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Production of π^+ Mesons in $p-p$ Collisions at 480–660 Mev.*

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The energy distributions of π^+ mesons from the reactions $p + p \rightarrow \pi^+ + n + p$ and $p + p \rightarrow \pi^+ + d$ have been measured at four laboratory system angles utilizing an external beam of 660 Mev protons. The differential cross section for π^+ meson production has been measured at eight laboratory system angles. The total cross section for the first reaction is (10.9 ± 1.1) mb and the angular distribution is proportional to $(0.66 \pm 0.14) + \cos^2\theta$. The excitation function of this reaction is given in the energy region 480–660 Mev approximately by the power law $0.67\,P_{\max}^{4.7}$ mb, where $P_{\max}$ is the maximum meson momentum in units of $m_{\pi^+}c$ in the center-of-mass system.

INTRODUCTION

MESON PRODUCTION in nucleon-nucleon collision is a more involved process than meson nucleon scattering or meson photoproduction on free nucleons. Thus, experiments on meson production in nucleon-nucleon collisions can yield important information on the characteristics of mesons and of the meson-nucleon interaction.

The reactions that lead to charged meson production in $p-p$ collisions at 600 Mev are

$$p + p \rightarrow \pi^+ + n + p; \qquad (1)$$

$$p + p \rightarrow \pi^+ + d. \qquad (2)$$

Production of two mesons which is energetically possible for proton energies above 600 Mev contributes only very little to the total cross section and is not considered in the present work. Lately there have appeared several papers on π^+ production in the energy region of this investigation. The reaction (2) was studied in detail in Ref. 1, using counter telescopes for the angles 30° to 90° in the center of mass system (c.m.s.). The excitation functions for this reaction were obtained in the energy range 510 to 660 Mev. In Ref. 2 reaction (1) was studied at an energy of 660 Mev in the same angular range by emulsion techniques.* The spectrum

* This paper was presented at the High Energy Particle Conference in May, 1956, in Moscow.

* An error contained in Ref. 2 should be pointed out. In that paper the differential cross section was obtained using for the energy loss the formula $(dE/dR)_E = 0.067\,R_\pi^{0.419}$ which has been derived empirically from range–energy measurements in emulsions. This formula is valid for small energies. However, extrapolating it into the energy region around 170 Mev one will obtain values for $(dE/dR)_E$ which are for meson energies 100–150 Mev lower by 20–40% than the actual energy loss of mesons in copper which was used to slow down the π^+ mesons. After correcting for this error both the energy and the angular distributions obtained in Ref. 2 change and the agreement of the angular distributions of Ref. 2 with the ones obtained in the present work is considerably improved.

of the π^+ mesons was investigated in Ref. 3 with a magnetic spectrometer at an angle of 24° in the laboratory system (l.s.) at 660 and 560 Mev.

In the present work reactions (1) and (2) were studied at the angles 10° to 40° in the c.m.s. This angular region had not been investigated earlier. Furthermore additional information was obtained on the angular distribution of π^+ mesons from reaction (1) at angles 50° to 80° in the c.m.s., and the excitation function for this reaction was determined for the energies 480 to 660 Mev.

EXPERIMENTAL ARRANGEMENT

It follows from kinematical considerations that nucleons emerging from reaction (1) are restricted to certain angles in the l.s. For 660 Mev incoming protons this angle is about 50°. Since protons scattered elastically into large angles have small energies, it is possible to distinguish these from mesons at angles larger than 70° by means of a thin filter.

Experiments on π^+ production in the energy range 660–480 Mev became feasible after an external high intensity 657-Mev proton beam was produced with the synchrocyclotron of the Institute for Nuclear Problems of the U.S.S.R. Academy of Sciences. The proton beam passed through steel collimators inserted into the concrete shielding wall and fell on the target after passing through an ionization chamber (see Fig. 1). The average intensity at the target position was 3×10^8 protons/cm^2 sec and was monitored by the ionization chamber. The chamber was calibrated with respect to proton current in absolute terms by means of a Faraday cup. Proton energies below 657 Mev were obtained by placing polyethylene absorbers in the path of the beam before it passed through the collimators. The energy uncertainty at 500 Mev was ± 7 Mev.

