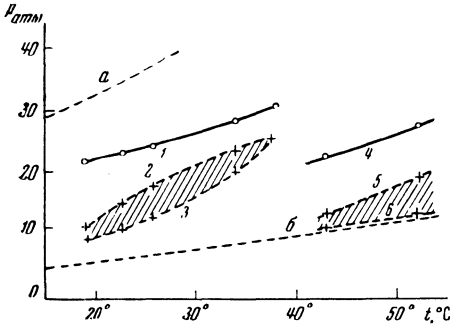

Dash lines—pressures of the unsaturated vapors of pure: (a) Freon-13, (b) Freon-12. Solid curves—same for the mixtures: (1) 40% Freon-12, (4) 75% Freon-12. Curves 2 and 3 bound the working region for the first mixture; curves 5 and 6 bound the region for the second mixture.

observed in the chamber at temperatures of 34 and 38° C for the first mixture and 52° C for the second mixture; at other temperatures there was no fog formation since it was impossible to realize a greater pressure drop due to boiling of the mixture in the separator.

The chamber was expanded every 10 sec. In all experiments the pressure in the chamber between expansions was 35 atmospheres. The sensitivity time was determined from photographs exposed with different delays relative to the beginning of pressure relief. The sensitivity time was found to be 40 microseconds.

Thus satisfactory operation of a bubble chamber is possible with a mixture which at room temperature has a saturated vapor pressure of about 21 atmospheres. The suggested mixture by virtue of its high density (~ 1.0 g/cm^3) is convenient for many nuclear investigations. The use of mixtures makes it easier to select the best "working substance" for a given physical problem. Of particular interest would be hydrogen-containing mixtures, for example, a mixture of methane with propane, which has been used in a "gas" bubble chamber,[2] or a mixture of ethane and propane.

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Two-Electron Green Function in the Bloch–Nordsieck Approximation

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THE ELECTRON GREEN FUNCTION in the Bloch–Nordsieck approximation[2] has been considered in an earlier work.[1] As is well known, the Bloch–Nordsieck approximation, in which the matrix γ^μ is replaced by the c-number u^μ, allows a consideration of the interaction of an electron with long-wave photons. In the present work we consider the two-electron Green function in this same approximation.

In the Bloch–Nordsieck approximation the Schwinger equation[3] has the following form:

$$\left\{iu^\mu\frac{\mu\partial}{\partial x^\mu} - m + V\overline{4\pi}\,eu^\mu A_\mu(x) + iV\overline{4\pi}\,eu^\mu\int D_{\mu\nu}(x,\xi)\frac{\delta}{\delta A_\nu(\xi)}\,d\xi\right\}G(x,x';y,y'\mid A)$$
$$= \delta(x-y')\,G(x',y\mid A) - \delta(x-y)\,G(x',y'\mid A), \qquad (1)$$
$$u^2 = (u^0)^2 - \mathbf{u}^2 = 1, \qquad m \to m - i\varepsilon\,(\varepsilon > 0,\ \varepsilon \to 0).$$

In this case the one-electron Green function satisfies the equation

$$F_x G(x,y\mid A) = -\delta(x-y), \qquad (2)$$

where F_x is the operator which appears in the curly brackets in the left-hand part of Eq. (1).

The solution of Eq. (1) is given by

$$G(x, x'; y, y' \mid A) = G(x, y \mid A)\, G(x', y' \mid A) - G(x, y' \mid A)\, G(x', y \mid A) + G_{12}(x,x'; y, y' \mid A) - G_{12}(x, x'; y', y \mid A) \tag{3}$$

and for the function G_{12} we get the following equation:

$$F_x G_{12}(x, x'; y, y' \mid A) = -i\sqrt{4\pi}\, eu^{\mu} \int D_{\mu\nu}(x, \xi)\, \frac{\delta G(x', y' \mid A)}{\delta A_{\nu}(\xi)}\, G(x, y \mid A)\, d\xi. \tag{4}$$

To solve this equation we use the same method as in the earlier work. The following expressions follow from the homogeneity of space and time:

$$\begin{gathered} G_{12}(x, x'; y, y' \mid A) = G_{12}(x' - x, y - x, y' - x \mid T_x A), \\ G(x, y \mid A) = G(y - x \mid T_x A), \qquad D_{\mu\nu}(x, \xi) = D_{\mu\nu}(\xi - x) \end{gathered} \tag{5}$$

where $T_x A_\mu(\xi) = A_\mu(\xi + x)$.

