MAGNETORESISTIVE PHENOMENA IN n-Ge TYPE SEMICONDUCTORS IN STRONG MAGNETIC FIELDS

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Submitted to JETP editor November 19, 1956

J. Exptl. Theoret. Phys. (U.S.S.R.) 33, 77-87 (July, 1957)

The equilibrium electron concentration, Hall constant, and electrical resistance of n-Ge type semiconductors in a strong magnetic field are considered. The dependence of these quantities on the field strength is determined. It is found that the quantities under consideration are anisotropic, the nature of this anisotropy being determined by the anisotropy of the electron mass and by the number and mutual arrangement of the constant-energy ellipsoids.

In the past, a number of papers[1,2] have appeared in which the electric resistivity in a magnetic field ρ_H and the Hall constant R in n-Ge type semiconductors have been considered. It is necessary to take into account here the anisotropy of the electron mass the the presence of several constant-energy ellipsoids in the Brillouin zone (six for Si and eight for Ge). However, the analysis was made by the usual kinetic-equation method without taking into account quantization of the electron energy by the field H, a procedure valid only for sufficiently small H.

In this connection, it is of interest to calculate ρ_H and R of n-Ge type semiconductors taking the electron mass anisotropy in strong magnetic fields into account. For this purpose it is necessary to take into account the above-mentioned electron energy quantization by the field H. The present paper is devoted to this.

1. CALCULATION OF THE CURRENTS

Let us assume that the electric field E is directed along the X axis and the magnetic field H perpendicular thereto is along the Z axis. To calculate ρ_H and R taking the energy quantization by the field H into account, we use the stationary states method.[3,4] Following this method, let us calculate the components of the electric current j in the presence of the crossed fields E and H. Inasmuch as several ellipsoids correspond to a given electron energy in the first Brillouin zone of n-Ge type crystals, the current j^i should be evaluated for each i-th ellipsoid and these currents should then be summed over all ellipsoids, taking their relative arrangement in the Brillouin zone into account. This is valid if the interellipsoid transitions caused by electron scattering are neglected, as can probably be done at low temperatures.

The energy spectrum of an electron with an anisotropic mass in the crossed fields E and H and its wave function (for the electron of the i-th ellipsoid) must be found in order to calculate the currents j^i. To do this, let us write the appropriate Hamiltonian in the system of the principal axes X_i, Y_i, Z_i of the i-th ellipsoid

$$\mathcal{H} = \frac{1}{2m_1}\left[\left(p_{x_i} - \frac{e}{c}A_{x_i}\right)^2 + \left(p_{y_i} - \frac{e}{c}A_{y_i}\right)^2\right] + \frac{1}{2m_2}\left(p_{z_i} - \frac{e}{c}A_{z_i}\right)^2 + e(\mathbf{Er}_i), \tag{1}$$

where A is a vector-potential, m_1 is the electron transverse mass, m_2 is its longitudinal mass (it is assumed that the constant-energy ellipsoids are ellipsoids of revolution, as in n-Ge). By transforming coordinates

$$x_i' = x_i,\ y_i' = y_i,\ z_i = z_i'/\sqrt{s},\ s = m_2/m_1$$

and introducing correspondingly the new quantities

$$A_{xi}' = A_{xi},\ A_{yi}' = A_{yi},\ A_{zi}' = A_{zi}/\sqrt{s};\quad E_{xi}' = E_{xi},\ E_{yi}' = E_{yi},\ E_{zi}' = E_{zi}/\sqrt{s} \tag{2}$$

the operator (1) can be reduced to the following form:

$$\mathscr{H} = \frac{1}{2m_1}\left(\mathbf{p}'_i - \frac{e}{c}\mathbf{A}'_i\right)^2 + e\mathbf{E}'_i\mathbf{r}'_i. \tag{3}$$

Here, it should be noted that the transformations (2) also lead to the introduction of the field $\mathbf{H}' = \text{curl}\ \mathbf{A}'$:

$$H_{xi} = H'_{xi}\sqrt{s}, \qquad H_{yi} = H'_{yi}\sqrt{s}, \qquad H_{zi} = H'_{zi}; \tag{4}$$

It is not difficult to verify that $E'_iH'_i = EH = 0$, i.e., $E'_i \perp H'_i$. Hence, the original problem (1) is reduced to finding the energy spectrum and wave function of particles of isotropic mass m_1 in the presence of the crossed fields E'_i and H'_i. To solve this problem, it is expedient to transform by rotation to a coordinate system X'_i, Y'_i, Z'_i in which $E'_i \parallel OX'_i$ and $H'_i \parallel OZ'_i$.

