which is known to satisfy Eq. (1) and the condition given in (7). In cases (I) and (II) we have

$$\omega = i\left[\left(\frac{g}{u}+\frac{\alpha}{\rho u}\right)(k_x^2+k_y^2)\right] = -uk_z, \tag{38.I}$$

$$k_y = 0;\ k_x = ik_z = k \leqslant 0, \qquad \omega = \frac{1}{u}\left(g+\frac{\alpha}{\rho}k^2\right) = -uk_x. \tag{38.II}$$

In case I we do not obtain stable waves of the form given in (37). In case II we obtain a wave which decays in the z-direction and which is characterized by a penetration depth

$$d = -1/k = (\rho u^2/2\alpha)(1 \pm \sqrt{1-4\alpha g/\rho u^4}). \tag{39}$$

Thus, the vortex magnetohydrodynamic surface waves have the same propogation and decay relations as the potential waves with the one exception that the current density is not zero; these waves are similar to the skin effect in this respect.

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11

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ON THE DYNAMICS OF A BOUNDED PLASMA IN AN EXTERNAL FIELD

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Some problems relating to the dynamics of a quasi-neutral plasma formation situated in the field of a plane electromagnetic wave are considered. The method of successive approximations is employed. It is shown that within the limits of the assumptions underlying the analysis a plasma bunch tends to spread out.

In connection with the proposal of a radiation method of accelerating particles[1] the question has arisen of the behavior of a quasi-neutral plasma condensation situated in the field of an electromagnetic wave and, in particular, the question of the stability of such a condensation. A more or less rigorous solution of this problem encounters formidable mathematical difficulties. Therefore it is not without interest to consider a simplified problem which would nevertheless allow one to obtain approximate estimates of the magnitude and the nature of the forces acting on the condensation. The present paper is devoted to the examination of one of these simplest cases.

We consider a system of electrons and ions subjected to an incident plane electromagnetic wave with a propagation vector **k** parallel to the z axis. By using the hydrodynamic description of plasma,* which is

* We shall not consider here questions concerning the permissibility of applying the hydrodynamic approximation.

apparently quite permissible in order to obtain results of an exploratory nature, we shall describe the system by specifying the densities ρ_i, ρ and the velocities $\mathbf{v}_i$ and $\mathbf{v}$ of the ions and the electrons respectively, and we shall assume that the total number of electrons is equal to the total number of ions, i.e.,

$$\int \rho_i(\mathbf{r})\, d\mathbf{r} = \int \rho(\mathbf{r})\, d\mathbf{r}.$$

Since the mass of the ion is much larger than the mass of the electron, then in the first approximation we may consider the ions to be at rest, i.e., $\mathbf{v}_i = 0$, and to have a time-independent density distribution $\rho_i = \rho_0(\mathbf{r})$. If the variable external field is described by means of the vector-potential $\mathbf{A}_{ext} = \{A_0, 0, 0\} \times e^{ikz - i\omega t}$ while the self-field is described by means of the potentials $\mathbf{A}(\mathbf{r}, t)$ and $\varphi(\mathbf{r}, t)$ then the equations which determine ρ, $\mathbf{v}$, φ and $\mathbf{A}$ will have the form:

$$\partial\rho / \partial t = -\nabla(\rho\mathbf{v}); \tag{1}$$

$$\rho\left[\frac{\partial\mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\mathbf{v}\right] = -\frac{e}{m}\rho\left\{\nabla\varphi + \frac{1}{c}\frac{\partial\mathbf{A}}{\partial t} + \frac{1}{c}\frac{\partial\mathbf{A}_{ext}}{\partial t} - \frac{1}{c}[\mathbf{v}[\nabla\mathbf{A}]] - \frac{1}{c}[\mathbf{v}[\nabla\mathbf{A}_{ext}]]\right\} - v_T^2\nabla\rho; \tag{2}$$

$$\square\mathbf{A} = -(4\pi e/c)\rho\mathbf{v}; \quad \square\varphi = -4\pi e(\rho_0 - \rho). \tag{3}$$

