at positions j and k are equal to zero[2]. It is also known that the wave functions of the field of electrically charged mesons φ and φ^* are made conjugate in the plane of the base vectors $(1, i)$. It is possible to assume, as a hypothesis, the existence of some special conjugation of the wave functions of the field, corresponding to the description of two nucleons with opposite $\pm g_n$-charges (g_n will be called the nucleon charge). Such a conjugation of the wave functions of the field can be carried out in the $(1, j)$ plane, where j does not coincide with the base vector i. If we do not go outside the $(1, j)$ plane, the algebraic properties of the wave functions of the field do not differ from the algebraic properties of complex functions. Therefore, the theory of particles with the g_n-charge (in other respects, neutral) can be developed analogously to the theory of electromagnetically charged particles. One of the authors of this note developed exactly such an approach to the theory of interaction of mesons with fermion fields[5].

However that may be, it is possible to consider that, together with electromagnetic conjugation, it is reasonable to introduce g_n-conjugation. The conjugation in the $(1, k)$ plane has not yet been employed. In the most general case, a particle having three charges, e, g_n, g_μ, should be described by a quaternion $\Psi = u_0 + iu_1 + ju_2 + ku_3$. A particle with charges $-e$, $-g_n$ and $-g_\mu$ will be described by the conjugate field function $\tilde{\Psi} = u_0 - iu_1 - ju_2 - ku_3$, where the sign $\sim$ means total conjugation in the case of Ψ and $\tilde{\Psi}$.

It can easily be shown that all possible combinations of the 3, 2, 1 and 0 charges, which the particles can have, can be described by 27 different quaternion field functions which have 3, 2, 1, 0 components.

The authors believe that the possibility described above of constructing a theory of particles which possess simultaneously three charges, is a new possibility in mesodynamics, opened up because of the broadening of the algebraic class of functions used as field functions.

In Ref. 5 a specific program of introducing field functions of a new algebraic class was outlined. However, other variants for using quaternions as field functions are equally conceivable.

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Translated by G. E. Brown
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Simplification of the Equations for the Distribution Function of Electrons in a Plasma

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(Submitted to JETP editor December 13, 1956; resubmitted March 14, 1957)
J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 1237-1238 (May, 1957)

DAVYDOV, starting from the Boltzmann kinetic equation, showed that for the electron distribution function

$$f(\mathbf{r}, \mathbf{v}, t) = f_0(\mathbf{r}, v, t) + \frac{\mathbf{v}}{v}\mathbf{f}_1(\mathbf{r}, v, t) + \chi(\mathbf{r}, \mathbf{v}, t)$$

in a plasma located in electric and magnetic fields the following system of equations is correct:

$$\frac{\partial f_0}{\partial t} + \frac{v}{3}\operatorname{div}\mathbf{f}_1 - \frac{e}{3mv^2}\frac{\partial}{\partial v}(v^2\mathbf{E}\mathbf{f}_1) - \frac{1}{v^2}\frac{\partial}{\partial v}\left\{\nu v^2\frac{kT}{M}\frac{\partial f_0}{\partial v} + \nu v^3\frac{m}{M}f_0\right\} = 0, \quad \text{(1a)}$$

$$\frac{\partial \mathbf{f}_1}{\partial t} + v\operatorname{grad} f_0 + \frac{e\mathbf{E}}{m}\frac{\partial f_0}{\partial v} + \frac{e}{mc}[\mathbf{H}\mathbf{f}_1] + \nu\mathbf{f}_1 = 0. \quad \text{(1b)}$$

Here e and m are the charge and mass of an electron, M is the mass of a molecule (ion), k is the Boltzmann constant, T is the temperature of the plasma, E and H are the intensities of the electric and magnetic fields, $\nu = \nu(v)$ is the frequency of collision of electrons with molecules or ions (see Ref. 2, §59). Terms of order χ (in comparison with f_0) have been neglected in the derivation of Eqs. (1). The evaluation carried out in Ref. 1 has shown that in a spatially uniform plasma $\chi \sim \delta f_0$, while in the presence of irregularities $\chi \sim \delta f_0 + l\partial f_1/\partial z$, that is, Eqs. (1) are true when the conditions

$$\delta \ll 1, \quad l\partial f_1/\partial z \ll f_0. \quad (2)$$

We shall now show that the system of equations (1) can be simplified. For this purpose we shall first consider the case of a spatially homogeneous plasma. In this case the symmetrical part of the electron distribution function (f_0) can be essentially changed only in a time of order $1/\delta\nu$, since $\partial f_0/\partial t \leq \delta\nu f_0$ (see Ref. 3). At the same time, as is clear from Eq. (1b), the current (directed) part of the distribution function ($\mathbf{f}_1$) changes essentially in a time of order $1/\nu$, since $\partial \mathbf{f}_1/\partial t \geq \nu \mathbf{f}_1$. Hence the function $\mathbf{f}_1$ changes more rapidly in time than f_0. For this reason the dependence of the function f_0 on t may be neglected in the integration of Eq. (1b). The solution obtained will then be correct to an accuracy which includes up to terms less than or of the order of δ, *i.e.*, to the same degree of accuracy to which Eqs. (1) themselves are correct.

