bination of the diffusion and thermal conductivity coefficients and the thermal diffusion ratio.

The results obtained in the present work are correct for the temperature region $T \leq 1.6 - 1.8°$ K, in which the rotons may be regarded as constituting an ideal gas. At low temperatures the applicability of the theory is limited by the mean free paths of the excitations associated with the transport phenomena. As usual, the mean free paths must be much shorter than the characteristic dimensions of the containers. For high impurity concentrations, for which the mean free paths are short down to the lowest temperatures, the theory is applicable down to the temperatures at which the Fermi degeneracy of the impurities becomes significant.

In conclusion, the authors wish to express their deep indebtedness to Academician L. D. Landau for his helpful discussions.

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Translated by S. D. Elliott
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SOVIET PHYSICS JETP VOLUME 5, NUMBER 5 DECEMBER, 1957

Shock Waves of Large Amplitude in Air

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(Submitted to JETP editor June 10, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 1126-1135 (May, 1957)

The state of air compressed by strong shock waves is examined by taking dissociation and ionization into account. Approximate expressions are given for the density and temperature in this region. The radiation from the front of the shock wave is considered. With increasing shock-wave amplitude, the observed surface temperature passes through a maximum, owing to the formation of an opaque layer of air, preheated by the radiation, ahead of the front of the wave. A proof is given of the non-existence of a continuous solution and of the unavoidability of discontinuities in the velocity, density, and temperature in a strong shock wave with radiative heat exchange. The wave structure is investigated in a strongly-ionized gas with allowances for the slow energy transfer between the ions and electrons.

PHENOMENA OCCURRING in strong shock waves are very interesting from many points of view. In practice we encounter shock waves during explosions and during the motion of bodies at supersonic speeds. The principal interest lies in the peculiarities of the compression in the shock wave: the compression occurs rather rapidly, is accompanied by a sharp increase in the gas entropy, and is irreversible. Gas compression in a shock wave produces high temperatures, considerably higher than adiabatic compression to the same pressure.

It was already noted by Muraour[1] that the glow observed in an explosion is neither the chemiluminescence reaction of the decomposition of the exploding substance nor thermal glow of the explosion products. What really glows is the air, compressed by the explosion shock wave around the explosion substance.

O. I. Leipunskii and the author[2] observed rather high temperatures in a shock wave produced by the motion of a bullet in mercury vapor. Extensive investigations of strong shock waves are being carried at the present time in the U.S. with the aid of a so-called shock tube[3], in which a diaphragm separates the high-pressure gas from the low-pressure gas, and the wave is produced by rupture of the diaphragm* or with the aid of explosive substances[4].

The high temperatures of the strong shock waves are accompanied by new physical phenomena, such as dissociation, ionization, and emission of light. The influence of these phenomena on the properties of air compressed by a shock wave and on the structure of the wave is the subject of this article.

Ionization and radiation in a shock wave was considered in detail in an article by Prokof'ev[7], using compressed monatomic hydrogen as an example. Let us note that we are not in agreement with Prokof'ev's conclusions concerning the structure of the wave (see Sec. 5).

1. DISSOCIATION AND IONIZATION IN SHOCK WAVE

The temperatures reached at shock-wave pressures of 200–600 atmos are 5,000–10,000°, at which strong dissociation of the oxygen and the nitrogen molecules takes place[8,9]. The compressed air becomes monatomic. Because of the large amount of energy required to break up the molecule, the gas pressure is substantially less than 2/3 the energy density (subtracted from the molecule energy at 0°K). The density of the air compressed in the shock-wave therefore does not diminish to $4\rho_0$, but, to the contrary, increases to 9–12 ρ_0 at the above pressure and temperature ranges.

With increasing pressure, the dissociation energy, which is a constant term, becomes less and less important. However, noticeable ionization of the atoms occurs even before the dissociation is complete†. The pressure range, in which the density is on the order of 10 ρ_0, is therefore quite extensive. Substituting into the Sach equation the electron density corresponding to almost complete ionization of air compressed ten times atmospheric density, we obtain the following numerical expression for the average number of K-electrons (electrons of the innermost shell of the nucleus) in equilibrium at a temperature T:

$$n_K = 2/[1 + 60\,T^{3/2} e^{-E/T}], \qquad (1)$$

where T and E are given in electron volts, E being the binding energy, which is 650 ev for nitrogen and 850 ev for oxygen. The detachment of the K-electrons occurs not at $T \sim E$, but at $T \sim E/\ln(60\,T^{3/2}) \approx E/10$. The energy consumed in the ionization therefore constitutes a large component in the energy balance of the shock wave.