Here v_T plays the role of the speed of sound in the plasma. Equations (1) — (3) represent a system of non-linear differential equations. By utilizing the formalism of Green's functions we shall write the system of equations (2), (3) in the form of a single (non-linear) integro-differential equation. We then obtain:

$$\partial\rho / \partial t = -\nabla(\rho\mathbf{v}); \tag{4}$$

$$\frac{\partial\mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\mathbf{v} = \frac{e}{m}\left\{\mathbf{E}_{ext} + \mathbf{E}(\rho\mathbf{v}) + \frac{1}{c}[\mathbf{v}\mathbf{H}_{ext}] + \frac{1}{c}[\mathbf{v}\mathbf{H}(\rho\mathbf{v})]\right\} - v_T^2\frac{\nabla\rho}{\rho}; \tag{5}$$

where

$$\mathbf{E}_{ext} = -\frac{1}{c}\frac{\partial\mathbf{A}_{ext}}{\partial t}, \quad \mathbf{H}_{ext} = \operatorname{curl}\mathbf{A}_{ext},$$

$$\mathbf{E}(\rho\mathbf{v}) = -\frac{e}{c^2}\frac{\partial}{\partial t}\iint G(\mathbf{r}\mathbf{r}'tt')\,\mathbf{v}(\mathbf{r}'t')\,\rho(\mathbf{r}'t')\,d\mathbf{r}'dt' + e\,\operatorname{grad}\operatorname{div}\iint G(\mathbf{r}\mathbf{r}'tt')\,d\mathbf{r}'dt'\int\mathbf{v}(\mathbf{r}'t'')\,\rho(\mathbf{r}'t'')\,dt''; \tag{6}$$

$$\mathbf{H}(\rho\mathbf{v}) = \frac{e}{c}\operatorname{curl}\iint G(\mathbf{r}\mathbf{r}'tt')\,\mathbf{v}(\mathbf{r}'t')\,\rho(\mathbf{r}'t')\,d\mathbf{r}'dt', \tag{7}$$

$$G(\mathbf{r}\mathbf{r}'tt') = \delta(t' - t + |\mathbf{r} - \mathbf{r}'|/c)/|\mathbf{r} - \mathbf{r}'$$

is the Green function satisfying the equation:

$$\square G(\mathbf{r}\mathbf{r}'tt') = -4\pi\delta(\mathbf{r} - \mathbf{r}')\,\delta(t - t').$$

The exact solution of Eqs. (4), (5) is difficult;* however, under certain assumptions it is possible to obtain an approximate estimate of the magnitude of the forces and apparently to reach some conclusions with respect to the nature of the possible deformations. In particular, we shall tackle the problem on the assumption of small non-linearity and of weak self-fields. Thus, we assume: (1) that the particle density is not too high, i.e., $E_{ext} \gg E$, (2) that the external field and the wavelength are such that† $v/c \ll 1$, and we shall solve the system of equations (4), (5) by the method of successive approximations taking for our small parameter λ the ratios E/E_{ext} and v/c. Moreover, we shall assume that $\rho_0(\mathbf{r})$ satisfies the following conditions: (1) $\rho_0(\mathbf{r})$ is a function of the radius only, (2) $\rho_0(\mathbf{r})$ varies slowly for $r < a$, and falls off sufficiently rapidly for $r > a$ where a is a characteristic dimension of the system (for example, like $\exp\{-(r/a)^n\}$ where $n \geq 2$).

Now by representing ρ and $\mathbf{v}$ in the form

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* In general, the exact solution of (4), (5) does not appear to be required since it will not be able to give us a sufficiently detailed answer to the questions of interest to us because of the assumption made above that the ions are at rest; however, the consideration of the case of high densities is of some interest even within the limitations imposed on the problem considered above.

†The criterion for the fulfillment of the above conditions will be obtained below.

$$\rho = \rho_0(r) + \sum \lambda^n \rho_n(\mathbf{r}), \quad \mathbf{v} = \mathbf{v}_0(\mathbf{r}) + \sum \lambda^n \mathbf{v}_n(\mathbf{r}) \tag{8}$$

and substituting into (4) and (5) we shall obtain after restricting ourselves to terms up to the second order (in λ) inclusively:

$$\frac{\partial \mathbf{v}_0}{\partial t} = \frac{e}{m}\mathbf{E}_{\text{ext}}, \quad \frac{\partial \mathbf{v}_2}{\partial t} = \frac{e}{m}\Big[\mathbf{E}(\rho_0\mathbf{v}_1) + \mathbf{E}(\rho_1\mathbf{v}_0) + \frac{1}{c}[\mathbf{v}_1\mathbf{H}_{\text{ext}}]\Big] - (\mathbf{v}_0\nabla)\mathbf{v}_0, \quad \frac{\partial \mathbf{v}_1}{\partial t} = \frac{e}{m}\Big[\mathbf{E}(\rho_0\mathbf{v}_0) + \frac{1}{c}[\mathbf{v}_0\mathbf{H}_{\text{ext}}]\Big] - (\mathbf{v}_0\nabla)\mathbf{v}_0,$$

$$+ \frac{1}{c}[\mathbf{v}_0\mathbf{H}(\rho_0\mathbf{v}_0)]\Big] - (\mathbf{v}_0\nabla)\mathbf{v}_1 - (\mathbf{v}_1\nabla)\mathbf{v}_0 - v_T^2\frac{\nabla\rho_0}{\rho_0} \tag{9}$$

so that consequently the force acting on the electrons of the condensation is given by

$$\mathbf{F} = e\Big\{\mathbf{E}_{\text{ext}} + \mathbf{E}(\rho_0\mathbf{v}_0) + \mathbf{E}(\rho_1\mathbf{v}_0) + \mathbf{E}(\rho_0\mathbf{v}_1) + \frac{1}{c}[\mathbf{v}_0\mathbf{H}_{\text{ext}}] + \frac{1}{c}[\mathbf{v}_1\mathbf{H}_{\text{ext}}] + \frac{1}{c}[\mathbf{v}_0\mathbf{H}(\rho_0\mathbf{v}_0)]\Big\}$$

$$- m\Big[(\mathbf{v}_0\nabla)\mathbf{v}_0 + (\mathbf{v}_0\nabla)\mathbf{v}_1 + (\mathbf{v}_1\nabla)\mathbf{v}_0 - v_T^2\frac{\nabla\rho_0}{\rho_0}\Big], \tag{10}$$

where $\mathbf{v}_0$, $\mathbf{v}_1$, ρ are determined by (9). We note that if we start to calculate the next approximations we shall obtain secular terms, and that consequently the series for ρ and $\mathbf{v}$ diverge. However, for sufficiently small times $t < \tau \approx c\omega/v\omega_0^2$ the expressions for ρ, $\mathbf{v}$ and consequently for $\mathbf{F}$, remain valid, and since for the reasons given above (see second footnote) we are not interested in an exact solution of our problem, the result obtained above appears to be satisfactory. Moreover, we regarded the term $v_T^2\nabla\rho_0/\rho_0$ as being of second order; in the opposite case (i.e., at sufficiently high temperatures) instability apparently occurs.

Since we are interested either in constant forces, or in forces varying slowly in time (in comparison with the frequency of the external field) the expression for $\mathbf{F}$ may be averaged over a time of the order of several periods $T = 2\pi/\omega$, and it may be easily shown that in the course of this averaging a number of terms drops out and we obtain:

$$\overline{\mathbf{F}} = \overline{\frac{e}{c}\{[\tilde{\mathbf{v}}_1\mathbf{H}_{\text{ext}}] + [\tilde{\mathbf{v}}_0\mathbf{H}(\rho_0\tilde{\mathbf{v}}_0)]\} - m[(\tilde{\mathbf{v}}_1\nabla)\tilde{\mathbf{v}}_0 + (\tilde{\mathbf{v}}_0\nabla)\tilde{\mathbf{v}}_1]} - mv_T^2\frac{\nabla\rho_0}{\rho_0}, \tag{11}$$

where $\tilde{\mathbf{v}}_0$ and $\tilde{\mathbf{v}}_1$ are determined by

$$\partial\tilde{\mathbf{v}}_0/\partial t = \frac{e}{m}\mathbf{E}_{\text{ext}}, \quad \partial\tilde{\mathbf{v}}_1/\partial t = (e/m)\,\mathbf{E}(\rho_0\tilde{\mathbf{v}}_0), \tag{12}$$

while the bar indicates averaging in time.

The physical nature of the first two terms in the expression for the force is clear: they take into account the interaction of the particles with the external and the self magnetic field; the term $\overline{\mathbf{v}\cdot\nabla\mathbf{v}}$ has a form analogous to the force acting on a dipole in an inhomogeneous field. Indeed, since $\partial\mathbf{v}/\partial t \sim \mathbf{E}$ and $\mathbf{v} \sim \partial\mathbf{P}/\partial t$, then

$$(\mathbf{P}\nabla)\mathbf{E} \sim \Big(\int \mathbf{v}\,dt\nabla\Big)\partial\mathbf{v}/\partial t,$$

and since

$$\overline{\frac{\partial}{\partial t}\Big(\Big(\int \mathbf{v}\,dt\nabla\Big)\mathbf{v}\Big)} = 0,$$

then it follows from this that $\overline{\mathbf{v}\cdot\nabla\mathbf{v}} \sim \overline{\mathbf{P}\cdot\nabla\mathbf{E}}$ (here $\mathbf{P}$ denotes the dipole moment).

Thus the problem has been reduced to finding $\mathbf{E}(\rho_0\mathbf{v}_0)$ and $\mathbf{H}(\rho_0\mathbf{v}_0)$, or by virtue of the relations

$$\mathbf{E} = -\nabla\varphi - \frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}, \quad \mathbf{H} = [\nabla\mathbf{A}], \quad (\nabla\mathbf{A}) + \frac{1}{c}\frac{\partial\varphi}{\partial t} = 0$$

to finding the vector potential

$$\mathbf{A}(\mathbf{r}t) = \frac{e}{c}\iiiint G(\mathbf{r}\mathbf{r}'tt')\,\rho_0(r')\,\tilde{\mathbf{v}}_0(\mathbf{r}'t')\,d\mathbf{r}'dt'. \tag{13}$$

Substituting $\tilde{\mathbf{v}}_0$ from (12) (the solution of which is elementary) into (13) and integrating we obtain:

$$\mathbf{A} = \{A, 0, 0\},$$

$$A = -4\pi k A_0 \frac{e^2}{mc^2} \sum_{n=0}^{\infty} i^{n+1} (2n+1) I_n(kr) P_n(\cos\theta) e^{-i\omega t}, \qquad I_n(kr) = h_n^{(1)}(kr) \int_0^r \rho_0(r') j_n^2(kr') r'^2 dr'$$
$$+ j_n(kr) \int_r^{\infty} \rho_0(r') j_n(kr') h_n^{(1)}(kr') r'^2 dr'. \tag{14}$$

Here r is the radius drawn from the center of the condensation to the given point, θ is the angle between the z axis and r, $j_n(kr)$ and $h_n^{(1)}(kr)$ are the spherical Bessel and Hankel functions, P_n are the Legendre polynomials.

Now by determining E and H from (14) and $\tilde{v}_1$ from (12) and substituting into (11) we obtain (after averaging) the desired expression for the force. We can specify the conditions under which we can use the expressions obtained in this way by taking into account the conditions under which the method of successive approximations may be used:

$$\omega_0^2/\omega^2 \ll 1, \; eA_0/mc^2 (ka)^\alpha \ll 1, \qquad \text{where } \alpha = \begin{cases} 1 & \text{for } ka \ll 1 \\ 0 & \text{for } ka > 1, \end{cases} \quad \omega_0^2 = \frac{4\pi e^2}{m} \rho_0(0). \tag{15}$$

Thus the problem has been solved. However, the expression for the average force obtained in the form of an infinite series is extremely complicated, and this does not permit us to draw any direct conclusions with respect to the nature of the deformations obtained. Therefore we shall take as an example a particular case in which the characteristic dimensions of the system and the wavelength are such that $ka \ll 1$. In this case by expanding the expressions obtained above in powers of ka and by limiting ourselves to terms of order ka we shall obtain for $r \leq a$:

$$\begin{aligned}
\bar{F}_x &= -kx \frac{9\sigma_T N\overline{W}}{2(ka)^3} \left\{ \frac{24}{105} + \frac{\Delta}{(kr)^2} \left[5\frac{x^2}{r^2} - 3 \right] - \frac{\Delta'}{kr} \frac{x^2}{r^2} \right\} - \frac{mv_T^2}{\rho_0} \frac{\partial \rho_0}{\partial x}, \\
\bar{F}_y &= -ky \frac{9\sigma_T N\overline{W}}{2(ka)^3} \left\{ \frac{32}{105} + \frac{\Delta}{(kr)^2} \left[5\frac{x^2}{r^2} - 1 \right] - \frac{\Delta'}{kr} \frac{x^2}{r^2} \right\} - \frac{mv_T^2}{\rho_0} \frac{\partial \rho_0}{\partial y}, \\
\bar{F}_z &= -kz \frac{9\sigma_T N\overline{W}}{2(ka)^3} \left\{ \frac{24}{105} + \frac{\Delta}{(kr)^2} \left[5\frac{x^2}{r^2} - 1 \right] - \frac{\Delta'}{kr} \frac{x^2}{r^2} \right\} - \frac{mv_T^2}{\rho_0} \frac{\partial \rho_0}{\partial z},
\end{aligned} \tag{16}$$

and for $r \geq a$:

$$\begin{aligned}
\bar{F}_x &= -kx \frac{9\sigma_T N\overline{W}}{2(ka)^3} \left\{ \frac{24}{105} \frac{a^3}{r^3} + \frac{\tilde{\Delta}}{(kr)^2} \left[5\frac{x^2}{r^2} - 3 \right] - \frac{\Delta'}{kr} \frac{x^2}{r^2} - \Phi_x \right\} - \frac{mv_T^2}{\rho_0} \frac{\partial \rho_0}{\partial x}, \\
\bar{F}_y &= -ky \frac{9\sigma_T N\overline{W}}{2(ka)^3} \left\{ \frac{32}{105} \frac{a^3}{r^3} + \frac{\tilde{\Delta}}{(kr)^2} \left[5\frac{x^2}{r^2} - 1 \right] - \frac{\Delta'}{kr} \frac{x^2}{r^2} - \Phi_y \right\} - \frac{mv_T^2}{\rho_0} \frac{\partial \rho_0}{\partial y}, \\
\bar{F}_z &= -kz \frac{9\sigma_T N\overline{W}}{2(ka)^3} \left\{ \frac{24}{105} \frac{a^3}{r^3} + \frac{\tilde{\Delta}}{(kr)^2} \left[5\frac{x^2}{r^2} - 1 \right] - \frac{\Delta'}{kr} \frac{x^2}{r^2} - \Phi_z \right\} - \frac{mv_T^2}{\rho_0} \frac{\partial \rho_0}{\partial z},
\end{aligned} \tag{17}$$

where

$$\begin{aligned}
\Phi_x &= \frac{a^3}{r^3}\left(1 - \frac{a^2}{r^2}\right)\left\{\frac{3}{70}\left(9 - 5\frac{a^2}{r^2}\right) - \frac{3z^2 + x^2}{2r^2}\left(1 - \frac{a^2}{r^2}\right) + \frac{x^2 z^2}{2r^4}\left(5 - 9\frac{a^2}{r^2}\right)\right\}; \\
\Phi_y &= \frac{a^3}{r^3}\left(1 - \frac{a^2}{r^2}\right)\left\{\frac{1}{70}\left(9 - 5\frac{a^2}{r^2}\right) - \frac{x^2 + z^2}{2r^2}\left(1 - \frac{a^2}{r^2}\right) + \frac{x^2 z^2}{2r^4}\left(5 - 9\frac{a^2}{r^2}\right)\right\}; \\
\Phi_z &= \frac{a^3}{r^3}\left(1 - \frac{a^2}{r^2}\right)\left\{\frac{3}{70}\left(9 - 5\frac{a^2}{r^2}\right) - \frac{3x^2 + z^2}{2r^2}\left(1 - \frac{a^2}{r^2}\right) + \frac{x^2 z^2}{2r^4}\left(5 - 9\frac{a^2}{r^2}\right)\right\}; \\
\sigma_T &= \frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2, \; \overline{W} = \frac{(kA_0)^2}{8\pi}, \; \Delta = 1 - \frac{\rho_0(r)}{\rho_0(0)}, \; \tilde{\Delta} = \frac{a^3}{r^3} - \frac{\rho_0(r)}{\rho_0(0)}, \; \Delta' = \frac{\partial \Delta}{\partial kr},
\end{aligned} \tag{18}$$

N is the total number of electrons in the condensation.