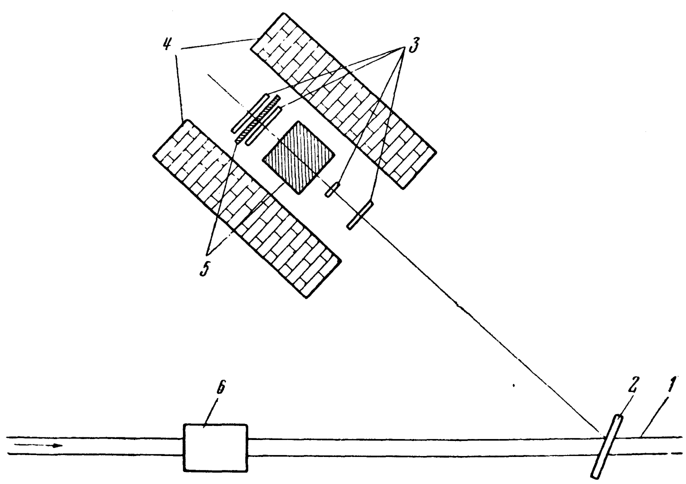

FIG. 1. Experimental setup. 1 – proton beam; 2 – target; 3 – scintillation counters; 4 – lead shield; 5 – copper or aluminum filters; 6 – monitor.

The π^+ production cross section on hydrogen at 60° –160° l.s. was obtained as the difference of charged particle yields from polyethylene and carbon targets, equal with respect to energy loss and with a thickness of about 1 g/cm^2. The excitation function was obtained with a liquid hydrogen target. The Dewar was made of glass. The effective target volume was defined by lead collimators which provided a shield for the detector against charged particles generated by the incoming protons in the glass walls of the Dewar.

The charged particles coming from the target were detected by a counter telescope. The first three scintillation counters were in coincidence, the fourth in anticoincidence. Between the second and the third counters was placed a copper or aluminum filter of different thickness. The interval of accepted ranges was given by the thickness of the filter placed between the third and the fourth counters. To produce a count a charged particle had to pass through the first three counters and stop in the filter between the third and the fourth counter. The first three counters consisted of 2 mm thick crystalline tolane (diphenyl acetylene) scintillators which were provided with special reflectors made of aluminum foil to increase the light collection efficiency. The counting efficiency of the telescope had a plateau of 200 v thus assuring a good stability of the setup. The efficiency of the telescope was determined by measuring the angular distribution of the charged particle yield from p–p collisions.

The integral

$$\int_0^{2\pi}\int_0^{\pi} \frac{1}{2}\frac{\partial\sigma}{\partial\omega}\sin\theta\, d\theta\, d\varphi \qquad (3)$$

in the case of p–p collisions has to be equal to the total interaction cross section. Here $\partial\sigma/\partial\omega$ is the differential cross section for charged particle emission in the direction θ in the l.s. This is due to the fact that in the energy region of interest, where multiple meson production is still unimportant, only two charged particles are always formed in the final state of a p–p collision, because of charge conservation. In the present investigation the value obtained for the integral was (42.9 ± 1.5) mb, in good agreement with the total p–p cross section of (41.4 ± 0.6) mb obtained in Ref. 4. This shows that the efficiency of the counter telescope was close to 100% despite the small thickness of the scintillators. Because of the thinness of the counters it was possible to detect mesons with energies down to 15 Mev, and furthermore the number of chance coincidences was also decreased. These constituted the bulk of the background in the present experiment. By surrounding the counter telescope with a 10 cm lead shield the accidental counting rate was further reduced and the background then did not exceed 0.5% of the true counting rate.

EXPERIMENTAL RESULTS

The differential cross sections for π^+ meson production according to reactions (1) and (2) at an energy of 657 Mev were determined at eight angles in the l.s., running at full beam intensity. The results are given in Table 1. They include mesons with energies from 15 Mev in the l.s. up. The given cross sections include corrections due to decay in flight (6–3%) and to nuclear capture both in the target and in the scintillators (1.5%).

