Letters to the Editor

The Nature of Field Fluctuations

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As IS WELL KNOWN, the neutral or charged nature of a given wave field, *i.e.*, the neutral or charged nature of the particles corresponding to that field, is closely connected with the character of the wave function. In the case of a neutral field, the wave function is real; in the case of a charged field, the wave function is complex. A neutral or charged nature means here, in fact, the absence or presence of interaction between the given wave field and the electromagnetic field. However, the wave fields do not interact with the electromagnetic field only. According to the contemporary meson theory, the meson fields interact with nucleons and with the electron-positron field.

However, whether a given wave field interacts with a given field of non-electromagnetic character cannot be ascertained from the form of the wave function, *i.e.*, from the algebraic construction of the wave function. This means that in the contemporary theory the existence of non-electromagnetic interactions between given wave fields does not impose conditions on the character of the corresponding field quantities, whereas in the case of interactions with the electromagnetic field such a condition exists (complex nature of the field function).

In fact, let us take, for example, a spinor field, characterized by a four-component spinor. If this field interacts with an electromagnetic field, then the wave function of the given spinor field should be complex. In the case of a real spinor (Majorana's theory for the neutrino) such an interaction cannot exist. There are no analogous conditions for the wave function relative to the interaction with the meson field. The interaction with the meson field is introduced in such a way that no conditions are imposed on the wave function of the spinor (or any other) field. This signifies that the electromagnetic field occupies a special position in the contemporary theory of wave fields.

This leads to one of two conclusions. Either it is to be acknowledged that the electromagnetic field occupies by its specific nature a special position distinguishing it from other wave fields, or it is necessary to admit that modern theory has not yet found an adequate apparatus for imposing supplementary conditions on the field quantities of interacting fields to reflect the particular characteristics of the existing wave fields.

The first conclusion is unacceptable for the reason that it does not lead out of the maze of contemporary meson theories. Even if it is valid, this does not give reason to deny the presence of particular characteristics of field quantities which describe interactions different from the electromagnetic one.

The second conclusion can serve as a basis for an attempt to generalize the concept of field functions. In other words, it is necessary to search for the conditions which must be imposed on the corresponding wave functions in order to carry through this or that interaction. As a hypothesis, it might be proposed that the field function of a wave field should be a hypercomplex number, in particular, a quaternion. Then the field function should have components in quaternion space with the base vectors $(1, i, j, k)$, where the base vectors satisfy the following conditions:

$$i^2 = j^2 = k^2 = -1, \quad ij = -ji = k,$$
$$jk = -kj = i, \quad ki = -ik = j.$$

In this space three independent rotations are possible; gauge transformations correspond to these rotations. Invariance under a gauge transformation (of the first type) will correspond to the conservation of certain three charges. We can thus expect that instead of the one known law of conservation of electromagnetic charge, two other laws of conservation of some charges g_n and g_μ (for example, nucleon and μ-meson charges) should exist.

Hypothetically, one can propose several ways of generalizing the concept of a field function. The simplest are the following assumptions: a) the field function is essentially a Hamiltonian quaternion[2,3] b) the field function is essentially a Dirac quaternion[4]. At present there is no need to employ more complex field functions.

It is known that the quaternion Q is equivalent to the complex number $Q = q_0 + iq_1$, if the components at positions j and k are equal to zero[2]. It is also known that the wave functions of the field of electrically charged mesons φ and φ^* are made conjugate in the plane of the base vectors $(1, i)$. It is possible to assume, as a hypothesis, the existence of some special conjugation of the wave functions of the field, corresponding to the description of two nucleons with opposite $\pm g_n$-charges (g_n will be called the nucleon charge). Such a conjugation of the wave functions of the field can be carried out in the $(1, j)$ plane, where j does not coincide with the base vector i. If we do not go outside the $(1, j)$ plane, the algebraic properties of the wave functions of the field do not differ from the algebraic properties of complex functions. Therefore, the theory of particles with the g_n-charge (in other respects, neutral) can be developed analogously to the theory of electromagnetically charged particles. One of the authors of this note developed exactly such an approach to the theory of interaction of mesons with fermion fields[5].

