π^0 Meson Production in p-p and p-n Collisions in the 390-660 Mev Energy Region*

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The total cross sections and angular distributions for the reactions of π^0 production in p-p and p-n collisions have been measured at various energies. The total cross sections at 660 Mev were found to be $\sigma_{pp}^{\pi^0} = (3.6 \pm 0.2) \times 10^{-27}$ cm^2 and $\sigma_{pn}^{\pi^0} = (7.0 \pm 1.1) \times 10^{-27}$ cm^2. In the energy range 390–660 Mev, the total cross section for the $p + p \to \pi^0 + p + p$ reaction is proportional to the 5.5 power of the maximum π^0 meson momentum. The π^0 meson angular distribution, which is appreciably anisotropic at the proton energy 450 Mev, becomes isotropic as the energy is increased to 660 Mev.

1. INTRODUCTION

BECAUSE THE LIFETIME of the π^0 is short, information about the total production cross section for π^0 mesons and their angular distribution can be obtained from a measurement of the absolute flux and angular distributions of secondary particles—γ-rays from the decay of the π^0 mesons. Previous experiments[1-8] have determined only the differential cross sections for the yield of γ—rays from the decay of π^0 mesons produced in the reactions:

$$p + p \to \pi^0 + p + p \quad \text{(I)};$$
$$p + n \to \pi^0 + \begin{cases} d & \text{(a)} \\ p + n & \text{(b)} \end{cases} \quad \text{(II)}$$

Since the angular distributions of the γ—rays were not investigated in these experiments, in order to determine the magnitude of the total cross section it was necessary to make various assumptions as to the nature of the angular distribution of the π^0 mesons, which materially reduced the reliability of the results obtained.

The investigation of reaction (I) is beset with great experimental difficulties, since the cross section for this reaction is small, due to the fact that the transition in the S state for the protons and p state for the π^0 mesons is forbidden by conservation laws. Measurements of the differential cross section at 340[1] and 430–480 Mev[4-7] have shown that the total cross section for reaction (I) increases sharply with energy. The rapid increase of the cross section for this reaction continues into the proton energy region 500–660 Mev[8]. The cross section for reaction (II) was measured at two proton energies: 340[2,3] and 660 Mev[8]. The angular distribution of the π^0 mesons produced in the reaction (II, a) was measured at the neutron energy 400 Mev[9]. The angular distribution of the π^0 mesons in reactions (I) and (II, b) have not yet been investigated.

The energy distribution of the π^0 mesons may be obtained from measurements of the energy spectra of the γ-rays produced upon their decay. Appropriate experiments have been conducted only for π^0 meson production in heavy nuclei[3,4,6]. A study of the γ-ray spectra also permits one to infer the character of the angular distribution of the π^0 mesons. Thus in Ref. 3 and 6 analysis of the γ-spectra obtained led to inferences concerning the anisotropy of the angular distribution of the mesons in reaction (II) at proton energies of 340 and 470 Mev.

We have measured the yield of γ-rays from the decay of π^0 mesons produced in reactions (I) and (II) at various angles, in the proton energy range 390–660 Mev. These measurements allowed us to determine the absolute total cross sections, excitation functions and π^0 angular distributions for the indicated reactions.

2. EXPERIMENTAL METHODS

The setup of the experiments described below is depicted in Figure 1. Neutral pions were produced in a target bombarded by protons of the internal cyclotron beam, with a maximum energy of 680 Mev. The γ-rays from the π^0 meson decays proceeded through a steel collimator in a four meter concrete wall to a lead diaphragm. A γ-ray telescope was placed at a distance of three meters from the diaphragm.

*The results of this paper were reported at conferences in Moscow (May, 1956) and Geneva (June, 1956).

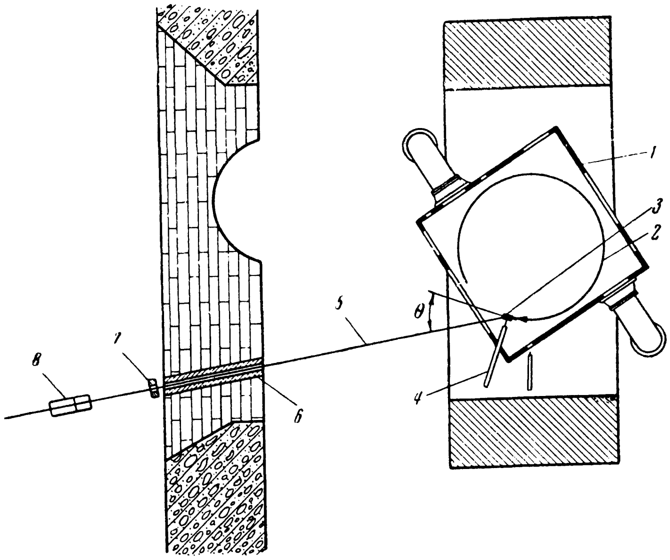

FIG. 1. The experimental setup. 1—vacuum chamber of the synchrocyclotron, 2—circulating beam of protons, 3—internal target, 4—trial rod, 5—γ-ray beam, 6—steel collimator, 7—lead diaphragm, 8—γ-telescope

The selection of the indicated experimental arrangement was dictated by the advantages associated with the use of the high intensity internal proton beam. The proton current through the graphite target reached $(1.6 \pm 0.1) \times 10^{13}$ (protons/sec)/(gm/cm^2). With so high a beam intensity, one can place the telescope at a large distance from the target ($\approx$20 m) and be adequately shielded from stray radiation. At this distance, the magnetic field of the accelerator clears the γ-ray beam thoroughly of charged particles. The high intensity of the internal beam makes possible the use of a calorimetric method for measuring the proton current. With this method, one need not consider the dimensions and density of the targets, and consequently one is rid of the errors associated with the measurement of these quantities.

Since the amplitude of the free radial oscillations of the particles in the cyclotron is large, the internal beam is appreciably non-monoenergetic. The mean energy dispersion of the particles bombarding the target reaches 10 Mev. Therefore, for an accurate determination of the cross section, it is necessary to know the energy spectrum of the internal beam protons. An attempt at the experimental determination of the spectrum was undertaken in Ref. 10. The spectra obtained, at proton energies of 400, 550 and 660 Mev, proved to be identical within the limits of experimental error. They are presented in Fig. 2. The mean square deviation of the energy from its maximum value was found to be 22 Mev. The dotted curve in Fig. 2 represents the normal distribution corresponding to this standard deviation.