The density reaches its limiting value $4\rho_0$ quite slowly, in accordance with the equation

$$\rho = 4\rho_0/(1 - 2Q\rho_0/p), \qquad (2)$$

where Q is the total ionization and dissociation energy, $2Q\rho_0 = 4 \times 10^{11}$ dyne/cm², so that, for example, $\rho = 6\rho_0$ is reached at $p = 10^{12}$ (10^6 atmos), when $T = 230$ ev $= 2.5 \times 10^6$ degrees.

One can roughly estimate that a ten-fold compression is reached in the shock-wave over the entire range T = 1–100 ev of interest to us; the speed of the shock-wave is then (D–cm/sec, p–dyne/cm²)

$$D = 28\sqrt{p} \qquad (3)$$

and the temperature (ev) is

$$T = 10^{-6.75}\, p^{3/4}. \qquad (4)$$

A 100 ev temperature is reached at approximately 4.5×10^{11} pressure, after which the temperature increases linearly with the pressure. The above equations and relationships are obtained by interpolating the results of actual numerical calculations.

2. RADIATION OF SHOCK-WAVE

The ionization of the gas results in a continuous spectrum of emission and absorption of light. A layer of compressed air of sufficient thickness becomes opaque and radiates as a black body. For a first estimate of the role of radiation, let us compare the work required to compress the air with the radiation

* Gershanik, Rozlovskii, and the author[5] studied the chemical reactions using the compression shock wave produced in a shock tube setup, while Shluapintokh and the author[6] studied the compression in the shock wave of a bullet.

† Ionization of hydrogen and argon was treated by Prokof'ev.[7]

energy, both quantities being taken per square centimeter of front surface and per second.

The work of compression, or more accurately that portion of the work that is converted into heat energy, is $W = pu/2 = 13\,p^{3/2}$ (all CGS quantities). If T is given in ev, the black-body radiation is $S = 10^{12}T^4$ in accordance with the Stefan-Boltzmann law.

Substituting the expression for T we obtain the radiation flux

$$S = 10^{-15}\,p^3. \qquad (5)$$

The ratio of the radiation flux to the compression work is $S/w = 8 \times 10^{-17}\,p^{3/2}$. This ratio becomes unity at $p = 5.5 \times 10^{10}$ and $T = 20$ ev. A conclusion suggests itself that the energy carried away by radiation in strong shock-waves can play a substantial role. An elementary calculation for the case of a strong point-source explosion, considered by Sedov[10], leads to a divergent integral. In Sedov's solution the radius of the spherical wave is $r \sim t^{2/5}$, and the pressure on the front is $p \sim r^{-3} \sim t^{-6/5}$, from which it follows that the total energy radiated

$$\int Sr^2\,dt \sim \int p^3 r^2\,dt \sim \int t^{-11/5}\,dt \qquad (6)$$

diverges at small values of t, *i.e.*, in the initial stage, when the temperature is high, if expression (5) is used for S.

It must be borne in mind that actually the air ahead of the wave front is transparent only to visible light and to the adjacent portion of the spectrum. Quanta with energies 7–10 ev and higher experience very strong absorption even in cold air. Consequently, the radiation flux of the shock-wave increases as K^4 only as long as the temperature does not exceed 2–3 ev. At higher temperatures, the Wien maximum occurs already in the region of the spectrum absorbed by the cold air ahead of the front.

If the wave front has a high temperature (above 8–10 ev) the radiation energy in the transparent region for cold air increases as T, according to the Rayleigh-Jeans law. The ratio of the thermal losses of the shock-wave front to the work of compression actually passes through a maximum not exceeding 1%.

3. STRUCTURE OF WAVE WITH ALLOWANCE FOR RADIATION

The energy emitted by the wave front and absorbed by the air ahead of the front does not enter into the equation for the final state of the air compressed by the shock wave. However, the radiation heat exchange exerts a substantial effect on the structure of the wave.

The structure of the wave with allowance for radiation was considered in detail by Prokof'ev[7] who derived equations, determined transparency coefficients (using atomic hydrogen as an example), and examined the structure of the wave in monatomic hydrogen and argon. His treatment of the structure of the wave contains disagreements with our work, which will be discussed in detail below in Sec. 5. We shall give here qualitatively, without equations, our ideas concerning the structure of the wave.