However that may be, it is possible to consider that, together with electromagnetic conjugation, it is reasonable to introduce g_n-conjugation. The conjugation in the $(1, k)$ plane has not yet been employed. In the most general case, a particle having three charges, e, g_n, g_μ, should be described by a quaternion $\Psi = u_0 + iu_1 + ju_2 + ku_3$. A particle with charges $-e$, $-g_n$ and $-g_\mu$ will be described by the conjugate field function $\tilde{\Psi} = u_0 - iu_1 - ju_2 - ku_3$, where the sign $\sim$ means total conjugation in the case of Ψ and $\tilde{\Psi}$.

It can easily be shown that all possible combinations of the 3, 2, 1 and 0 charges, which the particles can have, can be described by 27 different quaternion field functions which have 3, 2, 1, 0 components.

The authors believe that the possibility described above of constructing a theory of particles which possess simultaneously three charges, is a new possibility in mesodynamics, opened up because of the broadening of the algebraic class of functions used as field functions.

In Ref. 5 a specific program of introducing field functions of a new algebraic class was outlined. However, other variants for using quaternions as field functions are equally conceivable.

[1] G. Wentzel, *Quantum Theory of Fields* (Russian translation) GTTI, 1947.

[2] M. Lagalli, *Vector Calculus*, ONTI, 1949.

[3] Iu. V. Linnick, Usp. Mat. Nauk (Progr. in Math. Sci.) 4, 49 (1949).

[4] Iu. B. Rumer, *Spinor Analysis*, ONTI, 1936.

[5] V. V. Chavchanidze, Dokl. Akad. Nauk SSSR **104**, 205 (1955).

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Simplification of the Equations for the Distribution Function of Electrons in a Plasma

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DAVYDOV, starting from the Boltzmann kinetic equation, showed that for the electron distribution function

$$f(\mathbf{r}, \mathbf{v}, t) = f_0(\mathbf{r}, v, t) + \frac{\mathbf{v}}{v}\mathbf{f}_1(\mathbf{r}, v, t) + \chi(\mathbf{r}, \mathbf{v}, t)$$

in a plasma located in electric and magnetic fields the following system of equations is correct:

$$\frac{\partial f_0}{\partial t} + \frac{v}{3}\operatorname{div}\mathbf{f}_1 - \frac{e}{3mv^2}\frac{\partial}{\partial v}(v^2\mathbf{E}\mathbf{f}_1) - \frac{1}{v^2}\frac{\partial}{\partial v}\left\{\nu v^2\frac{kT}{M}\frac{\partial f_0}{\partial v} + \nu v^3\frac{m}{M}f_0\right\} = 0, \quad \text{(1a)}$$

$$\frac{\partial \mathbf{f}_1}{\partial t} + v\operatorname{grad} f_0 + \frac{c\mathbf{E}}{m}\frac{\partial f_0}{\partial v} + \frac{e}{mc}[\mathbf{H}\mathbf{f}_1] + \nu\mathbf{f}_1 = 0. \quad \text{(1b)}$$

Here e and m are the charge and mass of an electron, M is the mass of a molecule (ion), k is the Boltzmann constant, T is the temperature of the plasma, E and H are the intensities of the electric and magnetic fields, $\nu = \nu(v)$ is the frequency of collision of electrons with molecules or ions (see Ref. 2, §59). Terms of order χ (in comparison with f_0) have been neglected in the derivation of Eqs. (1). The evaluation carried out in Ref. 1 has shown that in a spatially uniform plasma $\chi \sim \delta f_0$, while in the presence of irregularities $\chi \sim \delta f_0 + l\partial f_1/\partial z$, that is, Eqs. (1) are true when the conditions

$$\delta \ll 1, \quad l\partial f_1/\partial z \ll f_0. \quad (2)$$