Since we are mainly interested in questions of stability it is easier to deal not with the Cartesian components, but only with the radial component of the force. For $r \leq a$ it has the form:

$$F_r = -\frac{3\sigma_T N\overline{W}}{2(ka)^3} \left\{ \frac{8}{35}\left[3 + \frac{y^2}{r^2}\right] kr + \frac{3\left[1 - \rho_0(r)/\rho_0(0)\right]}{kr}\left[3\frac{x^2}{r^2} - 1\right] + \frac{3}{\rho_0(0)} \frac{x^2}{r^2} \frac{\partial \rho_0(r)}{\partial kr} \right\} - \frac{v_T^2 m}{\rho_0(r)} \frac{\partial \rho_0(r)}{\partial r}; \tag{19}$$

and for $r \geq a$:

$$F_r = -\frac{3\sigma_T N\overline{W}}{2(ka)^3} \left\{ \frac{8}{35}\left[3 + \frac{y^2}{r^2}\right] \frac{a^3}{r^3} kr + \frac{3}{kr}\left[\left(\frac{a}{r}\right)^3 - \frac{\rho_0(r)}{\rho_0(0)}\right]\left[3\frac{x^2}{r^2} - 1\right] + \Phi_r(r) + 3\frac{1}{\rho_0(0)} \frac{x^2}{r^2} \frac{\partial \rho_0(r)}{\partial kr} \right\} - \frac{mv_T^2}{\rho_0(r)} \frac{\partial \rho_0(r)}{\partial r}, \tag{20}$$

where

$$\Phi_r = -3\frac{a^3}{r^3}\left(1-\frac{a^2}{r^2}\right)\left\{\frac{1}{70}\left(9-5\frac{a^2}{r^2}\right)\left(3-2\frac{y^2}{r^2}\right)kr - \frac{x^2+z^2}{2r^2}\left(1-\frac{a^2}{r^2}\right)kr + \frac{x^2z^2}{2r^4}\left(1-5\frac{a^2}{r^2}\right)kr\right\}. \tag{21}$$

Let us examine these expressions. First of all we emphasize once again that the results obtained above are only valid for a final time $t < \tau$ and consequently become invalid when the deformation of the condensation becomes appreciable. Therefore the present calculation does not pretend to give a more or less complete solution of the problem of stability, but is in the nature of only an approximate estimate, which, nevertheless, enables us to establish at least in general outline the tendency of the condensation to become deformed, and the nature of such a deformation.

Since we have assumed that $(ka) \ll 1$ the second and the third terms in the curly brackets will, generally speaking, be larger than the first one by a factor $1/(kr)^2$ and, consequently, in the region where Δ and Δ' differ from zero they will play a determining role. The first term is alway positive and consequently gives rise to a force directed towards the center of the condensation; on the other hand, the third term is always negative and gives the largest contribution at points of the most rapid rate of falling off of the function $\rho_0(r)$, i.e., at the "boundary" of the condensation. With respect to the second term we see that, depending on the value of x, it can be either positive or negative, i.e., it can give rise to a force directed either inwards or outwards with respect to the condensation, and that at sufficiently large distances (at points where $\rho_0 \to 0$ and $\partial\rho_0/\partial r \to 0$) it plays the decisive role.

We thus see that near the center of the condensation the force acting on the electrons (and, in particular, its sign) is completely determined by the behavior near zero of the function $\rho_0(r)$ and of its first derivative; while at values of r such that $\rho_0 \to 0$ and $\partial\rho_0/\partial\rho \to 0$ forces appear which are directed away from the center and which lead to the spreading of the peripheral particles of the condensation in the yz plane.

Thus, summarizing, we can say that the results obtained above show that the plasma condensation has a tendency to spread, at least for $ka \ll 1$. With respect to the case $ka \geq 1$ we have not succeeded in drawing any definite conclusions in view of the extremely complicated nature of the expressions obtained for **v**, **E**, **H** and **F**.

We note in conclusion that calculations made taking into account a constant magnetic field along the z axis have shown that in broad outline the picture remains the same with the only difference that the spreading in the yz plane is now replaced by spreading along the z axis.

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[1] V. I. Veksler, Proceedings CERN Symposium on High Energy Accelerators and Pion Physics, Geneva, June 1956, Vol. 1, p. 80.

Translated by G. Volkoff
12