TABLE I. Differential cross sections in mb/sterad for mesons with energy greater than 15 Mev in the l.s.*

θ, lab system	160°	140°	123°	108°	95°	86°	70°	60°
$\left(\frac{\partial\sigma}{\partial\omega}\right)_{lab}$, $\frac{\text{mb}}{\text{sterad}}$	0,36 ±0.03	0.40 ±0.03	0,49 ±0.03	0.61 ±0.03	0.74 ±0.06	0.84 ±0.06	0.88 ±0.12	0,87 ±0.19

*The given errors are standard deviations from several independent runs.

Further, the energy distribution of π^+ mesons at primary energy of 657 Mev were determined for the angles 160°, 140°, 123°, and 108° in the l.s. The results were corrected for decay in flight, and for nuclear interactions and multiple scattering in the filters and scintillators of the counter telescope. The resulting counting efficiency for π^+ mesons is shown in Fig. 2 as a function of meson energy. The accuracy of these corrections was checked by comparing the number of mesons obtained by integrating the obtained spectrum with the number of mesons counted in the absence of a filter in the first three scintillators. The results of this comparison are summarized in Table 2.

From comparison of lines 2 and 3 of Table 2 one may conclude that the uncertainty in the counting efficiency for π^+ mesons does not exceed 10%. The energy distributions of the π^+ mesons after applying the corrections were normalized with respect to the total cross sections given in Table 1. The obtained spectra are shown in Fig. 3. The indicated

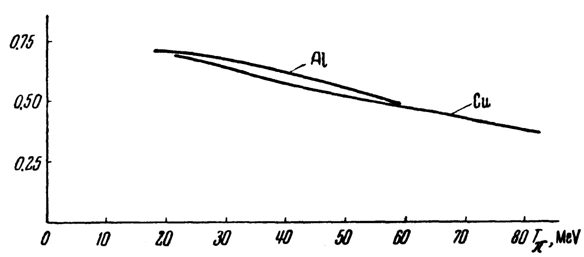

FIG. 2. Counting efficiency for π^+ mesons as a function of energy.

TABLE 2.

π^+ meson angle in the l.s.	160°	140°	123°	108°
Total number of mesons, N_1, from integrating the energy distributions	11300	7940	16090	18860
Number of mesons, N_2, counted in the first three scintillators	11200	8130	15840	16940
Ratio N_2/N_1	0.99±0.03	1.02±0.03	0.98±0.03	0.90±0.03

TABLE 3. Energy distribution of π^+ mesons from reactions (1) and (2).

E_p, Mev	485	521	557	584	597	609	621	633	646	657
$\left(\frac{\partial\sigma}{\partial\omega}\right)$ 95° l.s., in relative units	1.97 ±0.09	2.69 ±0.13	3.54 ±0.13	4.19 ±0.10	4.60 ±0.10	5.02 ±0.09	5.39 ±0.08	5.91 ±0.12	6.39 ±0.13	6.81 ±0.15

errors are the standard deviations due to counting statistics and the points are averages from two separate series of runs.

The excitation function for π^+ mesons from reactions (1) and (2) was obtained for a meson angle of 95° in the l.s. The results were corrected for the change of the ionization of protons at reduced energies, and for the admixture of slow particles to the beam generated in the polyethylene slabs which were used to slow down the protons. The latter correction was determined experimentally and did not exceed 4% under the most unfavorable conditions. The yield of mesons from reactions (1) and (2) for different proton energies is given in Table 3 in relative units.

DISCUSSION OF THE RESULTS

Each energy distribution of the π^+ mesons shown in Fig. 3 consists of two parts. The first one is broad and is associated with reaction (1); the second is a peak due to reaction (2). The dashed curves represent theoretical energy distributions for reaction (1). They were obtained under the assumption that the matrix element is proportional to the meson momentum and that one can neglect nucleon–nucleon interactions in the final state. Then the cross section is given by

$$\frac{\partial^2\sigma}{\partial E\,\partial\omega} \sim \frac{1}{\beta_\pi}\left[M\left(W-\sqrt{p^2+m^2}\right)-\frac{p^2}{4}\right]^{1/2} p^4, \tag{4}$$

where M and m are the nucleon and meson mass respectively, W is the kinetic energy of the protons in the c.m.s., and E, p, and β_π are the meson kinetic energy, momentum and velocity respectively also in the c.m.s. In deriving this formula the nucleons in the final state were assumed to be nonrelativistic. This is reasonable since for a primary energy of 660 Mev the maximum possible nucleon energy in the final state is 87 Mev. After transforming into the l.s. the theoretical cross sections were normalized to the experimental curves in the energy region where contributions from reaction (2) were fully excluded. The dash-dot curves were similarly calculated but with the assumption that the matrix element is independent of meson momentum. It is clearly evident that the other assumption leads to better agreement with the experiment.

