from Eq. (4) it follows that for a single particle

$$|S| \leqslant 2, \tag{5}$$

so that according to the d'Espagnat-Prentki theory the strangeness cannot have an absolute value greater than 2.

Relation (2) is of interest in connection with the slow secondary particles recently observed in K^--meson decay[4-6]. The analysis of these events shows quite definitely that they are the decays of some kind of negative "superheavy" mesons or hyperons, whose mass is greater than $M(K) \approx 965\, m_e$ and $M(\Xi) \approx 2586\, m_e$.

If we do not consider isotopic multiplets containing particles with charges greater than unity, then by using Eq. (1) it is not difficult to show that the only negative metastable particle heavier than the mesons and hyperons known at present can be the following isotopic singlets: the meson ω^- (with $S = -2$) and the hyperon Ω with $S = -3$). Expression (5) excludes the latter possibility.

Thus according to the d'Espagnat-Prentki theory, the observed[4-6] K^--meson decays may be considered the decays of "superheavy" ω^--mesons with strangeness $S = -2$. Applying the selection rule $\Delta S = \pm 1$, suggested by Gell-Mann for slow processes[1], one may suppose that in the decay of the ω^--meson, there appears in addition to the K^--meson a particle with strangeness $S = -1$. If the existence of negative metastable hyperons heavier than Ξ is nevertheless proved, this will mean either that the d'Espagnat-Prentki[2] interpretation of Gell-Mann's model is invalid, or that this hyperon belongs to an isotopic multiplet containing particles with charge greater than 1.

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The Interaction Cross Section of π-Mesons and Nucleons at High Energies

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IT IS WELL KNOWN that at high energies the interaction cross section of π-mesons with nucleons approaches a constant limit, which is a result of the finite dimensions of the nucleon (neglecting the Coulomb interaction). In order to calculate this limit let us make use of dispersion relations which connect the imaginary and real parts of the scattered amplitude for zero scattering angle. For instance, for scattering of negative mesons by protons we have[1]

$$\begin{aligned} \operatorname{Im} f_-(\omega) &= \frac{1}{2} \operatorname{Im} f_-(\mu)\left(1+\frac{\omega}{\mu}\right)+\frac{1}{2} \operatorname{Im} f_+(\mu)\left(\frac{\omega}{\mu}-1\right) \\ &+\frac{\omega^2-\mu^2}{\pi} \mathrm{P} \int_0^\infty \frac{d\omega'}{\omega'^2-\mu^2}\left[\frac{\operatorname{Re} f_+(\omega')}{\omega'+\omega}-\frac{\operatorname{Re} f_-(\omega')}{\omega'-\omega}\right] \\ &-\pi \sum_k \delta(\omega_k-\omega) \operatorname{Res} f_-(\omega_k); \end{aligned} \tag{1}$$

here we have accounted for the fact that the amplitude may have poles at the points ω_k (we have made use of the fact that the residues Res f_- are real). The symbol P indicates that we take the principal part of the integral not only at those points where the denominator vanishes but at all poles of the functions $f_\pm$. Letting ω approach infinity in Eq. (1), we obtain

$$\sigma_\infty = 4\mathrm{P} \int_0^\infty \frac{d\omega}{\omega^2-\mu^2} \operatorname{Re}[f_+(\omega)+f_-(\omega)-f_+(\mu)-f_-(\mu)]. \tag{2}$$

Eq. (2) is symmetric with respect to f_+ and f_-, so that in the limit the cross sections for positive and negative mesons are equal[2]. In deriving Eq. (2), we have used the well known relation $\sigma = (4\pi/\omega)\operatorname{Im} f(\omega)$, as well as the condition $\operatorname{Im} f_\pm(\mu) = 0$. The term $\operatorname{Re}[f_+(\mu) + f_-(\mu)]$ is added for convenience (this clearly does not destroy the equality since $\mathrm{P}\int_0^\infty d\omega/(\omega^2-\mu^2) = 0$).

Let us break up the integral in Eq. (2) into two integrals over the regions $0 \le \omega \le \mu$ and $\mu \le \omega \le \infty$. In the first region we make use of the relation[1]

$$\mathrm{Re}\,[f_+(\omega)+f_-(\omega)-f_+(\mu)-f_-(\mu)]$$
$$=\frac{\omega^2-\mu^2}{2\pi^2}\int_0^\infty\frac{dk'}{k'^2-k^2}[\sigma_+(k')+\sigma_-(k')] \qquad (3)$$
$$+\frac{2f^2}{M}\frac{\omega^2-\mu^2}{\omega^2-(\mu^2/2M)^2}.$$

Using the experimental values of the phase shifts[3], we obtain

$$\mathrm{Re}\,[f_+(\mu)+f_-(\mu)]=-0{,}04\lambda\!\!\!^{-}(1+\mu/M),$$

where $\lambda\!\!\!^{-}$ is the Compton wavelength of the meson, and μ and M are the meson and nucleon masses, respectively. Inserting (3) into (2), after some simple operations we obtain

$$\sigma_\infty=-1{,}5+I_0+I_1; \qquad (4)$$
$$I_0=\frac{1}{\pi^2}\int_0^\infty\frac{dE}{\sqrt{E(E+2\mu)}}\ln\frac{E+2\mu}{E}[\sigma_+(E)+\sigma_-(E)], \qquad (5)$$
$$I_1=4\lambda\!\!\!^{-2}\int_0^\infty\frac{dx}{x(x+2)}\left[\frac{1}{\lambda\!\!\!^{-}}\mathrm{Re}\,(f_+(x)+f_-(x))+0{,}04\,(1+\mu/M)\right],\quad x=\frac{E}{\mu} \qquad (6)$$

In order to calculate the integrals we make use of the experimental[4] values for $\sigma_\pm(E)$ and $\mathrm{Re}\,f_\pm(x)$. We obtain the following values: I_0 = 20 mb, I_1 = 11.5 mb. Inserting these values into (4), we obtain σ_∞ = 30 mb, which is in agreement with the experimental data[4],. The accuracy of σ_∞ is limited by the accuracy of the experimental data for $\sigma_\pm$ and $\mathrm{Re}\,f_\pm$.

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On the Second Approximation in the Problem of Slow Neutron Scattering by Bound Protons

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THE PROBLEM of scattering slow neutrons by protons bound in a molecule has been treated in the first approximation by Fermi[1]. In several other works[2-4] evaluations of the further approximations have been made. Of particular interest is the variational method developed by Schwinger and Lippmann[3], with the aid of which Lippmann calculated neutron scattering by a hydrogen molecule in the second approximation, and verified the results of Breit and Zilsel[2] who used a different model for the molecule. Soon, however, Ekstein's work[4] appeared, where it was proved that the second approximation of the Schwinger-Lippmann method for neutron scattering by bound protons always diverges, except if the proton is bound to an infinitely heavy nucleus. Ekstein commented, "whether the finite result found by Lippmann is due to the special choice of wave functions . . . or the limiting process used in the evaluation of the integral, remains undecided."

In this note we investigate the question of convergence of the second approximation in the problem of slow neutron ($E\approx 0$) scattering by a proton bound in a molecule of mass M.

In the Schwinger-Lippman method the scattering matrix T_{ba} for zero-energy neutrons from state a to state b is given in the second approximation by the equation

$$T_{ba}=-(4\pi\hbar^2a/m)\left\{\int\chi_b^*(\mathbf{r})\,\chi_a(\mathbf{r})\,d\mathbf{r}+aI\right\},$$
$$I\equiv\int\chi_b^*(\mathbf{r})\left[\sum_\gamma\chi_\gamma^*(\mathbf{r}')\,\chi_\gamma(\mathbf{r})\,\frac{(2\mu/m)\exp(ik_\gamma a\,|\mathbf{r}-\mathbf{r}'|)-1}{a\,|\mathbf{r}-\mathbf{r}'|}\right]\chi_a(\mathbf{r}')\,d\mathbf{r}d\mathbf{r}', \qquad (1)$$