probability is found from the hypothesis of isotopic invariance) and the probability w for the decay of the θ-meson according to one or another scheme. The ratio of the θ-meson decay probabilities can be found by assuming that in this decay the isotopic spin selection rule $|\Delta T| = 1/2$ is operative[5]. There exist two possibilities depending on the spin of the θ-meson.

1. The spin of the θ-meson is odd. Then the $\theta^0 \to 2\pi^0$ decay is impossible. According to the above, the ratio R of the τ' decay to that of the τ decay is

$$R = w(\theta^+ \mid 0+)/2w(\theta^0 \mid +-), \tag{3}$$

where, for instance, $w(\theta^+|0+)$ is the probability that the θ^+-meson decays into a π^0 and a π^+. According to Gatto[5], if the spin of the θ-particle is odd

$$w(\theta^+ \mid 0+) = 2w(\theta^0 \mid +-)$$

and $R = 1$.

2. The spin of the θ-meson is even. Then according to Gatto[5], $w(\theta^+| + 0) = 0$ (if the selection rule $|\Delta T| = 1/2$ is operative), and

$$w(\theta^0 \mid +-) = 2w(\theta^0 \mid 00).$$

We then obtain $R = 0.5$.

As is well known, the ratio R can be found from the selection rule $|\Delta T| = 1/2$ alone[5,6] and lies in the interval $1/4 \leqslant R \leqslant 1$. The case $R = 1/4$ occurs when the total isotopic spin of the system of three π-mesons is equal to unity in our case of a strong π-K interaction, R is subjected to another restriction. We note that the experimental values of R as found by various observers differ among themselves to a great extent. Recently Birge, Perkins, *et al.*,[1] have obtained $R = 0.39 \pm 0.096$.

Using the concept of a strong π-K interaction, we can also determine the ratio R_0 of the decays

$$\tau^{0\prime}(\to 3\pi^0) \text{ and } \tau^0(\to \pi^+ + \pi^- + \pi^0).$$

We obtain $R_0 = 0$ if the spin of the θ-particle is odd, and $R_0 = 0.5$ if the spin of the θ-particle is even.

Several experiments have indicated the existence of the so-called anomalous θ^0-decay[7], among which there are cases which have been interpreted according to the scheme

$$K^0_{\mu 3} \to \mu^\pm + \pi^\mp + \nu. \tag{4}$$

For $K^+_{\mu 3}$ and $K^0_{\mu 3}$ decays we may write (see also an earlier work by the present author[8])

$$K^+_{\mu 3} \to K^+_{\mu 2} + \pi^0 \to \mu^+ + \nu + \pi^0,$$
$$K^0_{\mu 3} \to K^+{}_2 + \pi^- \to \mu^+ + \nu + \pi^-. \tag{5}$$

The ratio R_μ of the $K^+_{\mu 3}$ and $K^0_{\mu 3}$ decay probabilities is then $R_\mu = 0.5$ independent of the spin of the $K_{\mu 2}$ (in this case the absence of a neutral μ-meson is relevant).

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Application of a Renormalized Group to Different Scattering Problems in Quantum Electrodynamics

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THE METHOD of renormalized group was applied[1,2] to obtain the assymptotic expressions for the quantum electrodynamic Green function and for the vertex part. The use of the renormalized group presents also a considerable interest in the case of concrete scattering processes.

For this purpose, let us first formulate the renormalized group for the transition probabilities. This is conveniently done by using the generalization of the Feynman diagrams proposed by Abrikosov[3]. Let us consider the transition probability for the process represented by the totality of the generalized diagrams. It can be written in the form $e^n W((p_i p_j), e^2)$, where p_i and p_j are 4-vectors corresponding to the i'th and j'th external lines and n is the number of vertices with external photon lines. The transition probability represented by the totality of the generalized diagrams is equal to the matrix element of some operator between the initial and final states of this generalized diagram. Let us denote this operator by the letter M. The operator M has an arbitrary multiplicative constant which is determined by the number of free ends of the generalized diagram. Let the number of external fermion lines be $2f$. The multiplicative transformation of the operator M has then the form $e^n M \to e'^n ZM$, where $Z = Z_2^f Z_3^{n/2}$. The charge transformation law is known and, in electrodynamics, because of the Ward identity, has the form $e^n \to e'^n = Z^{\;n/2} e^n$. The multiplicative transformation of the operator M is therefore determined by only the number of external fermion lines: $M \to Z_2^f M$.

Let the lowest order perturbation approximation of the operator M be M_0. In cases of practical interest, it is sufficient to limit oneself to the consideration of only those terms of M which are propor- to M_0, *i.e.*, $M = M_0 \mathfrak{M}$, where $\mathfrak{M}$ contains all the radiative corrections proportional to M_0. Including as usual the multiplicative constant of $\mathfrak{M}$ in its argument

$$\mathfrak{M}\left((p_i p_j)/\lambda^2,\; m^2/\lambda^2,\; e^2\right),$$

one obtains a functional, and then a differential equation for $\mathfrak{M}$. These equations are analogous to the equations for the vertex part in Ref. 2. They are cumbersome and we will not cite them here. One of the possible equations has the form (with the same notation):

$$\frac{\partial}{\partial x}\ln\mathfrak{M}(x, y, \ldots, z, u, e^2) = \frac{1}{x}\left[\frac{\partial}{\partial\xi}\ln\mathfrak{M}\left(\xi, \frac{y}{x}, \ldots, \frac{z}{x}, \frac{u}{x} e^2 d(x, u, e^2)\right)\right]_{\xi=1}. \tag{1}$$

Integrating this equation, we obtain

$$\ln\left[\mathfrak{M}(x, y, \ldots, z, u, e^2)/\mathfrak{M}(x_0, y, \ldots, z, u, e^2)\right] = \int_{x_0}^{x}\frac{dt}{t}\left[\frac{\partial}{\partial\xi}\ln\mathfrak{M}\left(\xi, \frac{y}{t}, \ldots, \frac{z, u}{t, t}, e^2 d(t, u, e^2)\right)\right]_{\xi=1}$$

We will be interested in those solutions of equations of the type (1) which, for smaller e^2, coincide with the results of perturbation theory.

The obtained equations of renormalized group can be used effectively to avoid the infrared catastrophe in scattering problems. As known[4], in the usual perturbation theory, in addition to the main process, one considers also—in order to avoid the infrared catastrophe—scattering with emission of an additional long-wave quantum, the energy of which does not exceed $\Delta\varepsilon$. But then the dependence of the total cross section on $\Delta\varepsilon$ is no more true. The probability of pure elastic scattering is infinite. To obtain a physically admissible result, one has to make the summation of an infinite series of diagrams corresponding to the emission of different numbers of long wavelength photons. A direct summation of such a kind was carried out by Abrikosov[3]. The use of the renormalized group drastically simplifies the problem of such a summation, The physically admissible results then arise as the result of a decomposition that is invariant with respect to the renormalized group.

Let us consider, for instance, the scattering of an electron in an external field. Using the results of perturbation theory[4], we obtain—by solving equations of type (1)—an exponential dependence of the effective cross section on $\ln\Delta\varepsilon$. The probability of pure elastic scattering turns out to be zero, which is physically correct. An analogous situation arises when one considers the Compton effect.

The equations of type (1) for the renormalized group are also useful when one considers different high energy effects: scattering of an electron in an external field, Compton effect, and electron-electron scattering. The investigation of these phenomena by the method of summation of a series of diagrams was carried out by Abrikosov[5]. The renormalized group method permits to extend this investigation to much higher energies, because there is a simple possibility of taking vacuum polarization into account.

In conclusion, I express my deep gratitude to N. N. Bogoliubov who supervised this work, as well as to D. V. Shirkov for discussion of the results.