The peak in the experimental distribution can now be separated out rather easily using the theoretical expression (4), taking into account the energy resolution of the detector. This is the dashed curve in Fig. 3. The curves shown on the right side for each angle in Fig. 3 have been obtained this way. The solid curves which have been drawn through the points represent the theoretical resolu-

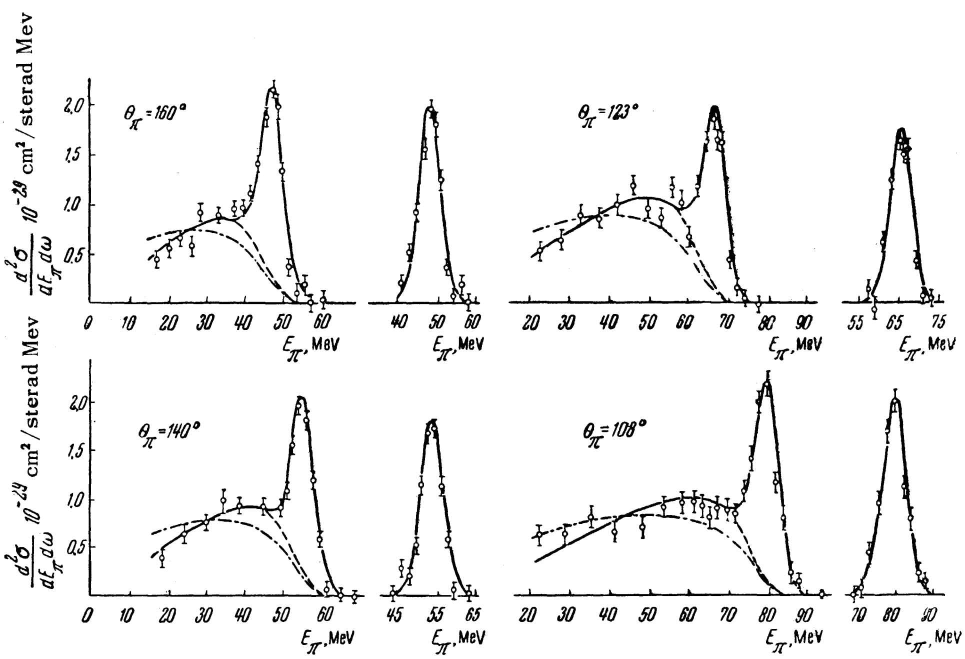

FIG. 3. Energy distributions of π^+ mesons from the reactions $p + p \rightarrow \pi^+ n + p$ and $p + p \rightarrow \pi^+ + d$; laboratory system energies and angles.

tion curves for the mesons from reaction (2). The form of the curves is given by

$$N(E) \sim \int_E^\infty \exp\left\{-\frac{(E_1 - E_0)^2}{\langle \Delta E^2 \rangle}\right\} \exp\left\{-\frac{(E_1 - E)}{\langle E \rangle}\right\} dE_1, \quad (5)$$

Here $\langle \Delta E^2 \rangle$ is the r.m.s. fluctuation of the meson energy including all factors contributing to the energy resolution of the setup; $\langle E \rangle = \langle \Delta R \rangle \, dE/dR$ with $\langle \Delta R \rangle$ the average range shortening of the mesons in the filters due to multiple Coulomb scattering, and E_0 is the kinetic energy of the mesons from reaction (2).

The differential cross sections for meson production in reaction (2), obtained from the area under the peaks and transformed into the c.m.s., are given in Table 4.

The obtained points are plotted in Fig. 4, which also shows the points obtained in Ref. 1. One sees that the angular distribution given in Ref. 1 also holds for small angles. This shows that contributions of d-wave mesons in reaction (2) are unimportant even up to a proton energy of 660 Mev.