Using these relations we can write Eq. (4) in momentum space in the form

$$\begin{aligned} &\left\{(up_1) + (up_2) + (up_3) - m - \int (uk)\, A_{\mu}(\kappa)\, \frac{\delta}{\delta A_{\mu}(k)}\, dk + \sqrt{4\pi}\, \frac{eu^{\mu}}{(2\pi)^2} \int A_{\mu}(k)\, dk \right. \\ &\left. + i\sqrt{4\pi}\, \frac{eu^{\mu}}{(2\pi)^2} \int D_{\mu\nu}(k)\, \frac{\delta}{\delta A_{\nu}(k)}\, dk \right\} G_{12}(p_1, p_2, p_3 \mid A) \\ &= -i\sqrt{4\pi}\, \frac{eu^{\mu}}{(2\pi)^2} \int D_{\mu\nu}(k) \left[\frac{\delta G(p_3 \mid A)}{\delta A_{\nu}(k)}\right]_{A \to T_x A} \cdot G(p_2 \mid A)\, e^{i(k - p_1 - p_3)x} dx\, dk, \end{aligned} \tag{6}$$

where $T_x A_\mu(k) = e^{ikx} A_\mu(k)$.

Using the proper-time method of Fock,[4] Eq. (6) can be solved exactly; the solution has the form

$$\begin{aligned} G_{12}(p_1, p_2, p_3 \mid A) = &-\sqrt{4\pi}\, \frac{eu^{\mu_1}}{(2\pi)^2} \int_0^{\infty} d\nu e^{-\varepsilon\nu} e^{-i(m - up_1 - up_2 - up_3)\nu + f(\nu)} \exp\left\{-\sqrt{4\pi}\, \frac{eu^{\mu}}{(2\pi)^2} \int \frac{e^{-i(up)\nu} - 1}{(up)}\, A_{\mu}(p)\, dp\right\} \\ &\times \int D_{\mu_1\nu_1}(k) \left\{\left[\frac{\delta G(p_3 \mid A)}{\delta A_{\nu_1}(k)}\right]_{A \to T_x A} G(p_2 \mid A)\right\}_{A \to A^{T_\nu}} e^{i(k - p_1 - p_3)x} dx\, dk, \end{aligned} \tag{7}$$

$$A_{\mu}^{T_\nu}(k) = A_{\mu}(k)\, e^{-i(uk)\nu} - i\sqrt{4\pi}\, \frac{eu^{\sigma}}{(2\pi)^2}\, D_{\sigma\mu}(k)\, \frac{e^{-i(uk)\nu} - 1}{(uk)},$$

$$f(\nu) = -i\, \frac{e^2}{4\pi^3} \int_0^{\nu} d\nu \int_0^{\nu} d\nu \int e^{-i(uk)\nu} u^{\mu} u^{\rho} D_{\mu\rho}(k)\, dk. \tag{8}$$