The radiation emitted by the wave front heats the air ahead of the front. This heating is accompanied by increased pressure. The shock wave in the narrow sense of the word—the discontinuity in pressure and density—now propagates in heated gas.

As is known, the compression in a shock wave occurs in a zone whose thickness is on the order of the mean free path of the molecules and atoms. A layer of gas of this thickness is transparent at all temperatures. Thus, the compression follows the classical Hugoniot adiabatic line. Radiant cooling of the gas and asymptotic assumption of the final state occur only behind the compression, at a distance on the order of the path length of the radiation.

The structure of the front is shown schematically in Fig. 1, which shows the temperature distribution, and in Fig. 2, where it is shown in the (p, v) plane. The indices 1, A, B, and 2 of Figs. 1 and 2 pertain to identical states. From the condition that the entire pattern of Fig. 1 has a stationary propagation with equal velocity it follows that all points of Fig. 2 lie on a straight line. What is remarkable in Fig. 1 is the temperature peak B. At a given wave velocity, the compression of the heated gas A leads to a state B, in which the temperature is higher than in the final state 2, which is reached by compressing cold gas 1. An entirely different diagram is obtained if the radiant heat exchange is set approximately equal to the heat conduction (Fig. 3), and is an isothermal density jump appears at the origin. Actually, the narrow temperature peak, drawn in Fig. 3 dotted from the point $T3$, should appear also at high temperatures, when the heat conduction approximation would seem permissible. The presence of a temperature peak at Figs. 1 and 3 is closely related to the finite range of the radiation and to the fact that in the case of abrupt changes

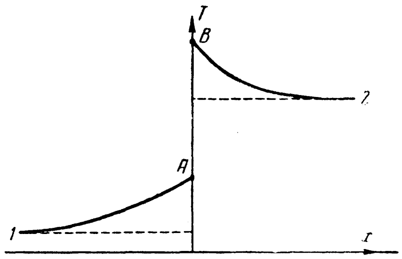

FIG. 1

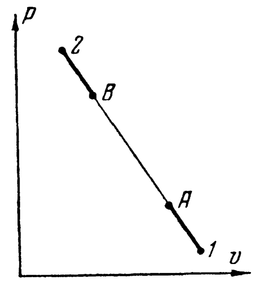

FIG. 2

of state, at distances less than the free path length, the differential equation of heat conductivity is not applicable. This will be treated in greater detail at the end of Sec. 5.

When we estimated the heat flux in the end of Sec. 2 we remarked on the absorption of ultraviolet by the cold air ahead of the front. In the visible portion of the spectrum, the cold air is transparent and the temperature of the wave can therefore be determined optically. However, as the temperature rises, the heating of the air ahead of the wave front, according to Fig. 1, causes the boundary of air absorption to shift towards the long-wave region; when the air ahead of the front becomes opaque, the visible temperature diminishes.

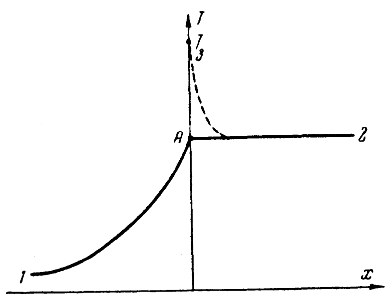

FIG. 3

Thus, in a monotonic increase of the shock wave temperature, the optically-measured temperature first coincides with the true temperature of the front and increases together with it, and then begins to lag the true temperature and passes through a maximum. These phenomena were observed by Model' in investigations[11] carried out in connection with our ideas. Similar phenomena were observed earlier by Vul'fson and his associates[14]. A quantitative examination of the theory of air radiation was made by Raizer[12].

4. ELECTRONIC HEAT CONDUCTION

Peculiar effects occur in strong shock waves with high ionization when allowances are made for the great difference between the electron and ion masses.

On one hand, the small mass of the electron means a high velocity of sound in the electron gas taken by itself. The shock wave is subsonic and slow relative to the electron gas. The electron and ion gases are bound to each other by electrostatic forces. The heat conduction of the electron gas is large, but the energy exchange between the electrons and ions is slow. If we specify the medium to be electrically neutral, we are left with finding density distribution at two different temperatures, T_i of the ions and T_e of the electrons.