To obtain the angular distribution of the π^+ mesons from reaction (1) the meson energy distributions were transformed into the c.m.s., utilizing the fact that the expression

$$(1/p)(\partial^2\sigma / \partial E\, \partial\omega) \quad (6)$$

is an invariant. Here p and E are the meson momentum and energy respectively in the appropriate coordinate system. The meson energy distributions in

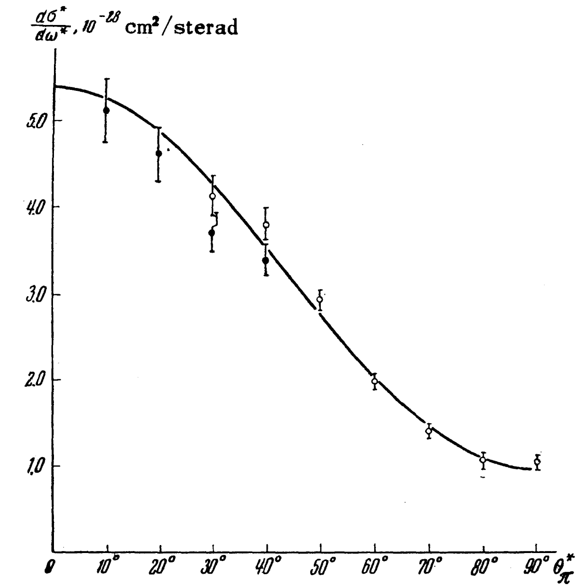

FIG. 4. Angular distribution of π^+ mesons from the reaction $p + p \rightarrow \pi^+ + d$; ●—results of this work; ○—results from Ref. 1.

TABLE 4.

Angle, c.m.s.	10°	20°	30°	40°
$\left(\frac{\partial\sigma^*}{\partial\omega^*}\right)$, $\frac{\text{mb}}{\text{sterad}}$	0.51 ± 0.04	0.46 ± 0.03	0.37 ± 0.02	0.34 ± 0.02

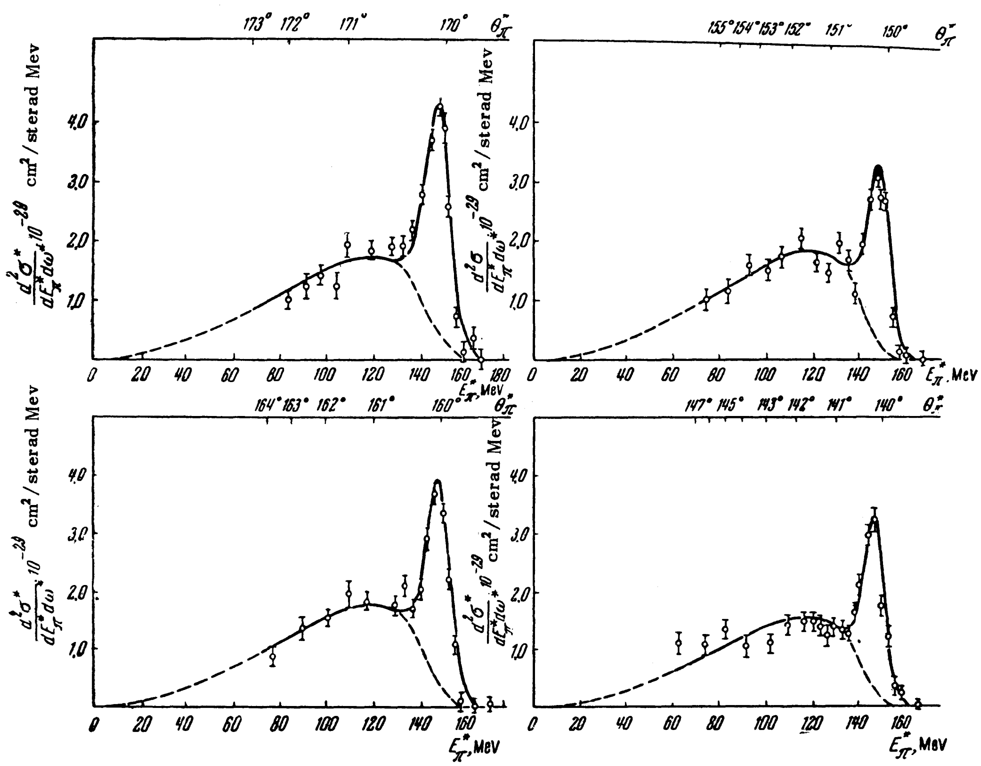

FIG. 5. Energy distribution of π^+ mesons from the reactions $p + p \rightarrow \pi^+ + n + p$ and $p + p \rightarrow \pi^+ + d$; center of mass angles and energies.