The electron wave function ψ_Q in this coordinate system is[3,4]

$$\Psi_{Qi} = \exp\left[\frac{i}{\hbar}(p'_{yi}y'_i + p'_{zi}z'_i)\right]e^{-\xi_i^2/2}\mathscr{H}_n(\xi_i), \qquad \xi_i = (x'_i - x'_{0i})\sqrt{m_1\omega'_{0i}/\hbar}, \quad \omega'_{0i} = eH'_i/m_1c, \tag{5}$$

where $x'_{0i} = -eE'_i/m_1\omega'^2_{0i} - p'_{yi}/m_1\omega'_{0i}$ and $\mathscr{H}_n(\xi)$ is the normalized Chebyshev-Hermite polynomial. The electron energy spectrum is known to be:

$$\mathscr{E}_{Qi} = \hbar\omega'_{0i}\left(n + \frac{1}{2}\right) + eE'_ix'_{0i} + p'^2_{zi}/2m_1. \tag{6}$$

Since we must calculate the components of the total current along the X, Y, Z axes in (5) and (6), a transformation should be made from the X'_i, Y'_i, Z'_i system to the X, Y, Z system. This transformation yields the following result:

$$\mathscr{E}_{Qi} = \hbar\omega'_{0i}\left(n+\frac{1}{2}\right) + eEx_{0i} + p_{zi}^2/2m_1\beta_i, \quad x_{0i} = -\frac{p_{yi}}{m_1\omega_0} - \frac{\beta_i^{-1}}{m_1\omega_0}p_{zi}(1-s)\sin\vartheta_i\cos\vartheta_i\cos\varphi_i - \frac{eE}{m_1\omega_0^2}\frac{(1-s)\sin^2\vartheta_i\sin^2\varphi_i}{s+(1-s)\sin^2\vartheta_i\sin^2\varphi_i},$$

$$\Psi_{Qi} = \exp\left\{\frac{i}{\hbar}\left[p_{yi}y_i + p_{zi}z_i + x_i(\sin\vartheta_i\cos\varphi_i\, p_{yi} + \cos\vartheta_i\, p_{zi})\frac{(s-1)\sin\vartheta_i\sin\varphi_i}{s+(1-s)\sin^2\vartheta_i\sin^2\varphi_i}\right]\right\}$$

$$\exp\left[-\frac{(x_i - x_{0i})^2}{2\hbar}\frac{m_1\omega'_{0i}s}{s+(1-s)\sin^2\vartheta_i\sin^2\varphi_i}\right]\mathscr{H}_n\left[\sqrt{\frac{m_1\omega'_{0i}}{\hbar}}(x_i - x_{0i})\frac{\sqrt{s}}{(s+(1-s)\sin^2\vartheta_i\sin^2\varphi_i)^{1/2}}\right], \tag{7}$$

$$Q = (n, p_{yi}, p_{zi}), \quad \omega_0 = eH/m_1c; \quad \omega'_0 = \omega_0\sqrt{\beta_i/s}, \quad \beta_i = \sin^2\vartheta_i + s\cos^2\vartheta_i.$$

Here φ_i and ϑ_i are the first two Eulerian angles of the X, Y, Z system with respect to the principal axes of the i-th ellipsoid.

Let us turn to the computation of the currents j^i. In order to calculate the y and z components, the average quantum-mechanical values of the corresponding carrier velocity components must be calculated and the quantities obtained must then be averaged over the electron equilibrium distribution:[3]

$$\overline{v^i_y} = \overline{\partial\mathscr{E}_{Qi}/\partial p_{yi}} = -cE/H, \qquad \overline{v^i_z} = \overline{\partial\mathscr{E}_{Qi}/\partial p_{zi}} = -(cE/H)\alpha_i, \qquad \alpha_i = \beta^{-1}(1-s)\sin\vartheta_i\cos\vartheta_i\cos\varphi_i, \tag{8}$$

where the bar denotes the statistical average. The currents are correspondingly determined by the following expressions

$$j^i_y = -eN^icE/H, \qquad j^i_z = -eN^ic(E/H)\alpha_i, \tag{9}$$

where N^i is the density of the number of electrons in the i-th ellipsoid. The currents (9) are not ohmic — they are not related to the electron scattering and are independent of the scattering mechanism; the current j^i_z appears exclusively because of the anisotropy of the electron mass.

The current j^i_z is purely ohmic, i.e., it is determined essentially by electron scattering. In reality, $\overline{v^i_z} = 0$; consequently, the current is determined as the flow of charge through a unit area of the x = 0 plane,[4,5] caused by scattering:

$$j^i_x = -e\sum_{QQ'}\sum_{\alpha=\pm1}(W^i_{QQ'}\chi^i_{Q'\alpha} - W^i_{Q'Q}\chi^i_{Q\alpha}), \qquad x'_{0i} = x_{0i}(Q') > 0, \qquad x_{0i} = x_{0i}(Q) > 0, \tag{10}$$

where $W^i_{QQ'}$ is the probability of a quantum transition of the current carrier from the Q state into the Q′ state under the influence of a scattering factor (within the i-th ellipsoid);* $\chi^i_{Q\alpha}$ is the electron equi-

* We neglect the probability of interellipsoid transitions because they occur through the agency of phonons with large f, which are few in number at the low temperatures under consideration (f is the wave vector of the phonon).

librium distribution function. Since we are limited to the case of such fields E for which $j_x^i \sim E$ (Ohm's law) and since a non-degenerate electron gas is assumed in the semiconductor,* we have (taking spin into account):

$$\chi_{Q\alpha}^i = \exp\left\{\frac{1}{kT}\left[\mu - \alpha\mu_B H - \frac{\hbar\omega_0}{\sqrt{s}}\sqrt{\beta_i}\left(n + \frac{1}{2}\right) - \frac{\beta_i^{-1}}{2m_1}p_z^2\right]\right\}, \tag{11}$$

where $\alpha = \pm 1$, μ is the chemical potential, and $\mu_B = e\hbar/2mc$ is the Bohr magneton.

Let us consider the interaction between electrons and long-wave longitudinal acoustic phonons (deformation potential)[6] as the electron scattering mechanism and let us disregard scattering by impurity ions for the following reasons. We are interested in how energy-level quantization by the field H affects the dependence of magnetoresistive phenomena in n-Ge type semiconductors on the direction of the field H and on H/T. Hence, it appears that a significant exponential anisotropy of ρ_H and R, dependent on the anisotropy of the equilibrium electron concentration N, occurs. It will be shown by further computations that precisely this large anisotropy determines essentially the dependence of ρ_H and R on the direction of H; the scattering factor (the interaction with phonons in our case) makes only an insignificant contribution to the anisotropy of ρ_H, i.e., the anisotropy of the effective mobility $u_0 = (Ne\rho_H)^{-1}$ is not very substantial. On the other hand, allowance for the scattering by impurity ions, within the framework of our method of calculating ρ_H, leads to an additional current δj_x (j_y and j_z are not determined by the scattering) and, therefore, to an additional resistance $\delta\rho \simeq E\delta j_x/j_y^2$ ($j_z = 0$, as will be shown). Hence, it can be shown that the exponential anisotropy of N(H) is fundamental, as before; while the dependence of u_0 is determined by the same functions φ_i and ϑ_i as in the case of scattering by phonons (by the functions α_i, β_i) and, as before, is not essential.

It is understood that $\delta\rho$ can be substantial in magnitude for a sufficient impurity concentration or in the case of compensated impurities. But we are interested not in the absolute value of ρ_H but in the ratio $\rho_H(H)/\rho_{H_0}$, where ρ_{H_0} is the resistance for a certain direction of H and E ($E \perp H$). This dependence is determined basically by the anisotropy of N(H). It must also be said that if the impurity concentration is small enough ($n_1 \lesssim 10^{15} - 10^{16}$) and if the impurity compensation can be neglected, then the energy gap between the impurity level and the bottom of the band will be large in comparison with kT at the low temperatures under consideration, so that only a very small portion of the impurities is ionized, the scattering by the impurity ions is insignificant and its influence on the resistance ρ_H can be neglected. Actually, an essential condition for the appearance of a sudden anisotropy in ρ_H and R is the absence of degeneracy (see Appendix). In connection with the above, we should like to emphasize that both the sharp dependence of ρ_H and R on the field strength H and the strong dependence of these quantities on its direction (exponential anisotropy) depend basically on the character of the change in the equilibrium conduction-electron concentration for $\hbar\omega_0 \gg kT$ in the absence of electron-gas degeneracy, and not on the character of the scattering mechanism.[5]

Consequently, we disregard scattering by the impurity ions and assume that the perturbation operator V that causes the scattering is the deformation potential:[6]

$$V = D\,\mathrm{div}\,\mathbf{R}(\mathbf{r}).$$

where $\mathbf{R}(\mathbf{r})$ is the displacement of the lattice at point $\mathbf{r}$.

Using (7) in the usual manner to determine the matrix elements of V, which are then substituted into

$$W_{QQ'}^i = \frac{2\pi}{\hbar}\sum_{(\overline{f})}|V_{QQ'}^{if}|^2\delta(\mathscr{E}_Q^i - \mathscr{E}_{Q'}^i \pm \hbar v f)$$

and into (10), we obtain the following expression for the current:

$$j_x^i = \frac{\pi e D^2 L^3}{Mv\,\hbar^4(2\pi)^5}\sum_{\alpha=\pm 1}\sum_{n,n'=0}^{\infty}\sum_{f_y,\,f_z}\int_{-f_0}^{f_0} df_x \iint_{(x'_{0i}>0,} dp_{zi}\,dp'_{zi}\iint_{x_{0i}<0)} dp_{yi}\,dp'_{yi,}f\overline{W}_{QQ'}^i\{[\chi_{Q\alpha}^i N_f - \chi_{Q'\alpha}^i(N_f+1)]$$
$$\times\Delta(p_{yi} - p'_{yi} + \hbar f_y)\,\Delta(p_{zi} - p'_{zi} + \hbar f_z)\,\delta(\mathscr{E}_Q^i - \mathscr{E}_{Q'}^i + \hbar v f) + [\chi_{Q\alpha}^i(N_f+1) - \chi_{Q'\alpha}N_f]\,\Delta(p_{yi} - p'_{yi} - \hbar f_y)$$
$$\times\Delta(p_{zi} - p'_{zi} - \hbar f_z)\,\delta(\mathscr{E}_Q^i - \mathscr{E}_{Q'}^i - \hbar v f)\}, \tag{12}$$