γ-*Telescope*

The γ-rays arising from the decay of π^0 mesons are distributed over a broad energy interval. A detector suitable for the investigation of π^0 meson

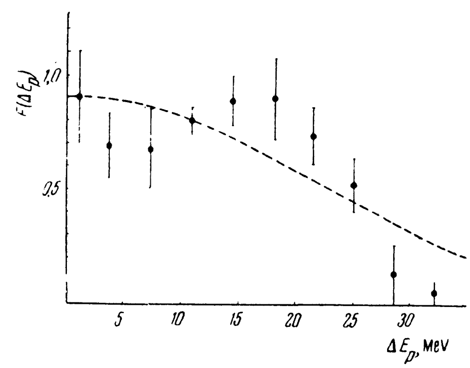

FIG. 2. The energy dispersion of the internal beam protons $F(\Delta E_p)$ (in relative units). ΔE_p is the deviation of the proton energy from the maximum possible.

production processes must efficiently count γ-rays with energies from 10–15 Mev and higher, *i.e.*, it must possess a very low energy threshold. In addition, it must not be sensitive to other forms of radiation—neutrons, protons, etc. A γ-telescope, which, in addition to scintillation counters, contains a Cerenkov counter, satisfies the above requirements. However, such a telescope has not been often applied to absolute γ-ray flux measurements, because the efficiency of the Cerenkov counter is appreciably less than unity and is difficult to calculate. Thus, in references 7 and 11, the Cerenkov counter was used only for measurements of the relative γ-ray flux; for getting absolute measurements, a telescope containing only scintillation counters was used. A γ-telescope will be described below whose efficiency can be experimentally determined.

The γ-telescope used contained a scintillation counter and a Cerenkov counter, placed in two separate blocks (Fig. 3). By varying the distance between the blocks, it was possible to vary the telescope angle from 3° to 45°. In the first block a tolane crystal, of dimensions 50 × 50 × 90 mm, was placed, with two photomultipliers attached; in the second block were a plexiglass radiator, of dimensions 105 × 115 × 40 mm, and two photomultipliers. A removable lead converter was set in front of the scintillation counter. Electrons and positrons, produced in the converter by high energy γ-rays and continuing through the telescope counters, are registered by a quadruple coincidence scheme with a resolution time of 2×10^{-8} sec[12].

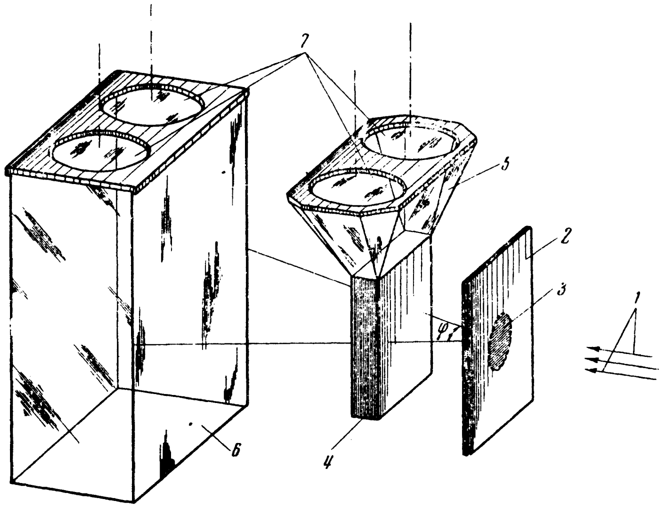

FIG. 3. The layout of the γ-telescope. 1—collimated γ-ray beam, 2—lead converter, 3—area of the converter irradiated by γ-rays, 4—crystal scintillator, 5—light conductor, 6—plexiglass radiator, 7—connection positions of the photomultipliers.

Since the fringing field of the accelerator satisfactorily clears the beam of charged particles, it was not necessary to place a counter, in anticoincidence with those of the γ-telescope, in front of the converter. The telescope count, without the converter, was 7–12% of that with the converter present.

At small angles with respect to the direction of motion of the bombarding protons, an appreciable quantity of high energy neutrons is contained in the γ-ray beam, which raises the telescope count somewhat. For an estimate of the background associated with the neutrons, a 5 cm thick lead absorber was placed in the beam, with "good geometry", clearing the γ-ray beam almost completely of neutrons. Measurements indicated that the γ-telescope is slightly sensitive to fast neutrons. At 660 Mev the neutron background was 3% of the total count.

Measurement of the Efficiency

The total efficiency of the γ-telescope is given by

$$F_\gamma = (1 - e^{-\mu d})\, \xi f_1 f_2. \tag{1}$$

where μ is the absorption coefficient in lead of the γ-rays studied, d is the thickness of the lead converter, ξ is a coefficient measuring the decrease in the efficiency due to scattering and absorption of electrons and positrons in the converter and counters, f_1 and f_2 are the efficiencies for the counting of electrons and positrons by the scintillation and Cerenkov counters. The magnitude of μ depends on the spectrum of the γ-rays counted, and therefore was measured for each value of the energy and angle of observation. For these measurements, a lead absorber was inserted into the γ-ray beam, and the telescope counts with and without absorber were compared. In this way the quantity $(1-e^{-\mu d})$ was determined to an accuracy of 2%.

One of the principal difficulties of a measurement of the absolute γ-ray flux is the determination of ξ. Due to the fact that the ratio of the counts "with converter/without converter" was large, and the efficiency of the telescope was high, the thickness of the converter was usually chosen to be close to a radiation length. As a consequence, the energy threshold of the telescope is high and is appreciably less than unity and can be calculated only by the Monte Carlo method. When d is decreased, ξ rapidly approaches unity and can be measured. The determination of ξ was carried out in the following way: the dependence of the telescope count on the thickness of the converter was measured, and a small thickness d_0 was chosen such that the deviation of the experimental points from the curve $(1-e^{-\mu d})$ was not large, *i.e.*, the telescope had a low energy threshold. This dependence is given in Fig. 4 at a proton energy of 660 Mev and for angles of observation of 0°, 30° and 180°. As is seen from Fig. 4, there is a strong dependence on the angle of observation, which is related to differences in the γ-ray spectra. The magnitude of d_0 was chosen to be 0.05 cm. For the determination of ξ, at this value of d, the dependence of the count on the magnitude of the telescope angle was measured (the distance between the telescope counters was increased). Typical curves, obtained at 660 Mev, are given in Fig. 5. In the above described manner $\xi(d_0)$ was determined to an accuracy $\approx 1\%$.

For the measurement of the relative γ-ray flux, we used a lead converter of thickness 0.2 cm. Under these conditions the spectral efficiency of the telescope $K_\gamma = (1-e^{-\mu d})$ is 0.170 ± 0.003 at 0° and 660 Mev; when the angle is increased to 180°, K_γ decreases to 0.134 ± 0.002.

In order to increase the efficiency of the counter, we used a photomultiplier with a large multiplication factor and operating with a non-uniform distribution of the voltage between the dynodes at a total voltage of 3.5 kv. Also, amplifiers with distributed parameters (70 Mc band-pass, amplification factor ~ 7) were introduced between the coincidence scheme and the photomultipliers. It should be remarked that due to the low intensity of the Cerenkov radiation, the efficiency f_2 is appreciably less than unity even in the sloping portion of the counter characteristic.

For the measurement of the efficiency f, we switched from a quadruple coincidence scheme to one of triple coincidence between any three of the four telescope photomultipliers. Under these conditions the sensitivity of the channels remained constant. Since two photomultipliers are adjoined respectively to the plexiglass radiator and to the scintillator, the exclusion of one of them does not

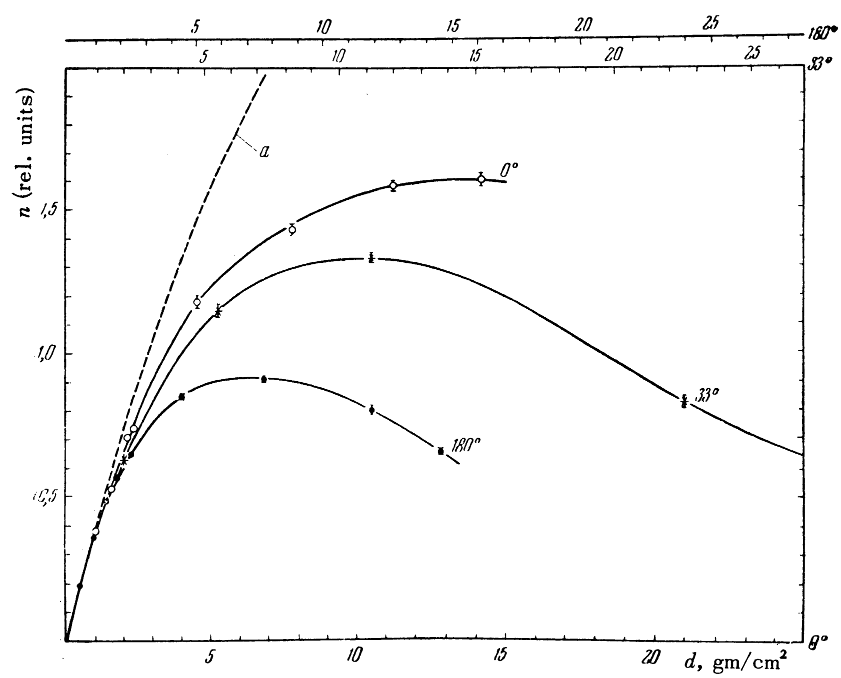

FIG. 4. The variation of the efficiency of the γ-telescope with the thickness of the lead converter d. The scales along the abcissa are chosen such that, for the angles indicated, the magnitudes of μd are the same. The values of μ used in the construction of the scales are given in Table 2. The curve a is that of $(1-e^{-\mu d})$.

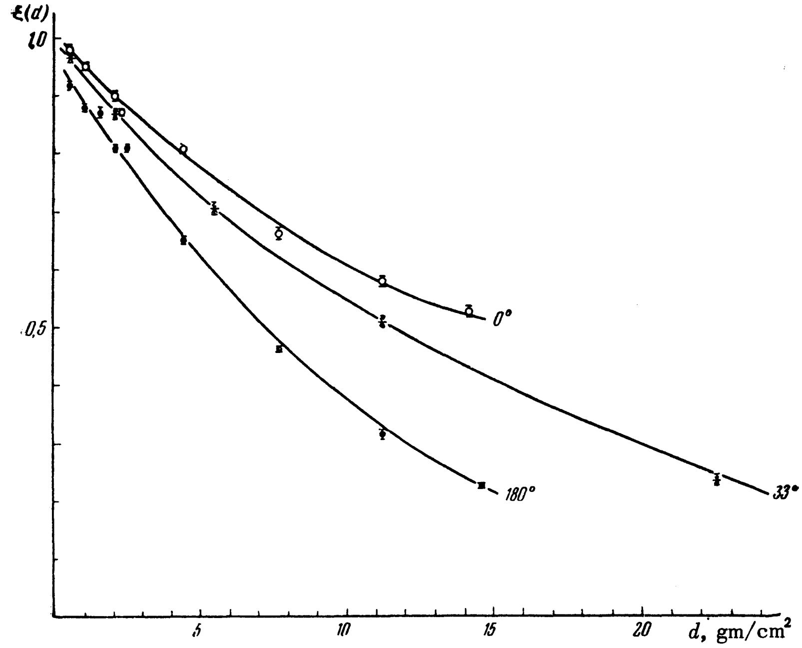

FIG. 5. The coefficient ξ as a function of d at the proton energy 660 Mev, measured at the angles 0°, 33° and 180° in the laboratory system.

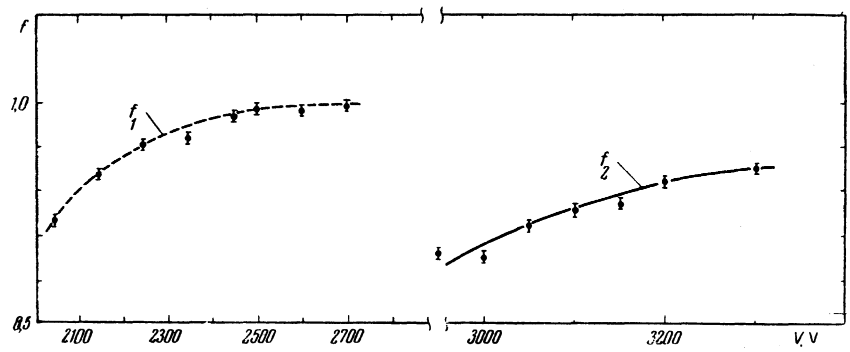

FIG. 6. The efficiency of the telescope counters as a function of the voltage on the photomultipliers.

change the geometry of the telescope. Consequently, by measuring the increase in the γ-ray count when one switches from a quadruple to a triple coincidence scheme, it is possible to determine the ineffectiveness of the excluded photomultiplier and its channel. With a view to checking the data, the method was used for the determination of the efficiency of a scintillation counter, which should not differ much from unity in the region of the plateau of the counter characteristic. We obtained: $f_1 = (99 \pm 1)\%$. The efficiency of the Cerenkov counter was appreciably less: $f_2 \approx 80\%$ (see Fig. 6). At 0° the magnitude of f_2 was somewhat larger than at 180°, which, seemingly, is explained by the difference in the average number of electrons coming out of the converter. By the above described method, the magnitude of $f_1 f_2$ was determined to 2%. The accuracy of the measurement of the total efficiency F_γ was ≈3%.

Measurement of the Proton Current

The proton current through the target, which was placed inside the accelerator chamber, was measured from the heating of the target by means of a calibrated battery of 40 copper-constantan thermocouples. The sensitivity of the "thermobattery" was measured in a special evacuated assembly to better than 1%, and was $(2.38 \pm 0.02) \times 10^{12}$ Mev/sec mv. The thickness of the heat conductor which held the target, was chosen to be sufficiently large so that the target was not heated more than 50°–70°. As the measurements carried out in the evacuated assembly indicated, the loss by radiation under these conditions is not appreciable.

For an estimate of the radiation loss, the ratio of the γ-ray flux to the heat flux through the target at different intensities of the internal beam was also measured. In the flux interval investigated this quantity was constant, to an accuracy $\approx 1\%$. Prior to this, it had been shown that the counting characteristic of the γ-telescope was linear.

