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Contribution to the Theory of Exchange Collisions between Fast Nucleons and Deuterons*

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It is shown that, under some general assumptions regarding the nucleon-nucleon scattering amplitude, comparison of the experimental data on exchange scattering of fast nucleons by deuterons with the results of computations carried out in the impulse approximation can be of aid in the phase shift analysis of scattering of neutrons by protons. The phase shift analysis carried out by Klein does not agree with the experimental data on the exchange scattering.

1. THE RESULTS OF EXPERIMENTS with unpolarized nucleons do not give the possibility to determine the spin dependence of nuclear forces. This determination requires experiments with polarized nucleons.

A few years ago, Pomeranchuk,[1] Chew,[2] and Shmushkevich[3] pointed out another way to get information on the spin dependence of nuclear forces. If one investigates the angular dependence of fast protons from exchange collisions of high energy neutrons with deuterons and compares it with the results of calculations in the impulse approximation, one can obtain some information on the spin dependence of exchange forces between the neutron and the proton.

Assuming that, for angles close to 180° ($\theta = 0°$), the amplitude for nucleon-nucleon collision has the form

$$M = a' + b'\sigma_1\sigma_2 = a + b(1 + \sigma_1\sigma_2)/2 = a + bP_S \quad (1)$$

(P_s is the spin exchange operator), and that the functions a and b do not depend on the spin. Pomeranchuk has shown how it is possible to obtain information on the magnitudes a and b from the experimental data on exchange *n-d* scattering. The spins of the particles of the deuteron being parallel, the appearance of fast particles, in the conditions where the momentum $\mathbf{k}_f$ of the fast proton after collision is close to the momentum $\mathbf{k}_i$ of the impinging neutron, is possible only as a consequence of exchange collisions without any spin exchange.

In order to draw more detailed conclusions on the cross section ratio for *n-d* and *n-p* collisions, Chew had to make a series of additional assumptions on the character of nuclear forces.

The theory of Pomeranchuk is in qualitative agreement with the experimental data[4,5] on exchange scattering of 380 Mev neutrons by deuterons. In the general case, however, when (1) is replaced by the relationship[6,7]

$$M = \alpha + \beta(\sigma_1\mathbf{n})(\sigma_2\mathbf{n}) + \gamma_1(\sigma_1\mathbf{P})(\sigma_2\mathbf{P}) + \gamma_2(\sigma_1\mathbf{K})(\sigma_2\mathbf{K}) + \mu(\sigma_1 + \sigma_2, \mathbf{n})$$
$$\mathbf{n} = [\mathbf{k}_i\mathbf{k}_f]/|[\mathbf{k}_i\mathbf{k}_f]|, \quad \mathbf{P} = (\mathbf{k}_i + \mathbf{k}_f)/|\mathbf{k}_i + \mathbf{k}_f|, \quad \mathbf{K} = (\mathbf{k}_i - \mathbf{k}_f)/|\mathbf{k}_i - \mathbf{k}_f|, \quad (2)$$

the expression for exchange scattering cross section becomes, as shown by Pomeranchuk, unwieldy and a simple interpretation of the experimental results becomes impossible.

It is shown in the present paper that, under certain general assumptions concerning M, one can still use the experimental data on exchange scattering of 300 to 400 Mev nucleons by deuterons with small momentum transfer to obtain additional information on the scattering of neutrons by protons for the phase shift analysis of the *n-p* scattering data.

2. The five quantities α, β, γ_1, γ_2, and μ of (2) depend, generally speaking, on the energy of the colliding nucleons as well as on the scattering angle. After averaging over the initial spins and summing over the final spins, the differential *n-p* scattering cross section (in the center-of-mass system) can be expressed in terms of these functions in the following way:

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$$d\sigma_{np}/do = {}^1/_4 \mathrm{Sp} M^+ M = |\alpha|^2 + |\beta|^2 + |\gamma_1|^2 + |\gamma_2|^2 + 2|\mu|^2. \quad (3)$$

For the differential cross section for exchange collision between neutrons and deuterons we have (in the laboratory system):[3]

$$\begin{aligned}\sigma_{nd} do\, d\mathbf{f} = 8\frac{k_f}{k_i}\{{}^1/_6[4|\alpha|^2 + 4|\beta|^2 + 4|\gamma_1|^2 + 4|\gamma_2|^2 + 10|\mu|^2 \\ + {}^1/_2|\alpha+\beta+\gamma_1+\gamma_2|^2]\,\left|\int \Phi_\mathbf{f}^t \exp\{-i\varkappa\rho\}\varphi_0 d\rho\right|^2 + {}^1/_6[{}^1/_2|\alpha-\beta-\gamma_1-\gamma_2|^2 \\ + |\alpha-\beta|^2 + |\gamma_1-\gamma_2|^2 + 2|\mu|^2]\,\left|\int \Phi_\mathbf{f}^s \exp\{-i\varkappa\rho\}\varphi_0 d\rho\right|^2\}\frac{d\mathbf{f}}{(2\pi)^3}do,\end{aligned} \quad (4)$$

where φ_0 represents the deuteron ground state wave function, $\Phi_\mathbf{f}^t$ and $\Phi_\mathbf{f}^s$ are functions which describe the motion of two particles with a relative momentum $\mathbf{f}$

$$\Phi_\mathbf{f}^t(r) = 2^{-1/2}(e^{i\mathbf{fr}} - e^{-i\mathbf{fr}}),$$

$$\Phi_\mathbf{f}^s(r) = 2^{-1/2}(e^{i\mathbf{fr}} + e^{-i\mathbf{fr}}) + (1 - e^{-i2\delta_0})e^{-ifr}/\sqrt{2}ifr, \cot\delta_0 = -\sqrt{\frac{\varepsilon}{E}}.$$

Denoting by S_1 the coefficient of the first integral in (4) and by S_2 the coefficient of the second integral, and performing an approximate integration over $d\mathbf{f}$, we have the following expression[3] for the differential cross section for fixed-angle scattering of a fast particle:

$$\frac{d\sigma_{nd}}{do} = 4\cos\theta\left\{(S_1 + S_2) - (S_1 - S_2)\frac{1}{\theta}\sqrt{\frac{2\varepsilon}{E}}\arctan\left(\theta\sqrt{\frac{E}{2\varepsilon}}\right)\right\}. \quad (5)$$

This expression does not depend on the relative motion of the two slow nucleons. In view of what follows, let us present M in a somewhat different form:

$$\begin{aligned}M = BS + C(\sigma_1 + \sigma_2, \mathbf{n}) + \{{}^1/_2 G[(\sigma_1\mathbf{K})(\sigma_2\mathbf{K}) + (\sigma_1\mathbf{P})(\sigma_2\mathbf{P})] \\ + {}^1/_2 H[(\sigma_1\mathbf{K})(\sigma_2\mathbf{K}) - (\sigma_1\mathbf{P})(\sigma_2\mathbf{P})] + N(\sigma_1\mathbf{n})(\sigma_2\mathbf{n})\}T.\end{aligned} \quad (6)$$

Here

$$S = {}^1/_4(1 - \sigma_1\sigma_2), \quad T = {}^1/_4(3 + \sigma_1\sigma_2)$$

are the singlet and triplet projection operators.

The functions B, C, ... of (6) can be expressed in terms of α, β, ... of (2). In order to obtain this relationship, let us transform (2) into the form (6), using relations of the type

$$(\sigma_1\mathbf{n})(\sigma_2\mathbf{n})T = (\sigma_1\mathbf{n})(\sigma_2\mathbf{n}) + S$$

etc., and the inequality

$$(\sigma_1\mathbf{P})(\sigma_2\mathbf{P}) + (\sigma_1\mathbf{K})(\sigma_2\mathbf{K}) + (\sigma_1\mathbf{n})(\sigma_2\mathbf{n}) = (\sigma_1\sigma_2),$$

which is valid for three mutually-orthogonal arbitrary vectors **P**, **K** and **n**. We get

$$\begin{aligned}\alpha = {}^1/_4(B + N + G), \gamma_1 = {}^1/_4(G - N - B - 2H), \beta = -{}^1/_4(G - N + B - 2N), \\ \gamma_2 = {}^1/_4(G - N - B + 2H), \quad \mu = C.\end{aligned} \quad (7)$$