* Conditions under which there is no degeneracy in the conduction band are analyzed in the Appendix.

where L is the linear dimension of the base region of the crystal, M is its mass, $\Delta(\eta) = 0$ for $\eta \neq 0$ and 1 for $\eta = 0$, v is the velocity of sound, and N_f is the number of phonons with the wave vector f:

$$N_f \to \overline{N}_f = [\exp(\hbar v f / kT) - 1]^{-1}, \qquad \overline{W}^i_{QQ'} = \left| \int_{-\infty}^{\infty} dx \varphi^*_Q (x - x_{0i}) \varphi_{Q'} (x - x'_{0i}) e^{ixf_x} \right|^2,$$

$$\varphi_Q (x - x_{0i}) = \Psi_Q (x - x_{0i}) \exp\left\{ -\frac{i}{\hbar} (y_i p_{yi} + z_i p_{zi}) \right\}.$$

Using the properties of the symbol $\Delta(\eta)$ and of the δ-function, transforming to the variables

$$x'_{0i} - x_{0i} = r, \quad x'_{0i} + x_{0i} = t, \quad p_{zi} - p'_{zi} = w, \quad p_{zi} + p'_{zi} = u,$$

$$r_{\max} \geqslant r \geqslant 0, \quad -r \geqslant t \geqslant r; \quad -\hbar f_0 \leqslant w \leqslant \hbar f_0, \quad -\hbar f_0 \leqslant u \leqslant \hbar f_0$$

and integrating over u and t, we reduce j^i_x to the following:

$$j^i_x = \frac{\pi e D^2 m_1 (m_1 \omega_0)^2 \beta_i}{\delta v \hbar^4 (2\pi)^5} e^{\mu/kT} \cosh\left(\frac{\mu_B H}{kT}\right) \sum_{n, n'=0}^{\infty} \int_{-f_0}^{f_0} df_x \int_{-\hbar f_0}^{\hbar f_0} dw \int_0^{r_{\max}} dr \frac{r f \sinh(eEr / 2kT)}{|w| \sinh(\hbar v f / 2kT)} \overline{W}^i_{QQ'} \exp\left(-\frac{w^2}{8 m_1 kT} \beta_i^{-1}\right)$$

$$\times \{\exp(-X^+_{0i} - X^+_{1i}) + \exp(-X^-_{0i} - X^-_{1i})\}, \tag{13}$$

where

$$X^{\pm}_{0i} = \frac{1}{kT} \left\{ \frac{\hbar \omega_{0i}}{2} (n + n' + 1) + \frac{m_1 \beta_i}{2 w^2} [\hbar \omega'_{0i} (n - n') \pm \hbar v f]^2 \right\},$$

$\delta = M/L^3$; f_0 is the maximum wave vector of the phonon; $-f_0 \leq f_x, f_y, f_z \leq f_0$; $X^{\pm}_{1i} \sim E$ and, consequently, we obtain in the region where Ohm's law holds

$$X^{\pm}_{1i} = 0, \quad \sinh(eEr / 2kT) \approx eEr / 2kT.$$

Let us introduce the symbol

$$P = \sum_{n, n'=0}^{\infty} \overline{W}^i_{QQ'} [\exp(-X^+_{0i}) + \exp(-X^-_{0i})].$$

Let us also make the change of variable

$$w = \hbar f_z, \quad r^2 = (\hbar / m_1 \omega_0)^2 (f_y + \alpha_i f_z)^2,$$

then

$$j^i_x = c \frac{E}{H} \frac{\pi e D^2 m_1 \beta_i}{\delta v \hbar kT (2\pi)^5} e^{\mu/kT} \cosh\left(\frac{\mu_B H}{kT}\right) \iint_{-f_0}^{f_0} df_x df_z \int_{-\alpha_i f_z}^{f_0} df_y \frac{f (f_y + \alpha_i f_z)^2}{|f_z| \sinh(\hbar v f / 2kT)} \exp\left(-\frac{\hbar^2 f_z^2}{8 m_1 kT} \beta_i^{-1}\right) P. \tag{14}$$

Analysis of (14) for arbitrary $\hbar\omega_0$ leads to very awkward and immense formulas so that the explanation of the dependence of j^i_x on $\hbar\omega_{0i}/kT$ requires in general numerical integration. We shall consider only the case $\hbar\omega_0 \gg kT$ (in fact, it is sufficient that $\hbar\omega_0 \gtrsim 3kT$), since it is the only one of interest to us.

