The latest measurements on copper[2] show that the change in the modulus of elasticity on irradiation does not amount to more than 1 – 2%.

We studied the effect of fast neutron irradiation in a nuclear reactor on the compressibility of aluminum and magnesium. Since this quality is directly connected with the elasticity and shear moduli, and since no change in these moduli was found in the materials investigated so far, it was to be expected that the compressibility, too, would not change appreciably, under the influence of neutron irradiation. Samples in the form of cylinders 6 mm in diameter and 6 mm high, were prepared from electrolytic materials of engineering purity. The compressibility measurement was made with apparatus developed in the ultra-high-pressure physics laboratory for measuring volume compressibility by the piston displacement method, which apparatus will be described in another communication. The effect of friction was allowed for by taking the piston displacement *vs.* pressure curves on both rising and falling pressure and plotting the mean curve. The measurements were carried out after first subjecting the sample to a maximum pressure of about 15,000 kg/cm^2.

The samples were irradiated in a nuclear reactor. The total neutron irradiation was 1.07×10^{19} neutrons/cm^2. After irradiation, the compressibility was measured under the same conditions as before irradiation, although, on account of the residual activity of the samples, the measurements could not be carried out until 72 hours after the irradiation.

The measurements showed that for magnesium and aluminium the piston displacement *vs.* pressure curves coincide completely before and after irradiation, *i.e.*, irradiation has no effect on the compressibility, to the accuracy of our measurements, about 5%. Since the experiments were carried out at ordinary temperature, the distortions produced by the irradiation may have been partly wiped out. Possibly at lower temperatures, with a preliminary annealing of the samples, the effect of irradiation would be considerably greater.

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Quadrupole Moments and Zero-Point Surface Vibrations of Axially Symmetrical Nuclei

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FOR SIMPLICITY WE SHALL consider even-even nuclei. In the collective model of the nucleus it is assumed that nucleons outside of filled shells can be described by a single-particle approximation and that the nucleons in the nuclear core of completely filled shells have only collective properties. As collective coordinates we shall use the three Euler angles which describe the orientation of the nucleus in space and β and γ[1], which define the deviation of the nucleus from a perfectly spherical shape.

In an adiabatic approximation we can regard the outer nucleons as moving in the field of a nuclear core of fixed shape. The interaction energy of the outer nucleons with the core, averaged over their states of motion is $\langle H_{\text{int}} \rangle = A\beta \cos\gamma$, which will depend on the coordinates β and γ and will act as additional energy to determine the equilibrium shape of the nucleus. A depends on the number of outer nucleons and their quantum numbers and can be either positive or negative.

In the collective model[1] this energy is defined by

$$E = \frac{B}{2}(\dot{\beta}^2 + \beta^2\dot{\gamma}^2) + \frac{C}{2}\beta^2 + \sum_{\lambda=1}^{3} \frac{M_\lambda^2}{8B\beta^2 \sin^2(\gamma - 2\pi\lambda/3)} + A\beta\cos\gamma + E_p. \quad (1)$$

For a given value of β the potential energy in (1) possesses a minimum at $\gamma = 0$ and π and becomes infinite for $\gamma = \pm\,\pi/3,\ \pm\,2\pi/3$.

Nuclei are evidently very stable with respect to variation of γ around the two possible equilibrium values 0 and π, which correspond to axial symmetry. Therefore we shall hereinafter consider only the vibrations which are associated with a variation of of β for the fixed values $\gamma = 0,\ \pi$.

For $\gamma = 0$ Eq. (1) becomes

$$E - E_p + \frac{C}{2}\beta_0^2 = \frac{B}{2}\dot{\beta}^2 + \frac{C}{2}(\beta - \beta_0)^2 + \frac{M^2}{6B\beta^2}, \quad (2)$$

where $\beta_0 = -A/C$. The corresponding Schroedinger equation for states with definite total angular momentum is for the S state ($j = 0$)

$$\left\{-\frac{\hbar^2}{2B\beta^2}\frac{\partial}{\partial\beta}\left(\beta^2\frac{\partial}{\partial\beta}\right)+\frac{C}{2}\left(\beta-\beta_0\right)^2-\varepsilon\right\}\Phi(\beta)=0. \tag{3}$$