Assuming the energy exchange between the electrons and ions to be the slowest process, we arrive in the limit at a following situation: at first the ions in the front of the shock wave heat up rapidly, and the electrons remain cold. The temperature is then gradually equalized in the compressed air—the ions become cooler and the electrons hotter. The ion temperature T_i is at first higher than the final temperature assumed by the ions and electrons after equalization (without heat losses).

If the energy exchange between electrons and ions is faster, we obtain the temperature distribution shown in Fig. 4. The front of the shock wave, *i.e.*, the density discontinuity, is at the origin. The electron temperature T_e is shown dotted, the ion temperature T_i is shown solid, with a discontinuity at the ordinate.

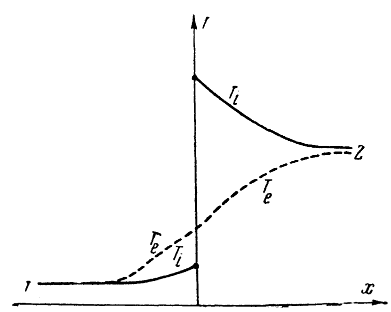

FIG. 4

The heating ahead of the front is due to the energy transferred by the electron heat conduction, while the electrons in turn heat the gases, so that $T_i < T_e$ at $x < 0$. The density discontinuity is accompanied by a sharp jump in the ion temperature T_i. The energy equation (in which the flow of heat transferred by the electron heat conduction must be allowed for) gives the connection between the sums of the electron and ion energies before and after the explosion. To make the system of equations complete, it is necessary to make use of the fact that the compression in a shock wave is slow relative to the electron gas.

Thanks to the electron heat conduction, T_e is not discontinuous on the density discontinuity surface. The work of compression of the electrons at the density discontinuity is given by the known expression $T_e \ln(\rho_2/\rho_1)$ (for one electron), and the equation of energy for the electron gas alone remains continuous

$$\lambda \left.\frac{dT_e}{dx}\right|_{\text{bef. raref.}} = \lambda \left.\frac{dT_e}{dx}\right|_{\text{aft. raref.}} + n_0 D T_e \ln \frac{\rho_2}{\rho_1}. \quad (7)$$

Here λ is the heat conduction, $\lambda dT_e/dx$ the heat flux, n_0 the electron density in the initial matter, and D the velocity of the wave. The energy equation for the ions is obtained by subtracting the equation for the electrons from the complete energy-balance equation. This results in a complete system of equations.

The substance remains electrically neutral because of electric fields that are particularly strong near the discontinuity. The potential difference is on the order of the temperature in the wave (T_e); a characteristic length is a quantity well known in electrochemistry, namely the thickness of the double layer, on the order of $\sqrt{T_e/n_0 e^2}$, from which we estimate the field to be $\sqrt{T_e n_0 e^2}$, where e is the electron charge. If $T_e = 300$ ev and $n_0 \sim 4 \times 10^{20}$ (ionized air), we obtain a thickness of 2×10^{-6} cm, and a field on the order of $1-2 \times 10^8$ v/cm.

The role of electrons pertains only to the extreme limiting case of shock waves of such intensity, that the ionization energy can be considered small. Only in this case is it possible to disregard the ionization process itself and the air prior to compression—evidently not ionized—can be considered as a mixture of cold electrons and ions. In this work we do not consider the considerably more complicated case of actual shock waves, in which the temperature is of the same order or lower than the ionization energy (see, for example, Ref. 3).

5. RIGOROUS THEORY OF WAVE STRUCTURE WITH ALLOWANCE FOR RADIATION

The previously cited article by Prokof'ev contains rigorous equations for the change in state of the substance in the wave front, and we shall make use of these equations. Let us remark immediately that the conclusions we reached by analysis of this equation differ from Prokof'ev's conclusions when it comes to the most important qualitative aspect of the problem. Prokof'ev proposes that in the presence of radiation the state of the gas varies continuously, whereas in our opinion a strong wave with radiation contains discontinuities in density, velocity, and temperature (see Figs. 1–4).

Using Prokof'ev's premises and notations, and using a coordinate system in which the wave is at rest, we have

$$\rho u = m, \quad p + \rho u^2 = p + mu = n, \quad (8)$$

$$m(\varepsilon_T + {}^1\!/_2 u^2) + pu + H = \quad (9)$$
$$m\varepsilon_T + nu - {}^1\!/_2 mu^2 + H = l,$$

where m, n, and l are constants, representing the flow of material, momentum, and energy, respectively. In the last equation ε_T is the internal (thermal) energy per unit mass of substance—a known single-valued function of p and ρ. H is the energy flux carried by the radiation. With the aid of the first two equations of (8) any two of the three quantities (u, p, ρ) can be expressed algebraically and uniquely in terms of the third.