Analogously, in the presence of spatial inhomogeneities the dependence of f_0 on t may be neglected in the integration of Eq. (1b) only on the condition that the function f_0 change much more slowly with time than $\mathbf{f}_1$, *i.e.*, that $\partial f_0/\partial t \ll \nu f_0$. On the other hand, it follows from Eq. (1a) that $\partial f_0/\partial t \lesssim \delta\nu f_0 + v\partial f_1/\partial z$, where z is the direction in which the function $|\mathbf{f}_1|$ changes most sharply. Hence the dependence of the function f_0 on t may be neglected in the integration of Eq. (1b) only if $\nu f_0 \gg \delta\nu f_0 + v\partial f_1/\partial z$, *i.e.*, if $\delta \ll 1$ and $l\partial f_1/\partial z \ll 1$, which coincides exactly with conditions (2).

Taking account of this circumstance (and knowing precisely how the fields E and H change with time), we can integrate Eq. (1b) without difficulty. The result obtained is a simple approximate expression for the function $\mathbf{f}_1$, which, as remarked above, is correct (*i.e.*, to the same degree of accuracy with which Eqs. (1) themselves are correct) when conditions (2) are fulfilled. Thus the problem of finding the electron distribution function reduces to the integration of the single equation (1a)*. For example, in the case of a spatially homogeneous plasma we have

$$\mathbf{f}_1 = -\mathbf{u}\frac{\partial f_0}{\partial v},$$

$$\frac{\partial f_0}{\partial t} - \frac{1}{v^2}\frac{\partial}{\partial v}\left\{v^2\left[\frac{e\mathbf{E}\mathbf{u}}{3m} + \nu\frac{kT}{M}\right]\frac{\partial f_0}{\partial v} + \nu v\frac{m}{M}f_0\right\} = 0.$$

* It should be noted that in the case of a constant magnetic field the system of equations (1) can also, generally speaking, be reduced to a single equation by means of an exact integration of Eq. (1b). However, such an integration does not lead to any simplification of the problem, since at the same time Eq. (1a) has its order increased and becomes integro-differential.

Here $\mathbf{u}$ is the velocity of the directed motion of the electron produced by external fields. The velocity $\mathbf{u}$ is determined by the Lorentz equation of motion of the electron

$$\frac{\partial \mathbf{u}}{\partial t} + \nu\mathbf{u} = \frac{e}{m}\left(\mathbf{E} + \frac{1}{c}\mathbf{u}\times\mathbf{H}\right), \tag{3}$$

which is ordinarily easily solved.

If spatial inhomogeneities are present in the plasma, the expression for the current function (in a constant magnetic field) takes the form

$$\mathbf{f}_1 = -\mathbf{u}\frac{\partial f_0}{\partial v} - \frac{l}{1+\gamma^2}\{\operatorname{grad} f_0 + \gamma^2 H^{-2}\mathbf{H}(\mathbf{H}\operatorname{grad} f_0) + \gamma\,[\mathbf{H}\operatorname{grad} f_0]/H\},$$

where $\gamma = eH/mc\nu$ and the velocity $\mathbf{u}$ is determined, as before, by Eq. (3).

In the case of a Maxwellian distribution of the electron velocities, the equations for the distribution function lead, as is well-known, to a system of equations for the temperature T_e and the density n of the electrons (the needed expressions for the current and for the energy flux carried by the electrons are calculated with the aid of the expression obtained above for the current function). These equations were obtained in Ref. 1 for a varying magnetic field ($\mathbf{E} = \mathbf{E}_0(\mathbf{r})\cos\omega t$) of low frequency ($\omega^2 \ll \nu^2$). For high frequency $\omega^2 \gg \nu^2$ the electron distribution is stationary. The equations for the temperature and density of the electrons become essentially simpler in this case. For example, if we assume that $l \neq l(v)$ (collision with neutral particles), then for $H = 0$

$$\theta - \frac{l^2}{3\delta}\operatorname{div}\operatorname{grad}\theta = 1 + \frac{e^2E_0^2(\mathbf{r})}{3kTm\delta\omega^2},$$

$$n = (n_0/\theta^{1/2})\,(V/\int_V \theta^{-1/2}dV),$$

where $\theta = T_e/T$ and n_0 is the electron density in a plasma undisturbed by an electric field.

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Translated by M. G. Gibbons
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