We put

$$\Phi(\beta)=\frac{1}{\beta}v(\zeta)\exp\left\{-\frac{\zeta^2}{2}\right\},\ \zeta=\delta\frac{\beta-\beta_0}{\beta_0},$$

$$\delta=\beta_0\left(\frac{BC}{\hbar^2}\right)^{1/4},\quad \omega=\sqrt{\frac{B}{C}}.$$

Then $v(\zeta)$ will satisfy the equation

$$v''(\zeta)-2\zeta v'(\zeta)+2\nu v(\zeta)=0,\quad -\delta\leqslant\zeta<\infty,$$

$$\nu=\frac{\varepsilon}{\hbar\omega}-{}^1/_2. \tag{4}$$

The function $v(\zeta)$ must satisfy the boundary conditions

$$e^{-\zeta^2/2}v(\zeta)\to 0,\ \text{ if }\ \zeta\to\infty, \tag{5}$$

$$v(-\delta)=0. \tag{6}$$

For nonintegral ν the general solution of (4) can be expressed in terms of Hermite functions[2]

$$H_\nu(\zeta)\equiv\{2\Gamma(-\nu)\}^{-1}\sum_{k=0}^{\infty}(-1)^k\Gamma\left(\frac{k-\nu}{2}\right)(k!)^{-1}(2\zeta)^k$$

by the relation

$$v_\nu(\zeta)=aH_\nu(\zeta)+bH_\nu(-\zeta).$$

The asymptotic behavior of the Hermite functions for large ζ gives $b = 0$ according to the boundary condition (5). Then boundary condition (6) leads to the transcendental equation $H_\nu(-\delta) = 0$, which determines the eigenvalues of ν and thus the energy levels ε_i of the nuclear S state for $\gamma = 0$. The energy ε_i corresponds to the wave function

$$u_i(\beta)=aH_{\nu_i}(\zeta)e^{\zeta^2/2}. \tag{7}$$

When $\gamma = \pi$ Eq. (1) becomes

$$E-E_p+\frac{C}{2}\beta_0^2=\frac{B}{2}\dot\beta^2+\frac{C}{2}(\beta-\beta_0)^2+\frac{M}{6B\beta^2},$$

for $0\leqslant\beta<\infty$. It is easily seen that the transformation $\beta\to-\beta$ can result in (4) without changing any of the preceding specifications only in the interval $-\infty<\zeta\leqslant-\delta$. From the requirement that the boundary conditions be satisfied at the ends of this interval it results that

$$v_\nu(\zeta)=bH_\nu(-\zeta)$$

and the transcendental equation which determines the eigenvalue of γ when $\gamma = \pi$ becomes $H_\nu(+\delta) = 0$.

The table contains the energy values of the zero-point surface vibrations of a nucleus in an S state when the states of the outer nucleons correspond to positive β_0 and consequently $\delta > 0$ for $\gamma = 0$ and π.

δ	0	0.1	0.2	0.3	0.5	0.8	1.0	2.0
$\frac{\varepsilon}{\hbar\omega}$ ($\gamma=0$)	1.5	1.39	1.29	1.21	1.03	0.81	0.72	0.50
$\frac{\varepsilon}{\hbar\omega}$ ($\gamma=\pi$)	1.5	1.61	1.73	1.84	2.16	2.66	3.02	6.5

A higher energy level is represented by $\gamma = \pi$. As δ increases the energy of the zero-point vibrations which corresponds to $\gamma = 0$ is reduced and the energy of the vibrations for $\gamma = \pi$ is increased.

The operator of the intrinsic electric quadrupole moment of the nucleus, which results from the collective degrees of freedom, is

$$\hat{Q}_0=\frac{3ZR}{\sqrt{5\pi}}\beta\cos\gamma.$$

In the ground state of an even-even nucleus $\gamma = 0$, the wave function is given by (7) and $\langle\beta\rangle > \beta_0$. The difference between $\langle\beta\rangle$ and β_0 is reduced as δ increases, but for small δ it can be of considerable magnitude*.

$\gamma = \pi$ corresponds to a special kind of excited states for which $\langle\beta\cos\pi\rangle < 0$. $\langle\beta\cos\pi\rangle$ is considerably smaller than β_0 in absolute value and tends to vanish as δ increases. Thus when the nucleus

*Our adiabatic approximation will be valid only when β_0 exceeds a certain critical value.

makes a transition from the ground state with $\gamma = 0$ to the S state which corresponds to $\gamma = \pi$ the quadrupole moment is changed in sign and magnitude.

In nuclei where the states of the outer nucleons correspond to negative values of β_0 the lowest energy levels occur for $\gamma = \pi$. However it appears from experiment that there are no nuclei with large negative values of β_0.

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Reactions Produced by μ-Mesons in Hydrogen

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AT PRESENT THERE IS EVIDENCE that at Berkeley[1] in a bubble chamber, filled with liquid hydrogen having a varying deuterium content, there could be observed a nuclear reaction catalyzed by μ-mesons. The possibility of such a type of reaction was first pointed out by Frank[2] in connection with the analysis of π-μ disintegrations in emulsions. This process was investigated in liquid deuterium by both of the authors of this article, independently of one another[3,4].

The presence of a μ-meson changes the form of the potential barrier which previously prevented nuclear reactions among slow proton and deuteron nuclei, increasing sharply the penetrability of the barrier and making possible the reactions

$$p + \mathrm{D} = \mathrm{He}^3,\quad \mathrm{D} + \mathrm{D} = \mathrm{He}^3 + n,\quad \mathrm{D} + \mathrm{D} = \mathrm{T} + p.$$

In the presence of tritium there are also possible the reactions

$$\mathrm{D} + \mathrm{T} = \mathrm{He}^4 + n,\quad \mathrm{T} + \mathrm{T} = \mathrm{He}^4 + 2n,\quad p + \mathrm{T} = \mathrm{He}^4.$$

The reaction $p + p = \mathrm{D} + e^+ + \nu$, catalyzed by mesons, is practically impossible, since in addition to the barrier there is also the factor of a low probability for the beta process.

It has been predicted[4] that the probability of the reaction in flight is low, the production of mesomolecules practically always leads to nuclear reactions, the rate of the process is determined by the production of mesomolecules, and the probability of mesomolecule formation during the lifetime of a meson may amount to several hundredths or even tenths, depending on the arrangement of the mesomolecule levels.

The experimental data of Alvarez[1] shows that in natural hydrogen (deuterium content ratio 1 : 7000), the reaction $p + d = \mathrm{He}^3$ occurs on the average once for each 150 mesons. If the deuterium ratio is 1 : 300, the reaction occurs once per 40 mesons, and if the ratio is 1 : 20, the reaction occurs once per 33 mesons. Furthermore, the energy of the resulting He^3 (5.4 Mev) is carried away by the μ-meson, so that monochromatic μ-mesons are observed while the reaction is taking place. The relatively high probability found for the reaction in the natural mixture is explained[1] by the transfer of the meson from the proton to the deuteron (charge exchange): $p\mu + d = p + d\mu$. Due to the difference in reduced mass, the energy of the Dμ bond (2655 ev) is greater by $\Delta E = 135$ ev than the energy of the $p\mu$ bond. Therefore the charge-exchange process appears to be irreversible under the experimental conditions.

We shall give a rough estimate of the probability of the transition. If ΔE is equal to zero, the cross section should be of the order of πa^2, where a is the radius of the Bohr orbit of the mesoatom, 2.5×10^{-11} cm.

Indeed, if the masses of the two nuclei are equal, with $\Delta E = 0$, the states of the systems $\sum_{g}^{+}$ and $\sum_{u}^{-}$ appear to be proper, and the cross section for charge exchange can be expressed by the scattering lengths a_g and a_u of these states in a continuous spectrum: $\sigma = \pi(a_g - a_u)^2$. When $\Delta E \neq 0$, but is still small with respect to the molecular dissociation energy, then

$$\sigma = \pi(a_g - a_u)^2 v_f/v_i,$$

where v_i is the velocity before collision and v_f is the velocity after collision.

In actual fact, ΔE is of the same order as the dissociation energy, so that the formula is corrected at least in order of magnitude. If v_i is small $\sigma \sim 1/v$ $\sigma \sim 1/v_i$. It follows that in order of magnitude,

$$\sigma \approx \pi a^2 v_*/v_i,$$