the c.m.s. from reactions (1) and (2) are plotted in Fig. 5. One sees that the mesons come out preferentially with high energy. The average energy of the mesons from reaction (1) is 110 Mev and 80% of the mesons have energies larger than 70 Mev. The transformation of such energy spectra from the l.s. into the c.m.s. depends very little on the meson energy, as can be seen from the scale of angles given at the top of the curves of Fig. 5, as well as from Table 5. The high energy part of the spectrum corresponds to an angular spread in the c.m.s. of ± 3°. Practically all mesons emitted in this angular spread were counted since the detector had an angular resolution of ± 2°. This peculiarity allows one to obtain the differential cross section in the c.m.s. by a measurement of the total meson yield in an arbitrary direction in the l.s. without first having to obtain the meson energy spectrum.

In Table 5, θ_π, E_π, θ_π^*, and E_π^* denote the angles and energies of the mesons in the l.s. and c.m.s. respectively.

In this manner the differential cross sections of π^+ meson production from reaction (1) were found for the angles 130° –100° in the c.m.s. According to the data of Ref. 1, reaction (2) does not contribute in this range. The differential cross section for the angles 171°, 162°, 152°, and 142° in the c.m.s. were obtained by integrating the smooth part of the respective energy spectra.

The low energy parts of the spectra inaccessible to the counter telescope were found by extrapolating the theoretical curves obtained by assuming a linear

TABLE 5.

	$\theta_\pi = 60°$				$\theta_\pi = 80°$			$\theta_\pi = 108°$		
E^*_π, Mev	E_π, Mev	θ^*_π	$\frac{d\cos\theta_\pi}{d\cos\theta^*_\pi}$	E^*_π, Mev	E_π, Mev	θ^*_π	$\frac{d\cos\theta_\pi}{d\cos\theta^*_\pi}$	E_π, MeV	θ^*_π	$\frac{d\cos\theta_\pi}{d\cos\theta^*_\pi}$
150	184	95°	0.72	150	130	116.5°	1.15	82	140°	2.06
90	111	99°	0.70	120	72	118°	1.15	61	141°	2.18
60	73	103°	0.69	70	59	124°	1.19	26	147°	2.59

dependence of the matrix element on the π^+ meson momentum. The uncertainty in the differential cross sections associated with the lack of knowledge of the actual energy spectra below the detection threshold is 5 to 15%, depending on angle.

The angular distribution of π^+ mesons created in reaction (2) is shown in Fig. 6. The curve in Fig. 6 corresponds to the expression

$$\partial\sigma^*/\partial\omega^* = (0.88 \pm 0{,}04)\,[(0.66 \pm 0{,}14) + \cos^2\theta^*_\pi]\cdot 10^{-27}\ \text{cm}^2/\text{sterad} \tag{7}$$

where the coefficients were obtained by a least-squares fit.

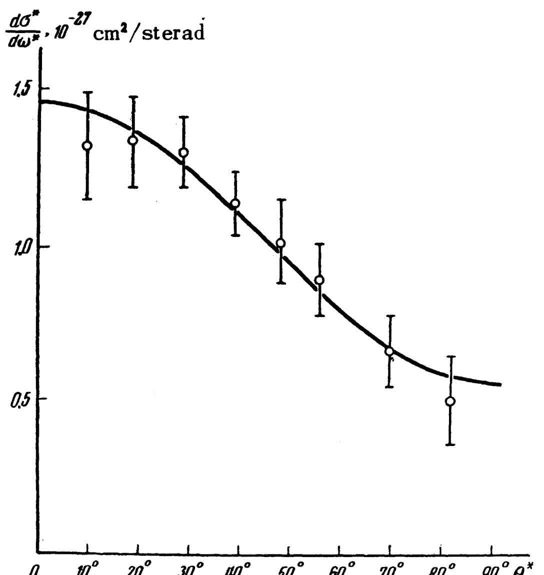

FIG. 6. Angular distribution of π^+ mesons from the reaction $p + p \to \pi^+ + n + p$.

The total cross section obtained by integrating (7) equals (10.9 ± 1.1) mb. In terms of the hypothesis of charge independence the cross section for reaction (1) is the sum of two independent parts[5]:

$$\partial\sigma/\partial\omega\,(p + p \to \pi^+ + n + p) = \partial\sigma_{10}/\partial\omega + \partial\sigma_{11}/\partial\omega, \tag{8}$$

where according to Ref. 6, $\partial\sigma_{11}/\partial\omega$ is isotropic at 660 Mev and has a value ~ 0.28 cm²/sterad. Therefore we have

$$\partial\sigma^*_{10}/\partial\omega^* = (0.88 \pm 0.04)\,[(0.34 \pm 0.15) + \cos^2\theta^*_\pi]\cdot 10^{-27}\ \text{cm}^2/\text{sterad}. \tag{9}$$

Since the contribution of $\partial\sigma_{11}/\partial\omega$ to the differential cross section of reaction (1) for the angles 170° – 160° in the c.m.s. is only about 20%, the energy spectrum of π^+ mesons at these angles is almost completely that of $\partial\sigma_{10}/\partial\omega$. One thus notices a remarkable difference in the form of the energy distributions of π^+ and π^0 mesons the latter of which have been obtained in Ref. 7. The maximum in the π^0 spectrum is at lower energies and lies around 75 Mev. This can be understood qualitatively remembering that σ_{11} is due predominantly to transitions of the Pp kind. Assuming that the form of the energy spectrum associated with σ_{10} does not depend on the angle, as appears to be the case with π^0 mesons, then the full spectrum of mesons from reaction (1) should become softer when going from 180° to 90° in the c.m.s. In particular, it should be softer than the spectrum measured for 170° to 160° in the c.m.s. This softening is indeed noticeable in the spectrum for 140° c.m.s. (Fig. 5), where the ratio of the cross sections $(\partial\sigma_{11}/\partial\omega)/(\partial\sigma_{10}/\partial\omega)$ approaches 1/2. Together with the above linear dependence of the matrix element on meson momentum, the angular distribution (9) indicates that at 660 Mev π^+ mesons are created in reaction (1) in the p-state. A similar observation has been made earlier.[3] It is well known that the angular distribution of mesons associated with the transition ${}^1D_2 \to {}^3S_1p_2$ is proportional to

$$^1/_3 + \cos^2\theta. \qquad (10)$$

The fact that the obtained angular distribution for σ_{10} is close to (10) indicates that at 660 Mev the meson creation connected with σ_{10} is due to a ${}^1D_2 \to {}^3S_1p_2$. However, the results of the present work do not allow to exclude the possibility of transitions of the Pp kind leading to a similar angular distribution.

As mentioned earlier the excitation function of reaction (1) was obtained utilizing the circumstance that the transformation of the energy distributions does not strongly depend on the meson energy. It was shown in Ref. 5 that if the angular distribution in the c.m.s. has the form

$$A + \cos^2\theta, \qquad (11)$$

then the differential cross section at θ_0 = arccos $(1\sqrt{3})$ (55° or 125°) in the c.m.s. and the total cross section are connected by the relation

$$4\pi\,(\partial\sigma^* / \partial\omega^*)_{\theta_0} = \sigma_t. \qquad (12)$$

The angle θ_0 is called the isotropic angle. From the kinematics of reaction (1) one has in the energy region 510 to 660 Mev that the isotropic angle changes only by 1° if one considers mesons in the energy interval 0.5 E_{max} to E_{max}. This energy interval contains 70 to 90% of all mesons. Furthermore, the transformation function for the solid angles from the l.s. to the c.m.s. for the same mesons changes only by 2% in the energy interval 510 to 660 Mev. These circumstances allow the determination of the excitation function of reaction (1) by using the meson yield as a function of proton energy as measured at 95° in the l.s. (~50° in the c.m.s.). The meson yield from reaction (2) was excluded according to the data of Ref. 1 where the excitation function was also obtained for 50° in the c.m.s. The results of the measurements were corrected for the change of the detection threshold with the c.m.s. energy of the mesons for decreasing proton energy. Since the angle 50° in the c.m.s. is not exactly the isotropic angle, the change of the angular distributions of the mesons from reaction (1) when going from 660 to 440 Mev proton energy[5] was also taken into account. The uncertainty in the total cross section due to these corrections at the lowest energy of 440 Mev did not exceed 15%. The excitation function normalized with respect to the total cross section at 657 Mev is given in Table 6.

The excitation function has also been plotted in Fig. 7. One sees that the excitation function is well represented by the power law

$$\sigma_t = 0{,}67\,P_{\max}^{4{,}7}\ \text{mb}, \qquad (13)$$

Here P_{max} is the maximum π^+ meson momentum in the c.m.s. in units $m_{\pi^+}c$.

TABLE 6. Energy distribution of π^+ mesons from reaction (1).

E_p, Mev	485	521	557	584	597	609	621	633	646	657
σ_t, mb	2.0 ±0.3	3.1 ±0.4	4.5 ±0.4	5.5 ±0.5	6.3 ±0.5	7.0 ±0.5	7.8 ±0.5	8.9 ±0.5	9.9 ±0.5	10.9 ±0.5

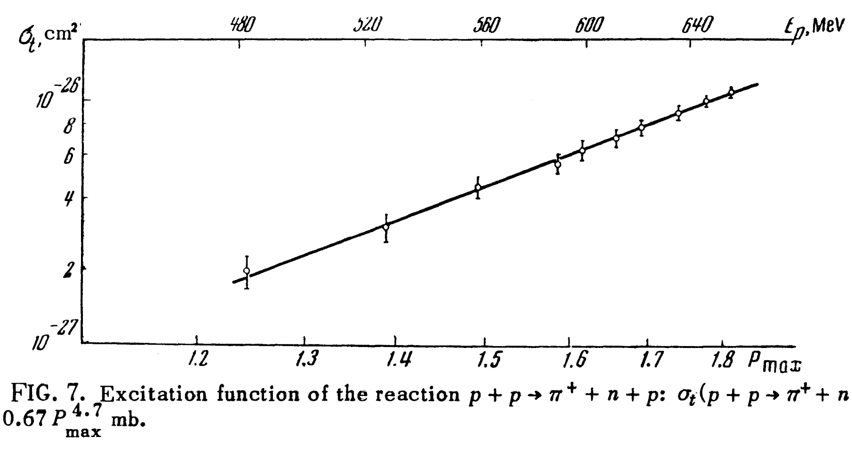

FIG. 7. Excitation function of the reaction $p + p \to \pi^+ + n + p$: $\sigma_t(p + p \to \pi^+ + n + p) = 0.67\,P_{\max}^{4.7}$ mb.

The excitation function for σ_{11} as measured in Ref. 6 has in the present energy region the form

$$\sigma_{11} = 0.11\, P_{\max}^{5,5} \text{ mb}, \tag{14}$$

where $P_{\max}$ is the maximum momentum of the π^0 meson in units of $m_{\pi^0}c$. From the difference of $\sigma_t(p + p \to \pi^+ + n + p)$ and σ_{11} one obtains for σ_{10} a power law approximation:

$$\sigma_{10} = 0{,}55\, P_{\max}^{4,4} \text{ mb}. \tag{15}$$

Assuming a linear dependence of the matrix element on the meson momentum and neglecting the nucleon-nucleon interaction in the final state, one obtains for the excitation function of σ_{10} an expression of the form

$$\sigma_{10} \sim \frac{1}{v} \int\limits_0^{P_{\max}} \left[M\left(W - \sqrt{p^2 + m^2}\right) - \frac{p^2}{4}\right]^{1/2} p^4 dp \sim P_{\max}^{5,6}, \tag{16}$$

where v is the velocity of the incoming protons in the c.m.s.

Evidently the experimentally obtained excitation function changes slower with momentum. This can be explained by the nucleon-nucleon interaction in the final state and, probably, by the resonance character of the matrix element for the meson production. One will expect these factors to be important if the transition ${}^1D_2 \to {}^3S_1p_2$ should predominate since it is connected with the $T = 3/2$, $I = 3/2$ state of the meson-nucleon system.

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