In the center-of-mass system of the colliding nucleons, we have

$$S_1 + S_2 = {}^1/_4 \mid B\mid^2 + {}^1/_4 \mid G - N\mid^2 + {}^1/_2 \mid N\mid^2 + {}^1/_2 \mid H\mid^2 + 2 \mid C\mid^2 = d\sigma_{np}/do, \tag{8}$$

$$6(S_1 - S_2) = 4(d\sigma_{np}/do) + \mathrm{Re}\,N^*(G - N - B) - \mid H\mid^2 - {}^1/_4 \mid B + G - N\mid^2. \tag{9}$$

Using (8) and (9), the expression (5) for the transverse cross section for exchange *n-d* collisions can be presented in the following form (in the laboratory system)

$$\begin{aligned}\frac{d\sigma_{nd}}{do} &= \left[1 - \frac{2}{3\theta}\sqrt{\frac{2\varepsilon}{E}}\,\mathrm{arc\ tan}\left(\theta\sqrt{\frac{E}{2\varepsilon}}\right)\right]\frac{d\sigma_{np}}{do} - {}^2/_3\cos\theta\,[\mathrm{Re}\,N^*(G - N - B)\\ &- \mid H\mid^2 - {}^1/_4 \mid B + G - N\mid^2]\frac{1}{\theta}\sqrt{\frac{2\varepsilon}{E}}\,\mathrm{arc\ tan}\left(\theta\sqrt{\frac{E}{2\varepsilon}}\right),\end{aligned} \tag{10}$$

which becomes for $\theta = 0°$ ($\theta_n = 180°$)

$$\frac{d\sigma_{nd}(0°)}{do} = \frac{1}{3}\frac{d\sigma_{np}(180°)}{do} - \frac{2}{3}[\mathrm{Re}\,N^*(G - N - B) - \mid H\mid^2 - {}^1/_4 \mid B + G - N\mid^2]_{\theta_n = 180°}. \tag{11}$$

It is interesting to compare the expressions for the exchange *n-d* collision cross-section with the corresponding expressions for exchange collisions of π-mesons with deuterons. For the π-*N* scattering, the matrix M can be written in the form $M = A + B(\sigma\mathbf{n})$, where the functions $A(\theta)$ and $B(\theta)$ can be expressed in the following way:[8]

$$\begin{aligned}A(\theta) &= \frac{1}{k}\sum_l [(l+1)\exp\{i\delta_l^+\}\sin\delta_l^+ + l\exp\{i\delta_l^-\}\sin\delta_l^-]\,P_l(\theta),\\ B(\theta) &= \frac{1}{ik}\sum_l [\exp\{i\delta_l^+\}\sin\delta_l^+ - \exp\{i\delta_l^-\}\sin\delta_l^-]\,P_l^1(\theta).\end{aligned} \tag{12}$$

In the same impulse approximation, one obtains for the cross section for exchange scattering of π-mesons by deuterons[9]

$$\frac{d\sigma}{do}(\pi^- - d \mid \pi^0\,2n) = \frac{d\sigma}{do}(\pi^- p \mid \pi^0 n) - (\mid A\mid^2 + {}^1/_3 \mid B\mid^2)\frac{2\alpha}{\mid \mathbf{k}_i - \mathbf{k}_f\mid}\,\mathrm{arc\ tan}\frac{\mid \mathbf{k}_i - \mathbf{k}_f\mid}{2\alpha}. \tag{13}$$

This process of interaction of π-mesons with deuterons leads to the formation of two slow identical nucleons in the final state. Because of the Pauli principle, the cross section for this process is characterized by a considerable decrease of the cross section for meson-deuteron interaction compared with the cross section for meson-nucleon interaction in the region of small angles of the outgoing fast particle. A similar decrease of the cross section for the case of exchange *n-d* scattering is given by the first term of (10). The presence of the term in square brackets in (10) and (11) and the absence of analogous expressions in (13) are explained by the absence of spin dependence in the π-*N* interaction for $\theta \to 0°$ and by the presence of spin dependence in (2) and (6) even for $\theta = 0°$ and $\theta = 180°$.