It is easy to show[5,7] that for $\hbar\omega_0 \gg kT$:

$$P \approx 2 \exp\left\{ -\frac{\hbar \omega_{0i}}{2kT} - \frac{m_1 v^2}{2kT} \frac{f^2}{f_z^2} \beta_i \right\}. \tag{15}$$

Substituting (15) into (14), we obtain

$$j^i_x = c \frac{E}{H} \frac{\pi e D^2 m_1 \beta_i}{\delta \hbar v kT (2\pi)^5} \exp\left[\frac{1}{kT}\left(\mu - \frac{\hbar \omega'_{0i}}{2}\right)\right] \cosh\frac{\mu_B H}{kT} I_i, \tag{16}$$

where I_i becomes after some manipulation and the introduction of spherical coordinates in the f-space:

$$I_i = \frac{\pi}{2} \int_0^{f_0} df \int_0^{\pi/2} d\theta [\tan\theta + (2\alpha_i^2 - 1) \sin\theta \cos\theta] \frac{f^4}{\sinh(\hbar v f / 2kT)} \exp\left\{ -\frac{\hbar^2 f^2 \cos^2\theta}{8 m_1 \beta_i kT} - \frac{\beta_i m_1 v^2}{\cos^2\theta \, 2kT} \right\}.$$

Introducing new variables

$$\lambda = \cos\theta, \quad \hbar v f / kT = \zeta, \quad 0 \leqslant \zeta \leqslant \hbar v f_0 / kT = T_D / T,$$

and using the notation $\epsilon = m_1 v^2/\alpha kT \ll 1$, we obtain for $T \ll T_D$:

$$I_i = \frac{\pi}{2}\left(\frac{kT}{\hbar v}\right)^5 \int_0^\infty d\zeta \int_0^1 d\lambda \left[\frac{1}{\lambda}(2\alpha_i^2 - 1)\lambda\right] \frac{\zeta^4}{\sinh \zeta/2} \exp\left\{-\frac{\zeta^2\lambda^2}{16\varepsilon\beta_i} - \frac{\varepsilon\beta_i}{\lambda^2}\right\}. \quad (17)$$

This integral can be evaluated approximately for $\epsilon \ll 1$:

$$I_i \approx 64\pi (kT/\hbar v)^5 (1 + 2\alpha_i^2 c_1), \quad c_1 = \text{const} \approx 1.$$

Substituting it in (16), we obtain finally

$$j_x^i = E \frac{2e^2 D^2 (kT)^4}{8\pi^3 \hbar^6 v^6 \omega_0} \exp\left(-\frac{\hbar\omega'_{0i}}{2kT}\right) \cosh\frac{\mu_B H}{kT} \beta_i (1 + 2\alpha_i^2 c_1) e^{\mu/kT}. \quad (18)$$

It was already indicated that to obtain the components of the total current the appropriate components of the partial single-ellipsoid currents must be summed over all the constant-energy ellipsoids. Hence, the relative arrangement of the ellipsoids in the Brillouin zone must be taken into account. As regards the y component of the total current,

$$j_y = \sum_i j_y^i = -ec\frac{E}{H}\sum_i N^i = -ec\frac{E}{H}N, \quad (19)$$

where N is the total density of the number of electrons in the conduction band

To calculate j_x and j_z, it is expedient to express the angles φ_i and ϑ_i by means of angles which the electric field E and the magnetic field H, as well as the Y axis, make with the cubic axes in the Brillouin zone (we denote the cosines of these angles by $\overline{m}_1\overline{m}_2\overline{m}_3$, $\ell_1\ell_2\ell_3$, and $n_1n_2n_3$ respectively).

A different number of ellipsoids, mutually arranged in a different way, can correspond to any one value of the energy in the Brillouin zone. We shall confine the analysis to two cases: (1) 8 ellipsoids arranged in the [111] directions in the reciprocal cell lattice, and (2) 6 ellipsoids arranged in the [100] directions. It is not difficult to confirm that $j_z = 0$ in both cases, as it should be for cubical symmetry of the reciprocal lattice.

In the first case, (n-Ge), after a number of simple transformations, we obtain:

$$j_x = \frac{2D^2 e m_1 (kT)^4}{8\pi^3 \hbar^6 v^6} c \frac{E}{H} \cosh\left(\frac{\mu_B H}{kT}\right) e^{\mu/kT} A_{Ge}, \quad (20)$$

$$A_{Ge} = \sum_{\alpha_1,\alpha_2,\alpha_3=\pm 1} \exp\left\{-\frac{\hbar\omega_0}{2\sqrt{s}\,kT}\left[1 + \frac{s-1}{3}(\alpha_1 l_1 + \alpha_2 l_2 + \alpha_3 l_3)^2\right]^{1/2}\right\}\left\{1 + \frac{s-1}{3}(\alpha_1 l_1 + \alpha_2 l_2 + \alpha_3 l_3)^2\right.$$

$$\left. + 2c_1(1-s)^2(\alpha_1 l_1 + \alpha_2 l_2 + \alpha_3 l_3)(\alpha_1 n_1 + \alpha_2 n_2 + \alpha_3 n_3)\left[1 + \frac{s-1}{3}(\alpha_1 l_1 + \alpha_2 l_2 + \alpha_3 l_3)^2\right]^{-1}\right\}. \quad (21)$$

In the second case, (n-Si), the expression for j_x remains the same as (20) but

$$A_{Si} = 2\sum_{t=1}^{3} \exp\left\{-\frac{\hbar\omega_0}{2\sqrt{s}\,kT}(1 + (s-1) l_t^2)^{1/2}\right\}\{1 + (s-1) l_t^2 + 2c_1 (1-s)^2 l_t^2 n_t^2 (1 + (s-1) l_t^2)^{-1}\}.$$

2. EXPONENTIAL ANISOTROPY OF THE CURRENT CARRIER CONCENTRATION

To determine completely the dependence of j_x and j_y on H and T, both $\mu(H, T)$ and $N(H, T)$ should be evaluated. Taking (7) into account, we obtain the following formula for the i-th ellipsoid, assuming no degeneracy (see Appendix)

$$N^i(H, T) = Z_0 \frac{\hbar\omega'_{0i}}{2kT} \exp\left\{-\frac{\hbar\omega'_{0i}}{2kT} + \frac{\mu}{kT}\right\} 2\cosh\frac{\mu_B H}{kT}, \quad (22)$$

in which we assume that $\hbar\omega'_{0i} \gg kT$; $Z_0 = 2(2\pi m_1 kT/h^2)^{3/2}$. Introducing again the quantities $\ell_1\ell_2\ell_3$ and $n_1n_2n_3$, we obtain

$$N = \sum_i N^i = Z_0 \frac{\hbar\omega_0}{\sqrt{s}\,kT} \cosh\left(\frac{\mu_B H}{kT}\right) e^{\mu/kT} B, \quad (23)$$

$$B_{Ge} = \sum_{\alpha_1\alpha_2\alpha_3=\pm 1} \exp\left\{-\frac{\hbar\omega_0}{2\sqrt{s}\,kT}\left(1 + (\alpha_1 l_1 + \alpha_2 l_2 + \alpha_3 l_3)^2 \frac{s-1}{3}\right)^{1/2}\right\}\left[1 + \frac{s-1}{3}(\alpha_1 l_1 + \alpha_2 l_2 + \alpha_3 l_3)^2\right], \quad (24)$$

$$B_{Si} = 2 \sum_{t=1}^{3} \exp\left\{-\frac{\hbar\omega_0}{2\sqrt{s}\,kT}\left(1 + (s-1)\,l_t^2\right)^{1/2}\right\}[1 + (s-1)\,l_t^2]. \tag{25}$$

To determine the chemical potential μ we use the neutrality equation and consider a Wilson type impurity semiconductor. In this case, the neutrality equation for any $\mu_B H$ is the following (see Appendix):

$$n_1\left[1 + 2\exp\left(\frac{\Delta E + \mu}{kT}\right)\cosh\frac{\mu_B H}{kT}\right]^{-1} = Z_0\,\frac{\hbar\omega_0}{\sqrt{s}\,kT}\cosh\frac{\mu_B H}{kT}\,e^{\mu/kT}B, \tag{26}$$

where n_1 is the concentration of monovalent donor impurities, ΔE is the gap between the impurity level and the bottom of the conduction band. Since $\Delta E \gg kT$ for low temperatures, then

$$e^{\mu/kT} \approx \frac{1}{2\cosh\dfrac{\mu_B H}{kT}}\left(\frac{n_1\sqrt{s}\,kT}{Z_0\hbar\omega_0}\right)^{1/2} e^{-\Delta E/2kT}B^{-1/2}. \tag{27}$$

Substituting (27) into (23), we obtain

$$N = {}^1/_2\,(n_1 Z_0\hbar\omega_0/\sqrt{s}\,kT)^{1/2}\,e^{-\Delta E/2kT}\,B^{1/2}; \tag{28}$$

It follows from (28) that, first, the number N decreases exponentially as $\hbar\omega_0/kT$ increases;[5] and second, that this number depends essentially on the direction of H. This second fact is very important; it is manifested in the significant "exponential" anisotropy of N. This anisotropy increases with the anisotropy of the electron mass and its character depends on the number and mutual arrangement of the ellipsoids in the Brillouin zone (B_{Si}, B_{Ge} depend differently on ℓ_t). This anisotropy is caused by the fact that in strong fields H the different ellipsoids contain different numbers of electrons which depend on the direction of E.

3. ELECTRIC RESISTIVITY AND HALL CONSTANT IN A STRONG MAGNETIC FIELD

Substituting (19), (20), and (23) into the known formulas of the stationary-states method[3] and keeping in mind that $j_x^2 \ll j_y^2$ in our case, we obtain the following expressions

$$R \approx -(2/ec)\,(kT\sqrt{s}/n_1 Z_0\hbar\omega_0)^{1/2}e^{\Delta E/2kT}B^{-1/2}; \tag{29}$$

$$\rho_H \approx AB^{-3/2}e^{\Delta E/2kT}D^2 m_1^{5/4} m_2^{3/4}\,(kT)^{11/2}/\,\delta\pi^3\hbar^6 v^6\,(\hbar Z_0)^{3/2}\,e^2\sqrt{n_1\omega_0}. \tag{30}$$

Hence, it follows that as $\hbar\omega_0/kT$ increases both R and ρ_H increase rapidly, basically as $\exp(\hbar\omega_0/4kT)$, in conformance with the behavior of N(H, T). Both R and ρ_H have considerable anisotropy, i.e., depend on the direction of H; ρ_H depends also on the direction of E.

The anisotropy of ρ_H is determined by the factor $AB^{-3/2}$, and $R \sim B^{-1/2}$. The mobility u_0 contained in $\rho_H = 1/Neu_0$ is also anisotropic; its anisotropy is determined by the factor B/A and is considerably less pronounced than the anisotropy of N(H, T). Consequently, the anisotropies of ρ_H and R increase with the anisotropy of the mass and depend on the number and mutual arrangement of the ellipsoids in the Brillouin zone (see Figs. 1 and 2).

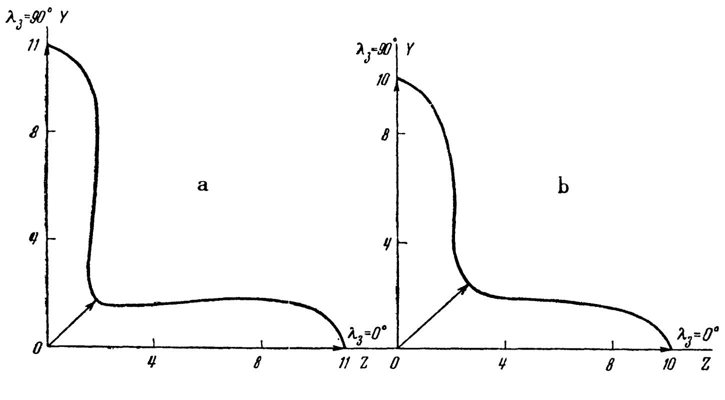

FIG. 1. Curve a: $A_{Ge}/B_{Ge}^{3/2}$ is the quantity defining the anisotropy of ρ_H for Ge. Curve b: $B_{Ge}^{-1/2}$ is the quantity defining the anisotropy of R for Ge. E is parallel to X and H in the YZ plane; $\ell_1 = 0$; $\ell_3 = \cos\lambda_3$; $s = m_2/m_1 = 19$.

Let us note, by the way, that if H is directed along one of the cubic axes all the above-mentioned quantities are isotropic (independent of the direction of E).

In conclusion, let us point out that it seems to us that an experimental investigation of ρ_H and R in semiconductors in strong magnetic fields at low temperatures affords the possibility of obtaining additional information on the carrier mass anisotropy and on the number, relative arrangement, and character of the constant-energy surfaces in the Brillouin zone near the energy extrema. In reality, if the dependences of $\rho_H(\ell_1\ell_2\ell_3;\ n_1n_2n_3)$ and $R(\ell_1\ell_2\ell_3)$ are established experimentally for a semiconductor in a strong field H, then, for a given number and arrangement of the constant-energy ellipsoids in the Brillouin zone, the exponential anisotropy of ρ_H and R would permit the quantities $\omega_0 = eH/m_1c$ and $s = m_2/m_1$, i.e., m_1 and m_2, to be determined by a comparison of experimental and theoretical results. The character of the dependences of $\rho_H(\ell_1\ell_2\ell_3;\ n_1n_2n_3)$ and $R(\ell_1\ell_2\ell_3)$ is determined by the arrangement of the constant-energy ellipsoids and by their number, which again affords a possibility of obtaining answers to this question by a comparison with experiment.

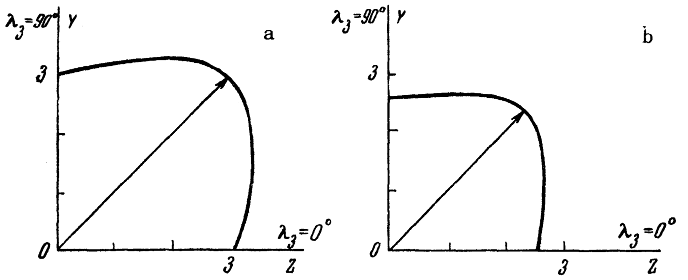

FIG. 2. Curve a: $A_{Si}/B_{Si}^{3/2}$. Curve b: $B_{Si}^{-1/2}$. E' is parallel to X and H in the YZ plane; $\ell_1 = 0$; $\ell_3 = \cos\lambda_3$; $s = m_2/m_1 = 5$.

It should be noted that the magnetic susceptibility of the conduction electrons, as is easy to see, is determined by the expression

$$\chi = \frac{\mu_B^2}{kT}\left(\frac{m^2}{m_1m_2}\right)^{1/2} B^{-1}\frac{\partial B}{\partial H} N(H, T),$$

and, therefore, depends on the direction of H; hence, the anisotropy of χ is again determined essentially by the dependence of N on the angles, i.e., on $\ell_1\ell_2\ell_3$. It can be assumed that other kinetic coefficients have a significant anisotropy for Ge type semiconductors in the presence of a strong field H.

All the results obtained above are valid for unipolar impurity semiconductors in the absence of electron gas degeneracy.

The case of large impurity concentration, when degeneracy of the conduction electrons, scattering by impurity ions, and impurity bands can play a substantial role, must be investigated especially as is being done at present.

We take the opportunity to express our gratitude to Prof. A. G. Samoilovich for very fruitful discussions.