The heat generated by the protons in the target is the sum of the ionization loss ϵ_{st}. The first quantity is well known[13, 14]. The quantity ϵ_{st} was calculated on the basis of the spectra of particles—light stars, measured by E. L. Grigor'ev and L. P. Solov'ev. For carbon, at 660 Mev, it was found that $\epsilon_{st}/\epsilon = 0.15 \pm 0.05$. The indicated error represents the maximum error of the calculation.

Since this gives the largest contribution to the error in the absolute cross section, we undertook an experimental determination of the ratio ϵ_{st}/ϵ. By placing an aluminum target, exposed to the protons, at various distances from the center of the accelerator, the energy dependence of the yield of the reaction $Al^{27}(p, 3pn)Na^{24}$ was measured. The products of the reaction were identified from the well known[15] half life of the Na. A comparison of the excitation function for the above reaction in the region 260–660 Mev with those measured in Refs. 16–18 gives, at 660 Mev and after converting to carbon, $\epsilon_{st}/\epsilon = 0.16 \pm 0.06$.

The differential cross section for the production of γ-rays is given by

$$d\sigma_A^{\gamma}/d\Omega = n\varepsilon(1 + \varepsilon_{st}/\varepsilon)\, A\, (1+\delta)/mF_{\gamma} N\Omega. \quad (2)$$

where n is the number of γ-rays counted per unit time by the telescope, m is the heat flux through the target, ϵ is the specific ionization loss (per gm/cm^2), δ is a small correction measuring the absorption of the γ-rays counted in the target and in air, Ω is the solid angle of the counter, A is the atomic weight of the target material, and N is Avogadro's number. The quantities N, A and ϵ are tabulated. The mean square error associated with the measurement of the remaining quantities in Eq. (2), after numerous measurements, comes to 5.2%.

Measurement of the γ-ray Production Cross Section in Hydrogen

By a difference method, the γ-ray production cross section was measured by means of the exposure of polythene $(CH_2)_n$ and graphite targets. The small thermal conductivity of polythene does not permit in this case the use of the calorimetric method. For the determination of the ratio of the cross sections for hydrogen and carbon, a measurement of the activity of the targets was employed. In these experiments the polythene and graphite targets were exposed one after the other and the flux of the γ-rays and the activity of the targets were compared. As in the calorimetric method, the measured ratio did not depend on the shape or density of the target:

$$(d\sigma_H^{\gamma}/d\Omega)/(d\sigma_C^{\gamma}/d\Omega) = [(n'/I') - 1]/p_H'. \quad (3)$$

where n and I' are the ratios of the γ-ray flux and the activities induced in the polythene and graphite targets and p_H' is the relative number of hydrogen nuclei in the polythene. For the polythene used, $p_H' = 2.05$.

The polythene and graphite targets were simultaneously placed in the accelerator chamber and, by means of a coil and current, were alternately introduced into the beam. Synchronously with the interchange of the targets, switching of the counting portions of the telescope is carried out by means of a special commutator, which takes advantage of the fact that distinct γ-ray counts are obtained from the polythene and carbon targets. The targets were switched once every minute. For so frequent an interchange of the targets, the efficiency of the counting devices would only be insignificantly changed, owing to the fact that the experimental error was reduced to 1–2%. When measuring the ratio of the target activities I', an alternate interchange of the targets with a simultaneous commutation of the counting apparatus was also carried out automatically.

The polythene and carbon used did not contain any heavy impurities. It was demonstrated that the γ-activity had a half life of (20.8 ± 0.2) min., which remained constant over a period of several hours. The energy of the γ-rays, measured by the absorption method, was shown to be 0.5 Mev. Thus it was demonstrated that the γ-activity of the targets was associated only with the annihilation of positrons arising from the decay of the C_6^{11} nucleus.

3. EXPERIMENTAL RESULTS

Determination of the Total Cross Sections

The angular distribution of the π^0 mesons produced in the reactions (I, II) can be represented in the center of mass system (CMS) of the colliding nucleons as a polynomial in powers of the cosine of the angle of emission of the mesons, ϑ. At proton energies ≈ 600 Mev and less, the terms of the polynomial containing high powers of the cosine must be comparatively small, as at these energies the contribution of final states with large momenta is still small. As was shown in Refs. 19 and 20, the γ-ray angular distribution function $F(\vartheta)$ has in this case a characteristic property: the magnitude of the function at the points $\vartheta^* = \arccos(\pm 1/\sqrt{3})$ is practically independent of the relations between the parameters, which determine the contribution of the various powers of the cosine in the angular distribution. It follows from this that the differential cross section for the production of γ-rays obtained at the "isotropic" angle ϑ^* is related to the magnitude of the total π^0 meson production cross section by

$$\sigma^{\pi^0} = 2\pi\, d\sigma^{\gamma}(\vartheta^*)/d\Omega. \quad (4)$$

Thus from a measurement of the γ-ray yield at the angle ϑ^*, it is possible to determine the total cross section without carrying out an investigation of the angular distribution.

The above simplifying circumstance was employed in the measurement of the absolute total cross sections for the π^0 meson production reactions (I) and (II). The measurement of the cross section was carried out at an angle of $\theta^* = 33°$, which, for protons with an effective energy of 660 Mev, corresponds to an "isotropic" angle of $\vartheta^* = 55°$ in the CMS. In determining the effective energy, the energy dispersion of the protons bombarding the target was teken into account (see Fig. 2). The absolute differential cross section for the production of γ-rays in carbon at 660 Mev was found to be

$$d\sigma_{\mathrm{C}}^{\gamma}/d\Omega = (8{,}1 \pm 0{,}4)\cdot 10^{-27}\ \mathrm{cm}^2/\mathrm{sterad} \quad (5)$$

At this angle the ratio of the differential cross sections for hydrogen and for carbon was also measured

$$(d\sigma_{\mathrm{H}}^{\gamma}/d\Omega)/(d\sigma_{\mathrm{C}}^{\gamma}/d\Omega) = 0{,}162 \pm 0{,}006.$$

Thus the total cross section for reaction (I) was found to be

$$\sigma_{pp}^{\pi^0}(660) = (3{,}6 \pm 0{,}2)\cdot 10^{-27}\ \mathrm{cm}^2.$$

Information about reaction (II) is usually obtained by a difference method, D–H. The cross section for the production of π^0 mesons in deuterium was measured by exposing targets of LiD, Li and C. The "deuterium/carbon" ratio was found to be

$$(d\sigma_{\mathrm{D}}^{\gamma}/d\Omega)/(d\sigma_{\mathrm{C}}^{\gamma}/d\Omega) = 0{,}48 \pm 0{,}05.$$