3. In Refs. 1–3 the functions α, β, ... were left arbitrary and no further analysis was possible without making additional assumptions.

Using the work of Wright[10] one can, as in the case of the π-*N* interaction, express these functions in terms of the phase shifts for nucleon-nucleon scattering and of Legendre polynomials and their derivatives (notation of Ref. 10):*

$$B = \frac{1}{2ik}\sum_l \alpha_{l,0}^N(2l+1)P_l = \frac{1}{2}[B_1(\theta) + B_0(\theta)] = \frac{1}{2ik}\left\{\sum_{l=2k}\alpha_{l,0}^N(2l+1)P_l + \sum_{l=2k+1}\alpha_{l,0}^N(2l+1)P_l\right\},$$

* Similar expressions were obtained by R. M. Ryndin and by Stech and Bakke.[11]

$$C = \frac{1}{2}\frac{\sin\theta}{4k}\left\{\sum_l\left[\beta_1^{l+1}\frac{(2l+3)}{l+1}P'_l - \beta_2^{l+1}\frac{(2l+3)}{l+2}P'_{l+2} - \alpha_{l,1}^N\frac{(2l+1)}{l(l+1)}P'_l\right] - \alpha_{0,1}^N\right\} = \frac{1}{2}\left[C_1(\theta) + C_0(\theta)\right], \tag{14}$$

$$N = {}^1/_2\left[N_1(\theta) + N_0(\theta)\right],\quad H = {}^1/_2\left[H_1(\theta) + H_0(\theta)\right],\quad G - N = {}^1/_2\left[G_1(\theta) - N_1(\theta) + G_0(\theta) - N_0(\theta)\right].$$

The isotopic invariance of nucleon-nucleon interaction is taken into account in the second halves of these equations. The matrix $M_1 = M_{T=1}$ is meant to be the M-matrix for p-p scattering without electromagnetic effects. The quantities B_0, C_0, N_0, and G_0 contain the contribution of the n-p state with $T = 0$. The functions B_1, C_1, N_1, and G_1 do not reverse signs as one makes the substitution $\theta \rightleftarrows \pi - \theta$, but the signs of the functions B_0, C_0, H_0, N_0 and G_0 are reversed by the substitution.

The functions B, N, H and $G - N$ are involved in (10) and (11) in combinations which are different from the combinations involved in the expression for the scattering cross section of an unpolarized beam by an unpolarized target; the investigation of exchange n-d scattering gives therefore additional information on the spin dependence of exchange forces between the neutron and proton. If one compares (10) and (11) with the expressions for the functions $D(\theta)$ and $R(\theta)$ which characterize the change of polarization after scattering a polarized beam by an unpolarized target:[12,13]

$$(d\sigma_{np}/do)\,(1 - D(\theta)) = {}^1/_4\,|G - N - B|^2 + |H|^2,$$

$$(d\sigma_{np}/do)\,R(\theta) = {}^1/_2\,\mathrm{Re}\left[(G - N + B)N^* + (G - N - B)H^*\right]\cos\frac{\theta}{2} + \mathrm{Im}\left[C^*(G - N + B)\right]\sin\frac{\theta}{2}, \tag{15}$$

it becomes clear that the experiments on exchange n-d scattering are, in the impulse approximation framework, close to the experiments on triple nucleon scattering. In this fashion, the experimental data on exchange n-d scattering together with the results of experiments with polarized nucleons can be used to obtain information on the spin dependence of the amplitude of exchange n-p scattering for the phase shift analysis of the n-p scattering data.