In the absence of viscosity, all the intermediate states are represented as points on a straight line in the plane p, $v = 1/\rho$, the same as in the case of the detonation waves[13].

Prokof'ev chooses as his variable the dimensionless velocity $\tilde{u} = u/u_1$. It is evident that $\tilde{u} = v/v_1$.

The quantity H, *i.e.*, the thermal flux needed to effect any one of the intermediate states, can be expressed in terms of $\tilde{u}$ through the third equation of (9). The thermodynamic equation of state gives the temperature T as a function of $\tilde{u}$. The typical form of the dependences $H(\tilde{u})$ and $T(\tilde{u})$, shown in Fig. 5, is based on Fig. 11 of Prokof'ev's article.

The fact that $H \leq 0$ when $\tilde{u}_2 < \tilde{u}_1 < 1$ and $H = 0$ when $\tilde{u} = 1$ and $\tilde{u}_1 = \tilde{u}_2$ is a general property of shock waves. The presence of a temperature maxi-

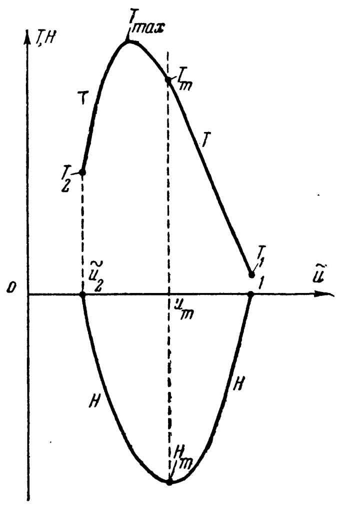

FIG. 5

mum is a characteristic of strong shock waves. Let us note that T reaches a maximum for a value of $\tilde{u}$ less than the value of $\tilde{u}$ corresponding to minimum H.

An examination of the emission and absorption of radiation yields the dependence of H on the temperature distribution; averaging over the frequencies and denoting $\sigma T^4 = \Theta$ (σ is the Stefan-Boltzmann-law constant) we can write down Eq. (20) of Ref. 7 [for the meaning of E_2 see Eq. (11) below] as

$$H = 2\int_{-\infty}^{\tau} \Theta(\zeta)\, E_2(\tau - \zeta)\, d\zeta - 2\int_{\tau}^{\infty} \Theta(\zeta)\, E_2(\zeta - \tau)\, d\zeta. \tag{10}$$

The coordinate x is replaced here by the optical thickness τ, $d\tau = \varkappa dx$, where $\varkappa$ is the coefficient of absorption.

The equation expresses the radiation flux through the τ plane as a sum of fluxes from individual layers $d\xi$, taken with a plus sign if $\xi < \tau$, and with a minus sign if $\xi > \tau$. The fraction of the transmitted flux is given by the function

$$2E_2(\tau - \zeta) = 2\int_{1}^{\infty} s^{-2} e^{-(\tau-\zeta)s}\, ds, \tag{11}$$

obtained by integrating the contribution of the oblique rays over all the angles. The assumptions under which it is possible to go from the integral expression for H to the differential equation (2) (Ref. 7), namely,

$$\frac{d^2H}{d\tau^2} = \frac{H}{\alpha^2} + \frac{2}{\alpha}\frac{d\Theta}{dx}, \tag{12}$$

are fully equivalent to replacing the function $2E_2(\xi)$ by the exponent $\exp(-\xi/\alpha)/\alpha$, where α is a dimensionless number that differs little from unity.