The total cross section was:,

$$\sigma_{pd}^{\pi^0}(660) = (10{,}6 \pm 1{,}3)\cdot 10^{-27}\ \mathrm{cm}^2.$$

Neglecting the nucleon binding in the deuterium nucleus, we find

$$\sigma_{pn}^{\pi^0}(660) = (7{,}0 \pm 1{,}1)\cdot 10^{-27}\ \mathrm{cm}^2.$$

Excitation Function

By the above methods, the energy dependence of the total cross section for the production of π^0 mesons by protons in hydrogen and deuterium in the proton energy range 390–660 Mev was obtained. The variation of the proton energy in these experiments was obtained by placing the targets at varying distances from the center of the accelerator. Simultaneously, the angle θ was varied from 33° to 36° in order that the angle of emission of the γ-rays remained equal to the "isotropic" angle $\vartheta^* = 55°$ in the CMS. The spectral efficiency of the γ-telescope was experimentally determined for each value of the energy. As was shown in Ref. 6, the γ-ray spectra are only insignificantly changed upon going from light to heavy elements. Because of this, the spectral efficiency K_γ was the same, within the limits of experimental error, for all the elements at fixed values of E_p and θ. The energy dependence of K_γ and the value of the absorption coefficient μ at various energies are presented below, in Table 1. To each value of μ there is juxtaposed an average γ-ray energy $\overline{E}_p$, for the determination of which the γ-ray absorption data obtained from Ref. 21 was used. Since the absorption coefficient is slightly dependent on $\overline{E}_p$, the obtained value must be only somewhat less than the average energy of the spectrum. Thus, for an energy of 660 Mev at an angle of 0°, we found, using the absorption method $\overline{E}_p = 160 \pm 20$ Mev, while the average energy of the spectrum measured in Ref. 22 was 190 Mev.

The obtained energy dependence of the total cross section for the reactions (I) and (II) is presented in Fig. 7. The errors indicated in Fig. 7 are those of the absolute measurements. Except at 660 Mev, they are everywhere little different from

TABLE 1*

E_p, Mev	270	355	445	550	660
$K_\gamma \cdot 10$	—	—	1.53±0.02	1.57±0.03	1.61±0.02
$\mu \cdot 10^2$ (cm²/gm)	9.20±0.25	9.35±0.33	9.20±0.25	9.56±0.10	9.40±0.17
E_γ (Mev)	100±10	105±15	100±10	115±5	110±10

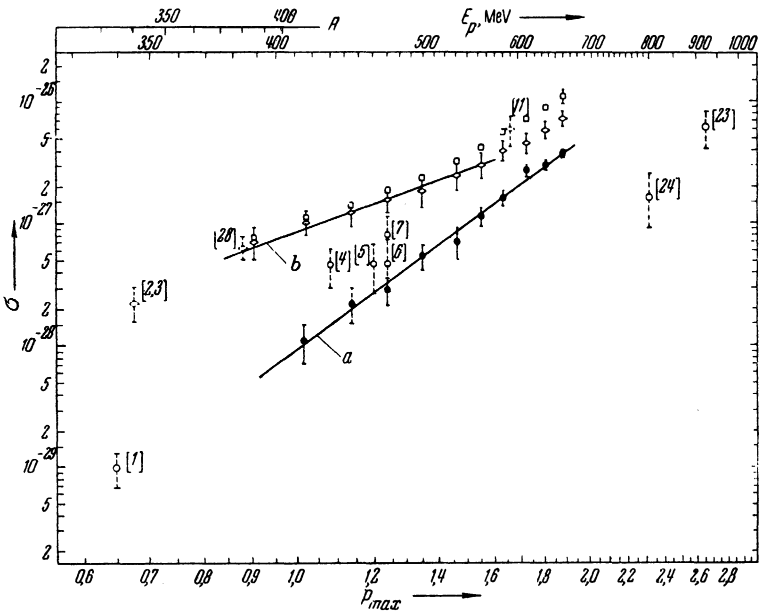

FIG. 7. The total cross sections for reactions (I) and (II). Along the abcissa is the maximum π^0 momentum in units of $m_\pi c$. To the p_{max} scale is compared that of the energy of the protons, for reactions (I) and (II, b), and scale A for reaction (II, a); the curve a is that of $9p_{max}^{5.5}\times 10^{-29}$ cm², $b - 8.2p_{max}^{3}\times 10^{-28}$ cm². In this and the following figures, solid lines indicate the results of our work, dashed lines that of others, presented in the references indicated.

the errors in the relative measurements. The values of the cross section $\sigma_{pn}^{\pi^0}$ presented are the differences of the cross sections for deuterium and hydrogen.

The Angular Distribution of the π^0 Mesons

Experiments to investigate the angular distribution of the particles in the final states of the reactions (I) and (II) are of special interest, as they allow one to ascertain the role of various transitions in greater detail than is possible from a study of the excitation functions. Angular measurements usually present complex experimental problems, and in the case of the angular distribution of π^0 mesons one must encounter an additional specific difficulty. It is associated with the fact that knowledge of the angular distribution of π^0 mesons can be obtained only from measurement of the angular (or energy) distribution of the γ-rays from the decay of the π^0 mesons. The angular distribution of the γ-rays and the π^0 mesons are connected by relations, the analysis of which shows that even at very high energies and with an anisotropic distribution of the π^0 mesons, the γ-ray angular distribution differs comparatively little from isotropic. Only at π^0 meson energies > 200 Mev does the angular distribution of the γ-rays approach that of the π^0 mesons. Thus, if the π^0 meson angular distribution is proportional to $\cos^2\vartheta$, then 50% of the γ-ray distribution is anisotropic at a proton energy of 660 Mev, and at 340 Mev already 10% of the total is. Therefore, when conducting experiments to investigate

*The magnitude of K_γ is given in Tables 1 and 2 for a lead converter of thickness 0.2 cm.

the angular distribution of the π^0 mesons, high accuracy is required, which requirement becomes all the more strict the lower the energy of the π^0 mesons.

As was already noted earlier, high powers of the cosine must be weakly represented in the π^0 meson angular distribution; to a first approximation, the angular distribution is of the form $a/3 + b\cos^2\vartheta$. To determine the ratio a/b, it is sufficient to compare the yield of γ-rays at two angles. By way of such angles, we chose θ^* and its complement $\pi - \theta^*$. The switch from θ^* to $\pi - \theta^*$ was accomplished by a change in the direction of rotation of the internal proton beam. Under these circumstances, naturally, the geometry of the experiment remained the same. The investigation of the π^0 angular distribution was carried out in the energy interval 450–660 Mev. When varying the proton energy, the angle ϑ_2 in the CMS, corresponding to the angle $\pi - \theta^*$, changed somewhat from 160° at 660 Mev to 157° at 400 Mev.

Since the efficiency of the γ-telescope depends on the spectrum of the γ-rays, it was measured for each value of the energy and angle of observation. Table 2 lists the values of the spectral efficiency K_γ, the absorption coefficient μ, and the effective energies E_γ obtained at the proton energy 660 Mev. In the next to last row are given, for comparison, the magnitudes of μ calculated from the γ-ray spectra measured in Ref. 22.

TABLE II

θ°	0	33	90	147	180
$K_\gamma \cdot 10$	1.70±0.03	1.61±0.03	1.45±0.06	1.32±0.04	1.34±0.02
$\mu \times 10^2$ (cm²/gm)	10.1±0.1	9.40±0.17	8.84±0.15	8.6±0.3	8.95±0.15
μ [22]	9.9±0.1	—	—	—	8.7±0.1
$\bar{E}_\gamma$, Mev	160±20	110±10	82±10	75±15	85±10

The ratio of the γ-ray yields for reaction (I), measured at 55° and 160° in the CMS and at proton energy 660 Mev, was found to be

$$\eta_{pp}(660) = 0.99 \pm 0.04.$$

From this it follows that the π^0 meson angular distribution is:

$$f_{pp}(660) \sim 1/3 + (0.01 \pm 0.06)\cos^2\vartheta.$$

Similar measurements were also carried out at lower energies (Table 3).

TABLE III

E_p (Mev)	445	500	555	610	660
η_{pp}	0.80±0.09	0.72±0.12	0.76±0.13	0.93±0.12	0.99±0.04
$f_{pp}(\vartheta) \sim$	$\left(0.3^{+0.6}_{-0.3}\right) + \cos^2\vartheta$	$\left(0.2^{+0.5}_{-0.2}\right) + \cos^2\vartheta$	$\left(0.3^{+1.1}_{-0.3}\right) + \cos^2\vartheta$	$1/3 + (0.1 \pm 0.4)\cos^2\vartheta$	$1/3 + (0.01 \pm 0.06)\cos^2\vartheta$

The angular distribution of the π^0 mesons in reaction (II) was measured at 660 Mev with less accuracy than was the case for hydrogen:

$$\eta_{pn}(660) = 0.98 \pm 0.17.$$

It follows from this that

$$f_{pn}(660) \sim 1/3 + (0.0 \pm 0.6)\cos^2\vartheta.$$

Similar measurements at 445 Mev gave

$$\eta_{pn}(445) = 0.90 \pm 0.25.$$

The large experimental error in the last instance, does not permit the determination of the coefficient b in a distribution of the type $1/3 + b\cos^2\vartheta$.

4. DISCUSSION

The $p + p \to \pi^0 + p + p$ *Reaction*

In the energy region 400–660 Mev the total cross section for reaction (I) measured by us goes, on the average, as $\sigma_{pp}^{\pi^0} \sim p_{max}^{5.3 \pm 0.5}$, where p_{max} is the maximum momentum of the π^0 in the CMS in units of the π^0 meson mass $m_\pi c$. As can be shown, it follows from this that the transition matrix element for reaction (I) in the indicated energy region varies directly as the momentum of the meson. This result differs from that obtained in Ref. 7, where the energy dependence of the differential cross section for reaction (I) at an angle $\approx 90°$ in the CMS was measured. If one considers the data in Table 3 concerning the angular distribution of the π^0 mesons, then, from the energy dependence given in Ref. 7, it follows that the excitation function for the total cross section for reaction (I) goes as $\sigma_{pp}^{\pi^0} \sim p_{max}^4$. This is not in agreement with the dependence obtained by us. Correspondingly, our conclusions as to the nature of the variation of the matrix element with the momentum of π^0 are also different.

In the energy interval 600-660 Mev some slowing down of the rate of increase of the matrix element for reaction (I), which becomes appreciable at higher values of the energy is observed. A comparison of our results with recently published data [23] indicates that in the region of 700-800 Mev the matrix element goes through a maximum.

As is seen from Fig. 7, the magnitude of the cross section for reaction (I), determined by us in the energy region 420-480 Mev, is on the average two times less than that measured earlier[4-7]. From a comparison of our cross sections with that obtained at 340 Mev[1], it follows that in this energy interval $\sigma_{pp}^{\pi^0}$ increases as $p_{max}^{5.5 \pm 1.5}$, and not as p_{max}^8.[25] This implies that, in reaction (I), along with the Pp transition* (which goes as $\sim p_{max}^8$), the Ss transition, the intensity of which is weakly dependent on energy ($\sim p_{max}^2$), plays a very appreciable role. The variation of the cross section (I) with energy in the region 340-400 Mev can be represented as follows:

$$\sigma_{pp}^{\pi^0} = (0{,}025\, p_{max}^2 + 0{,}12\, p_{max}^8) \cdot 10^{-27}\ \text{cm}^2.$$

The Ss transition can be appreciable only near threshold; with an increase in energy, the relative contribution of this transition quickly decreases, and at energies ≈ 400 Mev the transition in the Pp state is the principal one. The contribution to the reaction from the Ps transition is, apparently, not large. In view of this, the angular distribution of the π^0 mesons, obtained by us at 445 Mev and exhibiting an appreciable anisotropy, is affirmed.

*Here the letters indicate, respectively, the orbital angular momenta of the nucleons (in the nucleon-nucleon system) and of the pion.

The p + n → π^0 + p + n *Reaction*

At proton energies less than 400 Mev, the reaction (II, a) in which a deuteron is created in the final state, is the main contributor. Near threshold it is characterized by a cross section that goes as $\sim p_{max}^3$.[25] An analysis of the angular[26] and energy[22] distribution of the γ-rays produced in light nuclei by 450-480 Mev protons, indicate that, at these energies, the angular distribution of the π^0 mesons in reaction (II) is appreciably anisotropic. Together with the energy dependence obtained by us

$$\sigma_{pn}^{\pi^0} = (8{,}7 \pm 1{,}5)\, p_{max}^{3{,}0 \pm 0{,}5} \cdot 10^{-27}\ \text{cm}^2,$$

This attests to the fact that at 450 Mev and lower, the transition with a final S state for the nucleons and p state for the π^0 meson is the principal one for reaction (II).

With an increase of the energy in (II), reaction (II, b), proceeding without the production of a deuteron, begins to predominate (Fig. 8). The presence of three particles in the final state of the last reaction must lead to a stronger dependence of the statistical factor on the momentum of the π^0 meson.

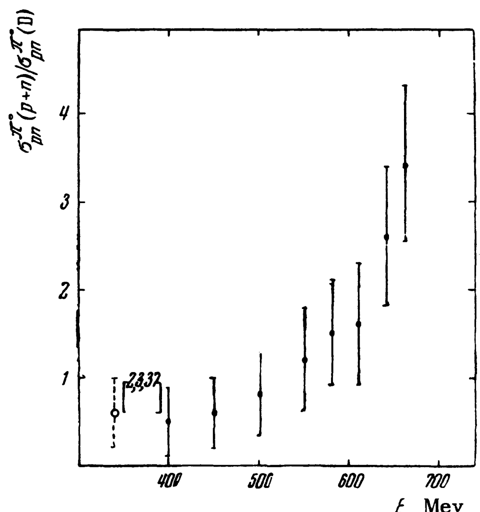

FIG. 8. The ratio of the cross sections for π^0 meson production in reactions (II, b) and (II, a) at various proton energies. The cross section $\sigma_{pn}^{\pi^0}(D)$ is gotten from the equation $2\sigma_{pn}^{\pi^0}(D) = \sigma_{pp}^{\pi^0}(D)$. The data used is that of Ref. 27.

There is, apparently, associated with this a more rapid growth of the cross section $\sigma_{pn}^{\pi^0}$ in the energy region 550-660 Mev than at lower energies. The relatively slow rate of increase of the cross section

$\sigma_{pn}^{\pi^0} = \sigma_{pd}^{\pi^0} - \sigma_{pp}^{\pi^0}$ in the low energy region can also be partially due to the presence of internal motion of the nucleons in the deuteron; with a decrease in the energy, this factor should play an increasingly significant role. At an energy ≈400 Mev the influence of the nucleon binding in the deuteron is, apparently, still not large, since the cross section obtained by us at 385 Mev agrees well with the π^0 production cross section in *n-p* collisions at 380 Mev recently published by Rosenfeld *et al.*[28]

Production of π^0 Mesons by Protons of Energy ~600 Mev

A comparison of the magnitudes of the relative cross sections for hydrogen and deuterium measured by us, shows that in the energy region up to 600 Mev the ratio $\sigma_{pp}^{\pi^0}/\sigma_{pn}^{\pi^0}$ increases rapidly, reaches a magnitude of ½ and remains constant upon further increasing the energy (Fig. 9). As Lapidus[29] has shown, the ratio of the cross sections for the reac-

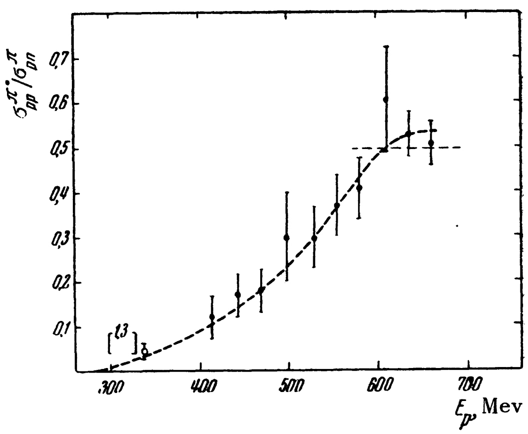

FIG. 9. The ratio of the cross sections for reactions (I) and (II) as a function of energy.

tions (I) and (II) must equal ½ if, in the final state of these reactions, the meson-nucleon system has an isotopic spin of $T = 3/2$. The data, presented in Fig. 9, shows that, in the 600–660 Mev energy region, the $T = 3/2$ transition is apparently the principal one.

We can also arrive at this conclusion if we consider the energy dependence of the ratio of the cross section $\sigma_{pp}^{\pi^0}/\sigma_{pn}^{\pi^+}$ (see Fig. 10). The rate of increase of this ratio is retarded at 600 Mev and thereafter approximately constant and close to ⅕ over a broad energy interval. As was shown in Refs. 29 and 30, such a ratio occurs if $T = 3/2$ for the meson-nucleon system in the final state of the indicated reactions.

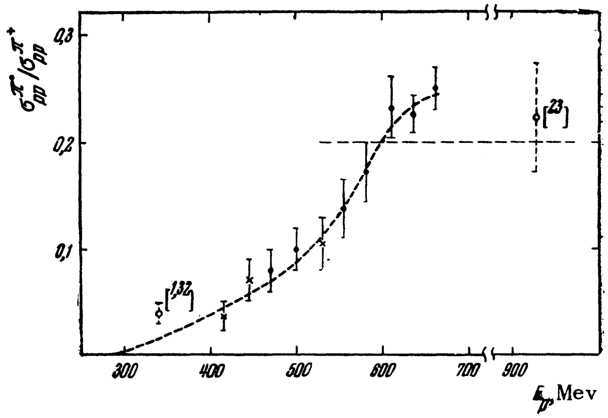

FIG. 10. The ratio of the cross sections for the production of neutral and charged pions in *p-p* collisions at various energies: ● indicate values obtained from a comparison of our results with those of Refs. 27 and 31; the data of Refs. 33 and 34 were used for the values marked ×.

The data given in Table 3 indicate that, with increasing energy, the angular distribution of the π^0 mesons becomes more and more isotropic, and at a proton energy of 660 Mev, the anisotropic portion of the π^0 distribution does not exceed 7%. For reaction (II) there occurs a similar transition from an anisotropic to an isotropic angular distribution with increasing collision energy. Such a change in the angular distribution is, apparently, associated with the fact that the more the energy is increased, the increasingly larger a role is played by a "resonance" transition in which the mesons interact strongly with one of the nucleons in the final state ($T = 3/2$, $J = 3/2$), and the orbital angular momentum of the second nucleon is equal to zero. The disintegration of such a system, and consequently the emission of the π^0 mesons, takes place isotropically.

Thus, all the conclusions from the above experimental data favor the assumption of a preeminent role for resonance transitions ($T = 3/2$, $J = 3/2$) in the energy region ≈600 Mev.

At proton energies 340–480 Mev, the pions are created for the most part with energies close to the maximum possible, as a consequence of the fact that the nucleons in the final states interact relatively strongly. In Refs. 8 and 22 it was established that when the proton energy is increased to 660 Mev, there occurs a relative "smearing" of the spectrum of the γ-rays from the decay of the π^0 mesons. The π^0 mesons, at this proton energy, carry away on the average only about half of the maximum possible energy.

If, when going from 450 to 660 Mev, the portion of the energy being carried away by the mesons were to remain unchanged, then the average energy of the γ-rays counted at 33° and at proton energies of 450, 560 and 660 Mev would be, respectively, 100, 120 and 145 Mev. A comparison of these values with the data in Table 1 indicates that the "smearing" of the π^0 spectrum occurs in the proton energy region of 560–660 Mev. It is possible that the indicated change in the nature of the π^0 spectrum is also a consequence of the strong interaction in the meson-nucleon system.

Comparison with the Production Cross Sections for Charged Pions

The total π^0 production cross sections obtained by us allow one to make, within the limits of the charge independence hypothesis, a crude estimate of the production cross section for charged mesons where the latter are unknown. Thus, the inequality

$$\tfrac{1}{2}\sigma_{pp}^{\pi^0} \leqslant \sigma_{pn}^{\pi^+} \leqslant \sigma_{pn}^{\pi^0} + \tfrac{1}{2}\sigma_{pp}^{\pi^0} \quad (5)$$

permits one to estimate the cross section $\sigma_{pn}^{\pi^+} = \sigma_{pn}^{\pi^-}$ at 660 Mev, experimental data about which is as yet lacking:

$$(1.8 \pm 0.1)\cdot 10^{-27}\ \text{cm}^2 \leqslant \sigma_{pn}^{\pi^+} \leqslant (8.8 \pm 1.1)\cdot 10^{-27}\ \text{cm}^2.$$

The magnitude of the cross section $\sigma_{pn}^{\pi^+}$ can be determined by means of a comparison of our results with the data of Refs. 27 and 31 if one uses the identity $\sigma_{pn}^{\pi^+} = \sigma_{pn}^{\pi^0} + \sigma_{pp}^{\pi^0} - (\tfrac{1}{2})\sigma_{pp}^{\pi^+}$:

$$\sigma_{pn}^{\pi^+}(660) = (3.5 \pm 1.3)\cdot 10^{-27}\ \text{cm}^2.$$

TABLE IV

E_p (MeV)	400[35]	500	555	580	610	640	660
$\sigma_{pn}^{\pi^+} \times 10^{27}$, cm²	0.16±0.04	⩽0.6	⩽0.8	0.8±1.1	1.6±1.1	2.1±1.2	3.5±1.3