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Simultaneous Creation of Λ and θ-Particles

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THE ANALYSIS OF ANGULAR and energy distributions of π-mesons from the decay $\tau^+ \to 2\pi^+ + \pi^-$ leads to the conclusion that the spin and parity of the τ-meson are 0^-. Such a meson cannot decay into two π-mesons, therefore there must be two kinds of K-mesons: τ^+ and θ^+. The experimental masses and lifetimes of the K^+-mesons coincide. The equality of the masses can be explained by the hypothesis of Lee and Yang[1] according to which the Hamiltonian of strong interactions is invariant with respect to the operation C_p which changes the parity. The equality of the lifetimes of the τ and θ-mesons remains, however, unexplained.

Another possible assumption is the hypothesis that the K-meson decay interaction does not conserve parity and that there exists only one K-meson.

We want to point out that the experiments on the pair production of Λ^0, K^0 can be used to answer the question on the number of K-mesons. Steinberger *et al.*,[2-4] observed the decays of Λ^0 and θ^0 particles, $\Lambda^0 \to p + \pi^-$, $\theta^0 \to \pi^+ + \pi^-$ produced in the process

$$\pi^- + p \to \Lambda^0 + \theta^0.$$

The lifetimes of these decays are $\tau \sim 10^{-10}$ sec. They determined the probabilities R_θ and R_Λ that the observed decay $\Lambda \to p + \pi^-$ will be followed by the fast decay $\theta \to \pi^+ + \pi^-$, and vice versa. The experiment gives $R_\theta \sim R_\Lambda \sim 0.3 - 0.4$.

Let us assume that there exists only the K-meson and that parity is not conserved. In order to explain the existence of the long-lived K^0-meson observed in the experiments of Lande *et al.*,[5] one has to assume that the Hamiltonian of the decay interaction is invariant with respect to C or CI, where C is the charge conjugation and I is the inversion[6]. The K-meson is the mixture

$$K^0 = (K_s^0 + K_a^0)/\sqrt{2},$$

where the wave function of K_s^0 is symmetric and the wave function of K_a^0 is antisymettric with respect to C or CI respectively K_s^0 decaying into two π-mesons with a lifetime of 10^{-10} sec and K_a^0 being long-lived. We then get for R_θ and R_Λ: $R_\theta = 0.5\, p_\theta$, $R_\Lambda = p_\Lambda$, where p_θ, p_Λ are the probability ratio

$$p_\theta = w(\theta^0 \to \pi^+ + \pi^-)/[w(\theta^0 \to \pi^+ + \pi^-) + w(\theta^0 \to \pi^0 + \pi^0)] < 1,$$

$$p_\Lambda = w(\Lambda \to p + \pi^-)/[w(\Lambda \to p + \pi^-) + w(\Lambda \to n + \pi^0)] < 1.$$

Comparing with experiment: $p_\theta = 0.6 - 0.8$, $p_\Lambda = 0.3 - 0.4$.

Experimentally, only the order of magnitude of these quantities is known at the present time. Osher and Mojer give p_Λ and $p_\theta \sim 0.5$ which, in view of the inaccuracy of these values, should be considered as not being in contradiction with the hypothesis of a single K-meson.

In the Lee and Yang scheme the τ and θ-mesons are produced in equal numbers and, taking into account that θ^0 is a mixture of symmetric and assymmetric components, we get

$$R_\theta = 0{,}25\, p_\theta < 0{,}25; \; R_\Lambda = p_\Lambda.$$

We come to a contradiction, as the experiment gives $R_\theta \sim 0.3 - 0.4$.

The contradiction can be avoided by taking into account the fact that, in the Lee and Yang scheme, even and odd Λ-particles should exist simultaneously with even and odd K-mesons, and by assuming that one of the Λ-particles is long-lived with a lifetime of $10^{-8} - 10^{-9}$ and that the θ-meson is produced only (or most of the time) with a short-lived Λ-particle. Such an assumption means that the θ and τ do not transform one into the other in strong interactions.

We then get, as in the case of a single K-meson:

$$R_\theta \sim 0{,}5\, p_\theta, \; R_\Lambda \sim p_\Lambda.$$