To prove this, let us write the corresponding approximate expression

$$H = F - G;\quad F = \int_{-\infty}^{\tau} \Theta(\zeta)\, \frac{1}{\alpha} e^{(\zeta-\tau)/\alpha}\, d\zeta, \qquad G = \int_{\tau}^{\infty} \Theta(\zeta)\, \frac{1}{\alpha} e^{(\tau-\zeta)/\alpha}\, d\zeta. \tag{13}$$

We obtain

$$dF/d\tau = \Theta/\alpha - F/\alpha;\quad dG/d\tau = -\Theta/\alpha + G/\alpha, \tag{14}$$

$$\frac{d^2F}{d\tau^2} = \frac{1}{\alpha}\frac{d\Theta}{d\tau} - \frac{\Theta}{\alpha^2} + \frac{F}{\alpha^2};$$

$$\frac{d^2G}{d\tau^2} = -\frac{1}{\alpha}\frac{d\Theta}{d\tau} + \frac{G}{\alpha^2} - \frac{\Theta}{\alpha^2}, \tag{15}$$

$$\frac{d^2H}{d\tau^2} = \frac{d^2F}{d\tau^2} - \frac{d^2G}{d\tau^2} = \frac{2}{\alpha}\frac{d\Theta}{d\tau} + \frac{1}{\alpha^2}H. \tag{16}$$

Let us note that α in the two terms of Eq. (16) must be the same, so that the radiation from a half space uniformly heated to a temperature T and adjacent to a vacuum is identically equal to $\Theta = \sigma T^4$.

One can notice that the expression H with the exponents (13) is the Green function of differential equation (12). It is evident from Eq. (13) that H is continuous and cannot experience finite jumps; in particular, if T and Θ are discontinuous, the first derivatives $dF/d\tau$, $dG/d\tau$, and $dH/d\tau$ become discontinuous, but not the quantities F, G, and H themselves.

It is further evident that the quantity $H_{\max}$ determined from Eqs. (10) or (13) satisfies the inequality $|H| < \Theta_{\max}$, where $\Theta_{\max}$ corresponds to the maximum temperature $T_{\max}$. Consequently, at low temperatures, when $\Theta_{\max} < |H_m|$ where H_m is the extremum of the flux of H as determined from Eq. (9), the equation $H(u) = F - G$ cannot be satisfied for all values of u and a discontinuity in the solution [a discontinuity in the function $u(\tau)$] is unavoidable. At low temperatures, this situation in unavoidable, since $H_{\max} \sim T$ and $\Theta_{\max} \sim T^4$.

In this respect, the case of small radiation differs from the case of low viscosity: if the viscosity μ is low, the curve $u(x)$ is continuous, decreasing u decreases the width of the zone, and $du/dx \sim 1/\mu$, but if μ is finite the derivative du/dx is everywhere finite.

In the case of a small but finite radiation, when $\Theta_{\max} < |H_m|$, the solution must have an infinitesimally thin discontinuity, if we disregard other dissipative factors (viscosity).

Let us now consider the general case, when $\Theta_{\max}$ is not small. Let us introduce the quantity

$$K = (F + G)/2. \tag{17}$$

From the definition of integrals F and G it follows that K, like H, is continuous; in addition

$$\begin{aligned} &\text{at} \quad \tau = -\infty, \ H = 0, \ K = \Theta_1 = \sigma T_1^4, \\ &\text{at} \quad \tau = +\infty, \ H = 0, \ K = \Theta_2 = \sigma T_2^4. \end{aligned} \tag{18}$$

From Eqs. (13)–(15) we get

$$dH/d\tau = 2(\Theta - K)/\alpha, \quad dK/d\tau = -H/2\alpha. \tag{19}$$

Dividing one by the other we eliminate τ and obtain an equation that can be investigated in the "phase plane" H, K (see Fig. 6):

$$dK/dH = H/4(K - \Theta). \tag{20}$$

From the integral definitions of H and K it follows that neither can be discontinuous.

A feature of the investigation is that H does not vary monotonically: if a continuous solution exists, H varies in the wave from zero in the initial state to H_m somewhere in the middle (region I), and then from H_m to 0 (region II).

Since $H(u)$ and $\Theta(u)$ are functions of a single parameter u, an expression can be found for $\Theta(H)$. This relationship, however, is not unique, and the curve $\Theta(H)$ has two branches with respect to the two regions of the variation of H.

We denote this curve by $\Theta_a(H)$ in the first region (between the initial state and the extremum H_m) and by $\Theta_b(H)$ in the second region (between the minimum and the final state). In particular, $\Theta_a(0) = \Theta_1$, $\Theta_b(0) = \Theta_2$, and $\Theta_a(A_m) = \Theta_b(A_m) = \Theta_m$.* Graphically, $\Theta(H)$ for a strong wave is shown solid in Fig. 6. In the K, H plane the line $K = \Theta(H)$ is the isocline of infinity. The differential equation (20) should be written separately for each region

$$\begin{aligned} dK_a/dH &= H/4(K_a - \Theta_a), \\ dK_b/dH &= H/4(K_b - \Theta_b) \end{aligned} \tag{21}$$

with boundary conditions (argument of function H):

$$K_a(0) = \Theta_1 = \Theta_a(0), \quad K_b(0) = \Theta_2 = \Theta_b(0). \tag{22}$$

The surface of Fig. 6 is thus bifoliate with a common intersection along the vertical $H = H_m$.

