Reflection of Gamma-Rays From Bent Quartz Plates

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The wavelength dependence of the coefficient of reflection from the $13\bar{4}0$ planes of quartz crystal bent to a cylinder having a 2 meter radius was investigated on a two-meter crystal gamma-spectrometer. The quadratic character of this dependence was substantiated. It was shown that in theoretical examinations of this effect it is essential to take into account distortion of the reflecting planes. In the treatment, the plate is subdivided in thickness into a series of independently reflecting laminae, the thickness of which is less than the primary extinction distance. The crystal plate is found to behave as a mosaic crystal with small primary extinction. Thus the hitherto inexplicable quadratic dependence of the reflection coefficient on the wavelength finds a natural explanation.

IN INVESTIGATING the coherent reflection of x- and γ-rays from the $13\bar{4}0$ planes of quartz plates cut from an α-quartz single crystal so that the $13\bar{4}0$ planes were normal to the plane of the plate and the side edges were parallel to the optical axis of the crystal, Lind, West and DuMond[1] found that the dependence of the integrated reflection coefficient on the wavelength for plates elastically bent to a cylinder of 2 meters radius is close to quadratic. Their measurements of the integrated reflection coefficient for the same plate in the unstressed state led to a dependence close to the linear, *i.e.*, the dependence characteristic of ideal crystals. Lind *et al.* do not attempt to give an explanation of the observed effect noting only that "... the quartz might, however, become mosaic-like in structure in some elastically reversible way not now understood."

We felt it would be of interest to verify the quadratic dependence of the reflection coefficient on the wavelength discovered by Lind, West and DuMond and to find an explanation of the effect. The measurements were carried out on the two-meter crystal-diffraction spectrometer of the All-Union Scientific Research Institute of Metrology.[2] We investigated the reflection from the $13\bar{4}0$ planes of a $50 \times 30 \times 1.6$ mm plate of α-quartz cut and bent just as in the measurements of Lind, West and DuMond. We determined the ratio of the area under the spectral line to the number of pulses from the direct beam, due to photons of the same energy. This ratio Γ_i, as can readily be shown, is proportional to the integrated reflection coefficient. Figure 1 (curve *a*) shows the results of measurements at energies of 191 kev (In^{114}), 279 kev (Hg^{203}), 412 kev (Au^{198}) and 1190 kev (mean energy of the group of hard lines in the spectrum of Ta^{182}). It will be seen that a dependence of the form $\log \Gamma_i = c - n \log E$ obtains. Processing of the data by the method of least squares yields $n = 1.85 \pm 0.05$, where 0.05 is the mean square error. Analogous measurements with reflection from the $11\bar{2}4$ planes of α-quartz (the $50 \times 30 \times 0.8$ mm plate was cut so that the plane of the plate was perpendicular to the $11\bar{2}4$ planes) also led to a dependence very close to the quadratic (Fig. 1, curve *b*).

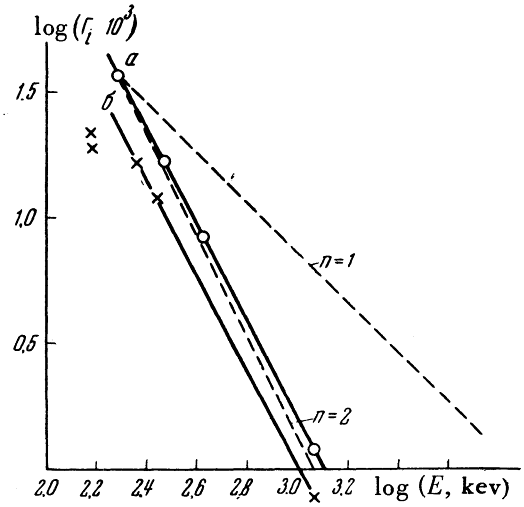

FIG. 1.

Lind, West and DuMond assumed that the initially flat reflecting planes remain plane when the plate is bent. This in general is not correct. Actually when a plate of arbitrary cross section is bent by the application of a bending moment M_1 acting in the *yoz* plane (Fig. 2) the displacements in the

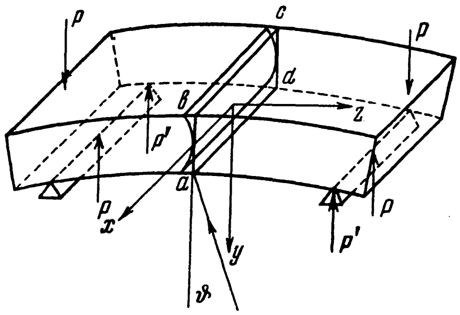

FIG. 2.

direction of the z-axis in the case of anisotropy of the general form are determined by the familiar relation[3]

$$W = (M_1 / 2I_1)[a_{35}xy + a_{34}y^2 + a_{33}y(2z - l)], \tag{1}$$

where M_1 is the effective bending moment, I_1 is the moment of inertia about the x-axis and the a_{ij}'s are strain coefficients. If a_{34} and a_{35} differ from zero, the cross sections are distorted by the bending into second-degree surfaces. With $a_{35} = 0$ the planes bend along the parabolas:

$$z = k_1 y^2, \quad k_1 = M_1 a_{34} / 2I_1.$$

If $a_{35} \neq 0$, Eq. (1) becomes invalid for cases of bending of the crystal plate between cylindrical mirrors or by application of moments by the method of Borovskii-Gil'varg.[4,5] Actually if $a_{35} \neq 0$, in addition to bending, the plate tends to twist. However, the design of the crystal holder prevents twisting with the result that in addition to the moment M_1 (couple PP in Fig. 2) the plate is subjected to a moment M_t opposing the torsion (forces PP' in Fig. 2). The deformation of a plate loaded in this manner is satisfactorily approximated by the following expression

$$W = \frac{a_{35}}{2\rho\left[a_{33} - a_{35}^2 / (a_{44}b^2/a^2 + a_{55})\right](a_{44}b^2/a^2 + a_{55})} \times \left\{a_{45}\frac{b^2}{a^2}x^2 - \left[a_{45} - \frac{a_{34}}{a_{35}}\left(a_{44}\frac{b^2}{a^2} + a_{55}\right)\right]y^2 + 2\frac{b^2}{a^2}a_{44}xy\right\}, \tag{2}$$

which is rigorously valid for a plate of elliptic cross section (in which case a and b are the axes of the ellipse) bent to a cylinder of radius ρ. Neglecting the terms with b^2/a^2 (in our case $(b/a)^2 \approx 2.5 \times 10^{-3}$), we obtain

$$W = \frac{1}{2\rho(a_{33} - a_{35}^2 / a_{55})}\left(a_{34} - \frac{a_{45}}{a_{55}}a_{35}\right)y^2. \tag{3}$$

The coefficients in Eq. (3) must be expressed through constants in crystallographic coordinates. Then, for an α-quartz plate with working planes $hk0$, Eq. (3) assumes the form

$$W = -\frac{a_{14}^2}{\rho(a_{11}a_{44} - a_{14}^2\sin^2 3\varphi)}\cos\varphi\sin 3\varphi\cos 2\varphi \cdot y^2, \tag{4}$$

where φ is the angle between the y-cut and principal (50×30) faces of the utilized plate. For plates cut as in our case $(13\bar{4}0$ working planes), $\varphi = 16°10'$ and Eq. (4) yields

$$W = -0.226 \times 10^{-3} y^2$$

with $a_{11} = 12.73 \times 10^{-7}$; $a_{14} = -4.23 \times 10^{-7}$; $a_{44} = 19.66 \times 10^{-7}$ cm²/kg (Ref. 3); $\rho = 200$ cm.

Thus, in this case, too, the reflecting planes proved to be bent and if we neglect displacements in the direction of the x- and y-axes the bent surface will be described by an equation of the form

$$z = k_2 y^2, \quad k_2 = -0.226 \times 10^{-3}\ \mathrm{cm}^{-1}.$$

The width of the diffraction peak is related to the number N of planes participating in the reflection by the familiar expression

$$\Delta\vartheta = \tan\vartheta / \pi N, \tag{5}$$

where ϑ is the Bragg angle. The angle of the parabolic surface relative to the plane cross section $abcd$ (Fig. 2) is $\Delta\vartheta' \approx \tan\Delta\vartheta' = 2k_2 y$. Congruous reflection can occur from planes for which $\Delta\vartheta'$ at the point of incidence does not exceed $\Delta\vartheta$. From the condition $\Delta\vartheta = \Delta\vartheta'$, and bearing in mind that

$$N = (y/d)\tan\vartheta,$$

where d is the interplanar distance between the reflecting planes, we obtain the following expression for the number of congruently reflecting planes:

$$N = \tan\vartheta / \sqrt{2\pi k_2 d}. \tag{6}$$

The thickness of the simultaneously reflecting lamina is given by

$$\Delta y = \sqrt{d / 2\pi k_2}. \tag{7}$$

Thus the plate is subvided in thickness into a series of independently reflecting laminae of thickness Δy. Let us assume that $\Delta y \ll t_x$, where t_x is the attenuation distance defined by the relation

$$t_x = [2r_0(F/v)\,pd\,\tan\vartheta]^{-1}, \quad (8)$$

where $r_0 = e^2/mc^2$, p is the polarization factor, F is the structure factor and v is the volume of the unit cell.