4. The experimental data of V. Dzhelepov, Kazarinov, and Fliagin[4,5] on exchange collision of 380 Mev neutrons with deuterons indicate a considerable decrease of the fast proton yield at angles close to 0°. The ratio $\sigma_{nd}(0)/\sigma_{np}(0)$ is 0.2 ± 0.035. Within the framework of Pomeranchuk's investigation,[1] this indicates a considerable probability of spin exchange in the exchange n-p collision. Using (11), the principal result of the experiment can be presented in the form

$$\left[\mathrm{Re}\,N^*(G - N - B) - |H|^2 - {}^1/_4\,|B + G - N|^2\right]\Big|_{\theta_n = 180^\circ} = {}^1/_5\,d\sigma_{np}(180^\circ)/do. \tag{16}$$

From the point of view of phase shift analysis, the expression (16) picks out, from among all the scattering phase shifts which agree with the experimental data on unpolarized n-p scattering cross section and on the polarization in n-p collisions, only those phase shifts which agree with the experimental data on n-d scattering. From the same point of view, there is another source of independent information on n-p scattering; it is the investigation of the polarization of the fast nucleons formed in exchange n-d collisions. For small momentum transfer, the polarization of fast nucleons can, in the impulse approximation, be expressed in terms of the amplitude of n-p scattering. The expression for the polarization differs from the corresponding expression for the case of collisions of a neutron with a free proton in the region of small angles.

If the functions B, C, ... are expressed in terms of the nucleon-nucleon scattering phase shifts, the comparison of the n-p scattering data with expressions of the type (10) show the applicability of the impulse approximation in similar problems, when the impinging particle interacts with a bound particle. For the same purpose, it can be profitable to compare the results of calculations with the experimental data on the exchange pion-deuteron collisions. In this case the situation is simplified by the fact that the meson-nucleon scattering phase shifts are now determined with a sufficient reliability.

The literature does not contain any uniquely determined n-p scattering phase shifts for the high energy region. If one still uses the phase shift analysis of Klein[14] for 330 Mev neutrons, the ratio $\sigma_{nd}(0)/\sigma_{np}(0)$ turns out to be close to unity. This is in strong contradiction with the experimental data on exchange n-d scattering, probably due principally to the fact that Klein's analysis (which underestimates the role of states with $L \geq 2$) agrees badly with the experimental data on n-p scattering in the angular region close to 180°.

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On the Magnetic Properties of Superconductors of the Second Group

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A study is made of the magnetic properties of bulk superconductors for which the parameter $\varkappa$ of the Ginzburg-Landau theory is greater than $1/\sqrt{2}$ (superconductors of the second group). The results explain some of the experimental data on the behavior of superconductive alloys in a magnetic field.

THE AUTHOR[1] has already noted that the quasimicroscopic Ginzburg-Landau theory[2] of superconductivity leads to the conclusion that there exist two groups of superconductors. For the first of these groups, the parameter $\varkappa$ entering into the Ginzburg-Landau theory is less than $1/\sqrt{2}$, and for the second group it is greater than $1/\sqrt{2}$.

This parameter $\varkappa$ determines to a great extent the surface energy at the normal – superconducting interface. It has already been mentioned,[2] in particular, that the calculated surface energy of a superconductor with $\varkappa > 1/\sqrt{2}$ is negative. Thus superconductors of the second group should have properties very different from those of the first group.

For pure metal, $\varkappa$ is found to be small. For instance for mercury, $\varkappa = 0.16$. In view of this, Ginzburg and Landau considered only the case in which $\varkappa \ll 1/\sqrt{2}$.

It has been shown by Zavaritskii,[3] however, that the properties of pure metal thin films condensed at liquid-helium temperatures are not described by such a theory. Zavaritskii and the present author have therefore suggested that such films correspond to $\varkappa > 1/\sqrt{2}$, and that superconductors can thus be divided into two groups. The critical field for superconductors of the second group has already been calculated[1] as a function of the film thickness. The agreement obtained with Zavaritskii's experimental data was not bad.

In the present work, a more detailed investigation of the magnetic properties of bulk superconductors of the second group (a cylinder in a longitudinal field) is undertaken. The results obtained show