If a solution exists in which all the quantities in the wave vary continuously, the transition from one branch to the other should occur at $H = H_m$.

$$K_a(H_m) = K_b(H_m) \tag{23}$$

should be satisfied for the values of K_a and K_b satisfying Eqs. (21) and (22).

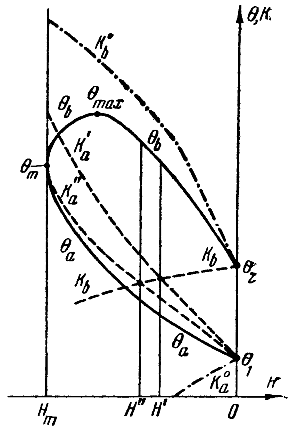

FIG. 6

An examination of the signs of the derivatives affords a ready qualitative picture of the field of the isoclines and of the behavior of the integral curves K_a and K_b. If $H = 0$, the equation has a saddle-point singularity in both cases (regions I and II). Two integral curves emerge from each saddle point.

The value of K is determined from Eqs. (17) and (13) and can be rewritten as a single integral

$$K(x) = \frac{1}{2} \int_{-\infty}^{\infty} \Theta(\zeta) \, e^{-|x-\zeta|/\alpha} \frac{d\zeta}{\alpha}, \tag{24}$$

from which it follows, by the theorem of the mean, that

$$\Theta_{\min} < K < \Theta_{\max}. \tag{25}$$

On this basis, it is possible to discard immediately the lower integral curve emerging from point Θ_1 in region I, shown in Fig. 6 by dash-dot lines, and

*Let us remark that the value of Θ at $H = H_m$, denoted Θ_m, is somewhat smaller than the maximum value of Θ reached in region II, denoted $\Theta_{\max}$.

marked K_a^0, since $K < \Theta_1 = \Theta_{min}$ on this curve. One can analogously discard the upper dash-dot integral curve in region II (K_b^0) emerging from the point $\Theta = \Theta_2$ on the $H = 0$ axis, for this curve is in its entirety above the line $\Theta_b(H)$; consequently, we obtain on the upper curve $K > \Theta_{max}$ in contradiction with condition (25).

The solution should therefore consist of an upper line K_a and a lower line K_b. The upper line K_a may reach the vertical line $H = H_m$ either at $\Theta > \Theta_m$ (see Fig. 6, K_a') or else at the point $\Theta = \Theta_m$ itself (see Fig. 6, K_a''), which is a singular point.

The realization of any one particular case depends on the numerical values of the function Θ. In any case, the only line K_a that can emerge from the saddle point $H = 0$, $\Theta = \Theta_1$ cannot intersect the line $\Theta_a(H)$, so that

$$K_a(H_m) \geqslant \Theta_m. \qquad (26)$$

Exactly the same way, the line K_b chosen in accordance with condition (25) does not intersect the corresponding line Θ_b

$$K_b(H) < \Theta(H), \ H_m < H < 0. \qquad (27)$$

But in this case it follows from the differential equation (20) that $dK/dH > 0$, and we obtain a stronger condition

$$K_b(H) < \Theta_2 = \Theta_b(0). \qquad (28)$$

A plot of K_b is also shown in Fig. 6.

Since $\Theta_2 < \Theta_m$ for strong shock waves, it follows from inequalities (26) and (28) that $K_a(H_m)$ and $K_b(H_m)$ cannot coincide, and consequently no solution exists in which H varies continuously from 0 to H_m on one branch and then from H_m to 0 on the other branch.

Actually H varies from 0 to H' or H'' along the lines K_a' or K_a'' and then (at the point of intersection with line K_b) there is a transition to line K_b and the value of H varies back from H' or H'' to zero.