In the Bloch–Nordsieck approximation there is no vacuum polarization; whence it follows that:

$$\begin{gathered} D_{\mu\nu}(k) = -g^{\mu\nu} / (k^2 + i\varepsilon) - g^{\mu\nu} / (M^2 - k^2 - i\varepsilon) \qquad (M \to \infty) \\ g^{\mu\nu} = 0\ (\mu \neq \nu), \qquad g^{00} = -g^{11} = -g^{22} = -g^{33} = 1, \end{gathered} \tag{9}$$

where M is the Pauli–Villars auxiliary mass. Carrying out the indicated integrations in (8) we find

$$f(\nu) = -i\, \frac{e^2}{2} M\nu + \frac{e^2}{\pi} \ln \frac{M}{m'} + \frac{e^2}{\pi} \ln m'\nu, \tag{10}$$

and from the previous work we have

$$G(p_1 \mid A) = i \int_0^{\infty} d\nu e^{-\varepsilon\nu} e^{-i(m - up_1)\nu + f(\nu)} \exp\left\{-\sqrt{4\pi}\, \frac{eu^{\mu}}{(2\pi)^2} \int \frac{e^{-i(up)\nu} - 1}{(up)}\, A_{\mu}(p)\, dp\right\}. \tag{11}$$

The divergence which appears in the function in (7) can be removed by renormalization. The first term in (10), which contains the auxiliary mass M, is removed by mass renormalization

$$m' = m + e^2 M / 2, \tag{12}$$

where m' is the observed mass of the electron. The other terms which contain M go into the renormalized constant in the Green function.

Using the present method it is also possible to determine Green functions for three and more electrons in the Bloch–Nordsieck approximation; in these cases, however, the expressions are much more complicated.

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Green Function in Scalar Electrodynamics In The Boch-Nordsieck Approximation

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A METHOD WHICH HAS been applied earlier[1] is used to consider the Green function in scalar electrodynamics in the Boch-Nordsieck approximation,[2] *i.e.*, when the particle recoil can be neglected. As is well known, the infra-red catastrophe does not arise in this case.

The Green function is scalar electrodynamics satisfies the equation

$$\left\{m^2 - \left[i\frac{\partial}{\partial x^\mu} + V\overline{4\pi}\, eA_\mu(x) + iV\overline{4\pi}e\int D_{\mu\nu}(x,\xi)\frac{\delta}{\delta A_\nu(\xi)}d\xi\right]^2\right\} G(x, y \mid A) = \delta(x-y). \tag{1}$$

Using the invariance of the Green function against translation, in the momentum representation Eq. (1) can be written in the form

$$\left\{m^2 - \left[p_\mu - \int k_\mu A_\nu(k)\frac{\delta}{\delta A_\nu(k)}dk + V\overline{4\pi}\frac{e}{(2\pi)^2}\int A_\mu(k)\,dk + iV\overline{4\pi}\frac{e}{(2\pi)^2}\int D_{\mu\nu}(k)\frac{\delta}{\delta A_\nu(k)}dk\right]^2\right\} G(k \mid A) = 1. \tag{2}$$

In the Boch-Nordsieck approximation Eq. (2) assumes the form

$$\left\{m^2 - \left[u^\mu\left(p_\mu - \int k_\mu A_\nu(k)\frac{\delta}{\delta A_\nu(k)}dk + V\overline{4\pi}\frac{e}{(2\pi)^2}\int A_\mu(k)\,dk + iV\overline{4\pi}\frac{e}{(2\pi)^2}\int D_{\mu\nu}(k)\frac{\delta}{\delta A_\nu(k)}dk\right)\right]^2\right\} G(p \mid A) = 1;$$

$$u^\mu = p^\mu / |p|, \quad m \to m - i\varepsilon \quad (\varepsilon > 0, \varepsilon \to 0). \tag{3}$$

In this same approximation the photon Green function is

$$D_{\mu\nu}(k) = -g^{\mu\nu}/(k^2 + i\varepsilon) - g^{\mu\nu}/(M^2 - k^2 - i\varepsilon) \quad (M \to \infty).$$
$$g^{\mu\nu} = 0 \ (\mu \neq \nu), \ g^{00} = -g^{11} = -g^{22} = -g^{33} = 1, \tag{4}$$

where M is the Pauli-Villars auxiliary mass.

Using the proper-time method of Fock,[3] Eq. (3) can be solved exactly. Terms containing the auxiliary mass M are removed by renormalization and the following expression is found for the renormalized Green function $G' = (M/m')^{e^2/\pi} G$: