$$S_\alpha(E_c, \geqslant E, \theta) = -\frac{\gamma\nu I_0}{2\pi i l}\int\limits_{s_0-i\infty}^{s_0+i\infty}\left(\frac{E_c}{E}\right)^s\int\limits_0^\theta \frac{\exp\{-f(D\alpha_s)(\theta-x)\}\,e^{-x/l}}{s(\gamma-s)}\,dx\,ds$$

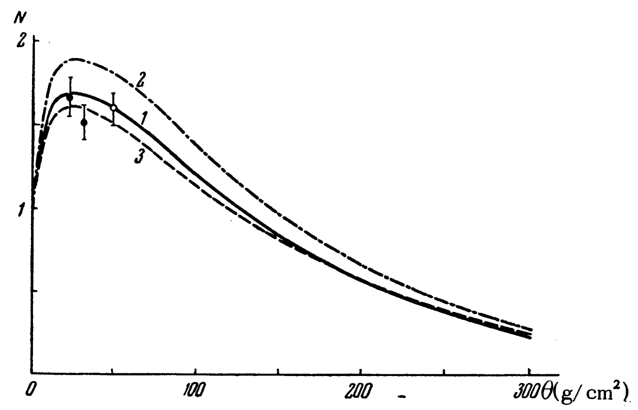
$$= \frac{\gamma\nu I_0}{2\pi i l}\int\limits_{s_0-i\infty}^{s_0+i\infty}\int\limits_0^\theta\left(\frac{E_c}{E}\right)^s\frac{\exp\{-f(D\alpha_s)\theta+f(D\alpha_s)x-x/i\}}{s(\gamma-s)[f(D\alpha_s)-1/l]}\,d[-f(D\alpha_s)\theta$$

$$+f(D\alpha_s)x-x/l]\,ds = \frac{\gamma\nu I_0}{2\pi i l}\int\limits_{s_0-i\infty}^{s_0+i\infty}\left(\frac{E_c}{E}\right)^s\frac{e^{-f(D\alpha_s)\theta}-e^{-\theta/l}}{s(\gamma-s)[1/l-f(D\alpha_s)]}\,ds.$$

Therefore

$$S_\alpha(E_c, \geqslant E, \theta) = \frac{\gamma\nu I_0}{2\pi i l}\int\limits_{s_0-i\infty}^{s_0+i\infty}\left(\frac{E_c}{E}\right)^s\frac{e^{-f(D\alpha_s)\theta}-e^{-\theta/l}}{s(\gamma-s)[1/l-f(D\alpha_s)]}\,ds.$$

The value $S_\alpha = (E_c, >E, \theta)$ was estimated by the saddle point method.



Altitude dependence of the number of stars and the flux of star-producing particles: 1- stars with account of primary α-particles, 2- flux of star-producing particles with account of primary α-particles, 3- stars without account of α-particles, ●- experimental data obtained in laboratory, O- point of normalization of the experimental data. All curves are normalized to unity.

We calculated the curves for the star-producing particles with energy larger than E = 100 Mev for for various atmospheric depths. The calculations were carried out with and without account of the primary α-particles. Results are given in the figure.

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Ponderomotive Forces in a Localized Plasma in the Electromagnetic Field of a Plane Wave

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WHEN THE FIELD of a plane wave is impressed on a localized plasma, there arise both a radiation pressure in the direction of the wave motion and ponderomotive forces tending to produce a deformation of the plasma. The present article is devoted to the explanation of the nature of these forces in the special case where the wavelength is much larger than the linear dimensions of the region of localization A quasi-neutral plasma may be described phenomenologically as a medium having dielectric constant ε, conductivity σ and magnetic permeability $\mu = 1$.