The intensity of reflection from the entire lamina will be proportional to

$$[r_0 p\,(F/v)\,\lambda\Delta y/\cos\vartheta]^2. \quad (9)$$

The width of the diffraction peak will obviously be

$$\Delta\vartheta_\Sigma = 2k_2 T, \quad (10)$$

where T is the thickness of the plate and the integrated reflection coefficient

$$R_\vartheta \sim 2k_2 T\left[r_0 p\left(\frac{F}{v}\right)\frac{\Delta y}{\cos\vartheta}\right]^2 \sim \frac{p^2}{\cos^2\vartheta}\,r_0^2\left(\frac{F}{v}\right)^2 dT\lambda^2. \quad (11)$$

This relation is identical with the expression for the integrated reflection coefficient for a mosaic crystal. Computing Δy from (7) with $k_2 = 0.226 \times 10^{-3}$ cm^{-1} and $d = 1.17 \times 10^{-8}$ cm, and t_x from (8) for $F\,(13\bar{4}0) = 21.0$, $v = 112$ A^3 (Ref. 1), $\lambda_{max} = 2d \sin\vartheta = 0.1$ A, we obtain $\Delta y = 2.87 \times 10^{-3}$ cm and $t_x = 2.3 \times 10^{-2}$ cm, *i.e.*, we see that Δy is really appreciably smaller than t_x [Eq. (9) and the following are valid only under this condition]. From Eq. (10) with $T = 0.16$ mm we then obtain for the width of diffraction peak

$$\Delta\vartheta_\Sigma = 7.2 \times 10^{-5} \text{ radians.}$$

An examination analogous to that above shows that for a plate cut with the $11\bar{2}4$ planes perpendicular to the large faces the dependence of R_ϑ on the wavelength must also be quadratic.

Thus we see that the quadratic dependence of the reflection coefficient of an elastically bent quartz plate on the wavelength can be explained naturally without recourse to any additional hypotheses. The bent plate actually does become similar to a mosaic crystal in consequence of bending of the reflecting planes.

[1] Lind, West and DuMond, Phys. Rev. 77, 475 (1950).
[2] P. I. Lukirskii and O. I. Sumbaev, Izv. Akad. Nauk SSSR, Ser. Fiz. 20, 903 (1956).
[3] S. G. Lekhnitskii, *Theory of Elasticity of an Anisotropic Body*, GTTL, Moscow 1950.
[4] I. B. Borovskii, Dokl. Akad. Nauk. SSSR 72, 487 (1950).
[5] A. B. Gil'varg, Dokl. Akad. Nauk. SSSR 72, 489 (1950).

Translated by E. J. Rosen
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up the crystals as infinitely long unidimensional or two-dimensional atom complexes, bound together by "small" forces of one nature, whereas in the complex itself the atoms are bound by "big" forces of another nature.

6. The difference between the typical molecular crystals (e.g., the CH_4 or C_6H_6 crystals) and the heteropolar molecular crystals (such as the NaCl, $HgCl_2$ or PbS crystals) lies: (1) in the degree of molecularity β; (2) in the nature of the forces in the molecules; (3) in the nature of intermolecular forces. The quantity β is defined as the ratio of the intramolecular energy $U^a \cong D$ (D is the energy of dissociation of the diatomic molecule into ions) to the intermolecular energy U^e per bond. For the substances for which β is given below, it is possible to take $U^e \approx 2S/l$. Example:

$\beta = 300\,(CH_4)$, $200\,(HCl)$, $22\,(HgCl_2)$, $10\,(NaCl)$ taking $l = 12$ in all four cases.

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ERRATA

Volume 5

Page	Line	Reads	Should Read
1043	Eq. (4)		$W = y^2 a_{14}^2 \sin 2\phi / 2\rho\, (a_{11} a_{44} - \alpha_{14}^2 \sin^2 3\phi)$ The coefficient k_2 equals $0.185 \times 10^{-3}\,\text{cm}^{-1}$.
1044	3 from bottom (l.h.)	$\Delta y = 2.87 \times 10^{-3}$ cm	$\Delta y = 3.18 \times 10^{-3}$ cm
	4 from top (r.h.)	$\Delta\vartheta_\Sigma = 7.2 \times 10^{-5}$ radians	$\Delta\vartheta_\Sigma = 5.9 \times 10^{-5}$ radians

Volume 6

Page	Line	Reads	Should Read
1090	4 and 5 from top	2—(d, 3n); and of the I_{53}^{127} cross section, 3—(d, 2n); 4—(d, 3n)	2—(d, 3n) on I_{53}^{127} and 3—(d, 3n); 4—(d, 3n) on Bi_{83}^{209}
1091	6 from bottom expression for determinant C(y)	ρ, γp, h, $1/\rho$	ρy_2, $\gamma p y_2$, $h y_2$, y_2/ρ
1094	7 from bottom	For $\gamma = 5/3$, μ has . . .	Here μ has . . .

Volume 7

Page	Line	Reads	Should Read
55	16 from bottom	Correct submittal date is April 5, 1957	
169	17 from bottom	Delete "Joint Institute for Nuclear Research"	
215	Table	Add: Note. Columns 2—9 give the number of counts per 10^6 monitor counts	
215	Table, column headings	1, 2, 3, 4-7, 8	1, 2, 3, 4, 8-7
312	Eq. (8)	$\ldots (1 \pm \mu/2M)^2$	$\ldots (1 \mp \mu/2M)^2$
313	2, r.h. col.	$\alpha_{33} = 0.235$	$a_{33} = 0.235$
692	Eq. (5)	$m_B/M_B = \ldots \mp [1 + \ldots]$	$m_B/M_B = \mp [1 + \ldots]$
461	Title	. . . Elastically Conducting	. . . Electrically Conducting