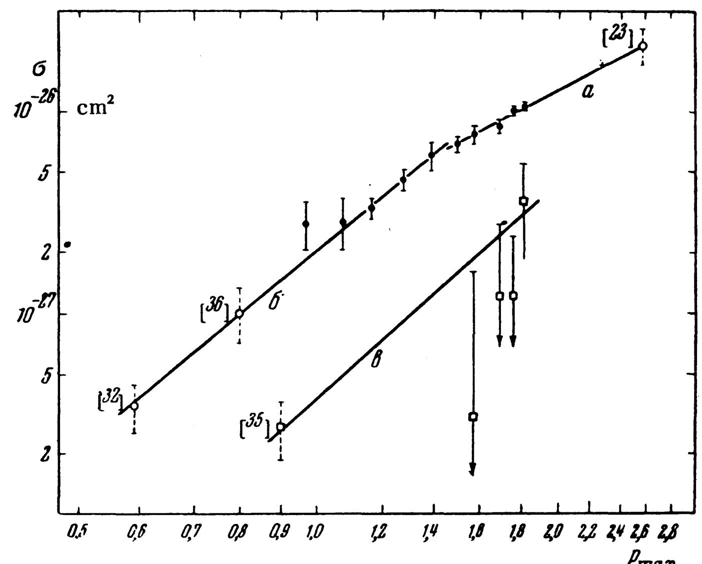

FIG. 11. The cross sections σ_{01} and σ_{10} as a function of the maximum pion momentum. The curve *a* is that of $3p_{max}^2 \times 10^{-27}$ cm², *b*–$2p_{max}^{3.2} \times 10^{-27}$ cm² and *c*–$3.6p_{max}^{3.5} \times 10^{-28}$ cm². In constructing the abcissa scale, the values of the pion masses used were: $m_{\pi^0} = 134.8$ Mev and $m_{\pi^+} = 139.3$ Mev.

Comparison with the data presented in a paper by G. Yodh[35] indicates that this cross section increases rapidly with increasing energy:

If one approximates the energy dependence of the

cross section by a function $\sim p^{a}_{max}$, then in the energy region below 650 Mev, we have
$\sigma^{\pi+}_{pn} = (0.3 \pm 0.1)\, p^{4\pm1}_{max} \times 10^{-27}$ cm^2.

Partial Cross Sections

Within the bounds of the charge-independence hypothesis all the cross sections for pion production in nucleon collisions can be represented as a linear combination of three partial cross sections σ_{02}, σ_{11} and σ_{10} (the notation of Ref. 25 is used here; the indices denote the magnitudes of the isotopic spins of the nucleons in the initial and final states). Using our data and that of Refs. 27 and 31, it is possible to determine the magnitude of these cross sections at the proton energy 660 Mev:

$$\sigma_{11} = (3{,}6 \pm 0{,}2)\cdot 10^{-27}\ \text{cm}^2,$$

$$\sigma_{01} = (3{,}6 \pm 1{,}8)\cdot 10^{-27}\ \text{cm}^2,$$

$$\sigma_{10} = (10{,}6 \pm 0{,}6)\cdot 10^{-27}\ \text{cm}^2.$$

The cross section $\sigma_{11} = \sigma^{\pi^0}_{pp}$ at various energies has been determined above (see Fig. 7).

A comparison of the magnitudes of the cross section σ_{01} at various energies indicates (Fig. 11) that σ_{01} varies as $p^{3.5\pm1.0}_{max}$, which is in good agreement with the conclusions of Rosenfeld[25].

The reaction $T_N = 1 \to T_N = 0$ is characterized by a slower rate of increase of the cross section with energy (see Fig. 11). In the region up to 580 Mev, the cross section σ_{10} is proportional to the third power of the momentum,

$$\sigma_{10} = (2{,}0 \pm 0{,}4)\, p^{3{,}2\pm0{,}6}_{max}\cdot 10^{-27}\ \text{cm}^2$$

in agreement with Ref. 25. At higher values of the energy, the rate of increase of the cross section is retarded and in the 550–1000 Mev region can be written as

$$\sigma_{10} = (3{,}0 \pm 0{,}2)\, p^{2{,}0\pm0{,}4}_{max}\cdot 10^{-27}\ \text{cm}^2.$$

From a comparison of the data of Refs. 38 and 39, it follows that in the energy region ≈1000 Mev the cross section σ_{10} goes through a broad maximum and thereafter slowly decreases with increasing proton energy.

In conclusion, we take the opportunity to express profound thanks to M. S. Kozodaev, B. M. Pontecorvo and L. I. Lapidus for the discussion of our results. We are indebted to M. M. Kuliukin for his aid in the construction of the apparatus.

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Asymptotic Meson-Meson Scattering Theory

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An asymptotic expression for the scattering amplitude for meson-meson interaction has been obtained from a theory of the Landau, Abrikosov, and Khalatnikov type by summation of an infinite number of graphs of a certain class. The problem reduces to an integral equation of a simple type, which can be solved exactly.

1. INTRODUCTION

NOT LONG AGO Landau, Abrikosov, Khalatnikov and Galanin[1,2] developed a new approach to the solution of field theory equations. While assuming that the coupling constant g_0 is small, i.e. $g_0^2 \ll 1$, and investigating the series expansion of all quantities for large momenta in powers of g_0^2 they do not however consider the quantity

$$\varkappa = g_0^2 \ln [\Lambda^2 / (-p^2)] = g_0^2 (L - \xi)$$

to be small (in contrast with the assumption used in conventional perturbation theory), here

$$L = \ln (\Lambda^2 / m^2) > 1, \quad \xi = \ln (-p^2 / m^2) > 1,$$

where Λ is the limit for momentum cut-off and p the momentum pertinent to the problem.

The result is that the asymptotic expressions of different field theory quantities are represented by series of the type:

$$f_0(\varkappa) + g_0^2 f_1(\varkappa) + g_0^4 f_2(\varkappa) + \ldots, \tag{1}$$

where the $f_n(\varkappa)$ are closed functions. In fact, the above authors[1,2] used the integral equations of field theory to construct the $f_0(\varkappa)$ term of the zero approximation for the case of a series expansion of the Eq. (1) type for propagation functions and functions for the vertex parts in quantum electrodynamics[1] and mesodynamics[2].

Essentially the condition that $g_0^2 < 1$ is not necessary for the existence of an expansion in a series of the Eq. (1) type. As has been shown by Pomeranchuk[3], when one introduces two cut-off limits[4]

$$L_p = \ln (\Lambda_p^2 / m^2), \quad L_p > L_k \gg 1$$

then in all equations and in particular in Eq. (1), g_0^2 is replaced by the quantity

$$\tilde{g}_0^2 = g_0^2 [1 + (g_0^2 / \pi)(L_p - L_k)]^{-1}, \tag{2}$$