A sphere of ionized gas with a uniform density of ionization may be considered as a rough model of a localized plasma. The electromagnetic field inside and outside such a plasma sphere is in accordance with the theory of the diffraction of a plane wave by a homogeneous dielectric sphere electromagnetic wave on a dielectric sphere as a whole has been investigated by Debye.[2]

We shall find the distribution of ponderomotive forces throughout the volume of the sphere by starting from the known expressions for the stress tensor of the electromagnetic field, according to which the time-averaged force per unit volume of the sphere is

$$\mathbf{f}^v = \frac{1}{c}[\mathbf{j}\times\mathbf{H}] - \frac{1}{8\pi}E_1^2 \operatorname{grad}\varepsilon + \frac{1}{8\pi}\operatorname{grad}\left(E_1^2\frac{\partial\varepsilon}{\partial\tau}\tau\right),$$

and the force per unit surface area is

$$\mathbf{f}^s = \frac{1}{4\pi}\varepsilon E_{1n}(\mathbf{E}_2-\mathbf{E}_1) - \frac{1}{8\pi}\left[E_2^2 - \left(\varepsilon - \frac{\partial\varepsilon}{\partial\tau}\tau\right)E_1^2\right]\mathbf{n},$$

where $\mathbf{E}_1$ and $\mathbf{E}_2$ are the electric fields inside and outside the sphere, $\mathbf{n}$ is the outward normal to the surface, and τ is the density of the medium.

Taking for the plasma $\varepsilon = 1-\omega_0^2/\omega^2$ (ω_0 is the plasma frequency) and, for simplicity, neglecting losses ($\sigma = 0$), we obtain

$$\mathbf{f}^v = \frac{\varepsilon-1}{8\pi}\operatorname{grad}E_1^2,$$

$$\mathbf{f}^s = \frac{1}{4\pi}\varepsilon E_{1n}(\mathbf{E}_2-\mathbf{E}_1) - \frac{1}{8\pi}(E_2^2 - E_1^2)\mathbf{n}.$$

If the radius a of the sphere is much smaller than the wavelength in space and in the plasma, then the time averaged volume density of the force within the sphere turns out, after rather tedious calculations[3], to be approximately equal to

$$f_x^v = \frac{E_0^2}{8\pi}k^2\frac{(\varepsilon-1)^2(35\varepsilon^4-87\varepsilon^3-1032\varepsilon^2-2075\varepsilon-1266)}{5(\varepsilon+2)^2(2\varepsilon+3)^2(7\varepsilon+12)}x,$$

$$f_y^v = -\frac{E_0^2}{8\pi}k^2\frac{(\varepsilon-1)^2(91\varepsilon+158)}{5(\varepsilon+2)^2(7\varepsilon+12)}y,$$

$$f_z^v = \frac{E_0^2}{8\pi}k^2\frac{(\varepsilon-1)^2(35\varepsilon^4-129\varepsilon^3-1209\varepsilon^2-2330\varepsilon-1392)}{5(\varepsilon+2)^2(2\varepsilon+3)^2(7\varepsilon+12)}z.$$

Here the x axis of a right-handed system of coordinates with origin in the center of the sphere coincides with the direction of polarization of the electric field of the plane wave (with wave vector k) which is being propagated along the z axis. The time-averaged force density on the surface has here the following form

$$\mathbf{f}^s = \frac{E_0^2}{8\pi}\,\frac{9}{2}\,\frac{(\varepsilon-1)^2}{(\varepsilon+2)^2}\,\frac{x^2}{a^2}\,\mathbf{n}.$$

It follows from the preceding formulae that for certain values of ε the forces inside the sphere are directed inward. The forces on the surface of the sphere are always directed outward. A comparison of the volume and surface forces shows (in view of of the fact that $ka \ll 1$) the predominant role of the surface forces on the periphery of the sphere.

Hence the ponderomotive forces lead to an unstable surface layer in a plasma sphere in the field of a plane wave for $ka \ll 1$ and thus make possible a dislocalization of the plasma in all directions, which in the present case is connected with the presence of a density gradient at the bounded plasma, as at every localized medium. Hence we may consider that a tendency to spread in the field of a plane electromagnetic wave is characteristic of a plasma concentrated in a localized region of dimensions considerably smaller than a wavelength.

We have limited our consideration to continuous oscillations of the plasma under the influence of a plane wave and have not been concerned with transient processes.

Analogous problems may be of interest in connection with the recently proposed new methods of accelerating clusters of charged particles[4].

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