer can have much larger bases than are permissible in visual observations.

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