The transition from line K_a to line K_b at the intersection point represents a shock wave in a limited sense of the word—a discontinuity of ρ, u, and Θ—the width of which is no longer dependent on the radiation (see above). H and K are conserved in this discontinuity. The conservation of H denotes that the quantities ρ, u, and Θ to the left and to the right of the discontinuity are related by the ordinary Hugoniot equation. The values of Θ to the left and to the right of the discontinuity are given by the intersection points of the curves Θ_a and Θ_b with the vertical $H = H'$ or $H = H''$ (see Fig. 6).

As can be seen from the inequalities and from Fig. 6

$$\Theta_2 > K_b(H') = K_a'(H') > \Theta(H'), \qquad (29)$$

i.e., the temperature, to which the gas can be heated by radiation ahead of the discontinuity front cannot exceed the final gas temperature $\Theta(H') < \Theta_2$ (the same result is obtained when K_a'' is realized also on the discontinuity $H = H''$).

This result is not as trivial as it appears at first glance, for behind the discontinuity front there is a temperature peak, in which $\Theta_b > \Theta_2$ (see Fig. 1).

How were the continuous solutions obtained in Prokof'ev's work? It can be shown that in the continuous solution proposed by him, at the point where $H = H_m$, $dH/d\tau$ experiences a discontinuity and H has a cusp. According to the initial equation (12), a discontinuity in $dH/d\tau$ is possible only in the point where the quantity Θ and the gas temperature uniquely related with it experience a discontinuity. Yet in Prokof'ev's solution the temperature is continuous.

The continuous solution derived by Prokof'ev is thus unacceptable.

Let us remark, finally, that replacing E_2 by the exponent [see Eqs. (10), (11), and (13)] (*i.e.*, a rough allowance for the oblique rays) is a poor approximation in the presence of a temperature discontinuity. One can assume that precise analysis will show that the fact itself that discontinuities are unavoidable when $\Theta_b > \Theta_2$ will remain in force*, but that the temperature discontinuities will be accompanied by singularities in the derivatives to the left and to the right of each discontinuity.

I take this opportunity to thank N. A. Dmitriev, the late S. P. D'iakov, A. S. Kompaneets, L. D. Landau, I. Sh. Model', V. A. Prokof'ev, and Iu. P. Raizer for valuable discussions.

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* Actually, if there is no discontinuity, the absence of a solution again becomes valid proof of a smooth distribution of temperature.

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Translated by J. G. Adashko
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SOVIET PHYSICS JETP VOLUME 5, NUMBER 5 DECEMBER, 1957

Resonant Pion-Nucleon Interaction and Production of Pions by Nucleons

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(Submitted to JETP editor June 11, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 1136-1142 (May, 1957).

The production of pions by nucleons is studied in an attempt to take approximate account of the strong pion-nucleon interaction. The calculation is based on the assumption that the probability of the process $N + N \to \pi + N + N'$ is determined by the energy of the created pion relative to one of the nucleons. Using experimental values for the matrix elements of the pion-nucleon interaction, we calculate the spectrum of pions and nucleons in the reaction $N + N \to \pi + N + N'$, and also the intensity of pion emission as a function of the angle between pion and nucleon. The results are compared with experiment.

IT IS NOW firmly established that the pion-nucleon interaction is strong in the P-state with total angular momentum $J = 3/2$ and isotopic spin $T = 3/2$. Upon this fact is based the hypothesis of a nucleon isobar state, formulated by Brueckner[1] and by Tamm and his collaborators[2]. To compare the consequences of this hypothesis with experiment, several properties of the production of pions by nucleons have been calculated[3]. Belen'kii and Nikishov[4] evaluated the relative magnitudes of single and multiple pion production, including both direct production and production through an intermediate isobar state. Many authors, for example Aitken *et al.*[5], have calculated pion production by nucleons taking the strong pion-nucleon interaction into account explicitly.

Instead of making such direct calculations, one can establish a phenomenological correspondence between two processes. Assuming that the matrix element for the process

$$N + N \to \pi + N + N', \tag{1}$$

depends only on the relative energy of the pion and one nucleon, the magnitude and the energy-dependence of the production cross-section can be obtained from the experimental values of the total cross-section for pion-nucleon scattering. In order that the strong pion-nucleon interaction shall appear in the process (1), it is only necessary[6] that the isotopic spin of the two nucleons in the initial state should be $T = 1$.

1. THE PION-NUCLEON INTERACTION MATRIX ELEMENT IN STATES WITH $T = 3/2$

The cross-section for a reaction

$$A + b \to C + d \tag{2}$$

is completely determined by a matrix element H according to the formula