APPENDIX

Let us determine the conditions under which the electron gas in the conduction band can be considered non-degenerate. To do this, let us write the neutrality equation for a Wilson type semiconductor:

$$n_1 - n = N, \tag{I}$$

where n is the density of the number of electrons in the impurity levels and N is the density of the number of electrons in the conduction band. As shown in Ref. 9, taking into account the Coulomb repulsion of electrons in the impurity level (monovalent impurity), we get

$$n = 2n_1 \exp\left(\frac{\mu + \Delta E}{kT}\right)\cosh\frac{\mu_B H}{kT}\left(1 + 2\exp\left(\frac{\mu+\Delta E}{kT}\right)\cosh\frac{\mu_B H}{kT}\right)^{-1}. \tag{II}$$

On the other hand, under the same conditions

$$N = \sum_i N^i = \sum_i \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} dp_z \frac{eH}{\hbar^2 c}\left\{\left[\exp\left(\frac{\hbar\omega'_{0i}}{2kT}(2n+1) + \frac{\beta_i^{-1}p_z^2}{2m_1kT} + \frac{\mu_B H - \mu}{kT}\right) + 1\right]^{-1}\right.$$
$$\left. + \left[\exp\left(\frac{\hbar\omega'_{0i}}{2kT}(2n+1) + \frac{\beta_i^{-1}p_z^2}{2m_1kT} - \frac{\mu_B H + \mu}{kT}\right) + 1\right]^{-1}\right\}, \tag{III}$$

where

$$\omega'_{0i} = \omega_0 s^{-1/2}(\sin^2\vartheta_i + s\cos^2\vartheta_i)^{1/2}, \qquad \beta_i = \sin^2\vartheta_i + s\cos^2\vartheta_i.$$

It is seen from (III) that the Boltzmann distribution can be used in this case if $\mu \ll \frac{1}{2}\hbar\omega'_{0i} - \mu_B H$, or, in order of magnitude

$$\mu \ll {}^1/_2 h\omega_0(\beta_i/s)^{1/2} - \mu_B H = \varepsilon_{\min}.$$

(The quantity $\hbar\omega'_{0i}/2 - \mu_B H$ determines the bottom of the zone for the i-th ellipsoid.) Let us assume that this condition is satisfied and let us explain what the parameters n_1, ΔE, m_1, and m_2 must be here.

Equation (I) becomes (for $\hbar\omega_0/2 \gg kT$)

$$\left[1 + 2e^{\tilde{\mu}/kT}\cosh\frac{\mu_B H}{kT}\right]^{-1} = \frac{Z_0}{n_1}\frac{\hbar\omega_0}{\sqrt{s}\,kT}e^{-\Delta E/kT}\cosh\left(\frac{\mu_B H}{kT}\right)Be^{\tilde{\mu}/kT}, \quad \text{(IV)}$$

where $\tilde{\mu} = \mu + \Delta E$. It is easy to find from (IV) that $\Delta E \gg kT$ for $\hbar\omega_0/2kT \gg 1$.

$$\frac{\mu}{kT} \approx -\frac{\Delta E}{2kT} - \frac{\ln B}{2} + \frac{1}{2}\ln\frac{n_1\sqrt{s}\,kT}{Z_0\hbar\omega_0} - \ln\left(2\cosh\frac{\mu_B H}{kT}\right). \quad \text{(V)}$$

It is not difficult to see from (24) and (25) that $\ln B \approx -\hbar\omega_0/2kT$ in order of magnitude. Using this, we reduce (V) to

$$\mu \approx \varepsilon_{\min} - \frac{\Delta E}{2} - \frac{\hbar\omega_0}{4} + \frac{kT}{2}\ln\frac{n_1\sqrt{s}\,kT}{Z_0\hbar\omega_0} + kT\left[\frac{\mu_B H}{kT} - \ln\left(2\cosh\frac{\mu_B H}{kT}\right)\right]. \quad \text{(VI)}$$

It is seen from (VI) that $\mu \ll \epsilon_{\min}$ if

$$\frac{\Delta E}{2} + \frac{\hbar\omega_0}{4} \gg \frac{kT}{2}\ln\frac{n_1\sqrt{s}\,kT}{Z_0\hbar\omega_0} + kT\left[\frac{\mu_B H}{kT} - \ln\left(2\cosh\frac{\mu_B H}{kT}\right)\right].$$

This inequality is satisfied for the values of ΔE and s frequently encountered at low T and not too large n_1. For example, if $\Delta E \approx 0.01$ ev, $T \sim 10°$ K, $s \sim 10$, $\hbar\omega_0/kT \sim 3$, then $\mu/kT \approx -4 + \ln(2n_1 \times 10^{-16})$; if $n_1 < 10^{20}$, μ is not only less than $\epsilon_{\min}$ ($\epsilon_{\min} \approx 3/2$ in this case) but $\mu < 0$. Even if it is assumed that a certain overestimate of $|\mu/kT|$ (which is not large, as shown in Ref. 6) is made by replacing (III) by the right side of (IV), the assumed non-degeneracy of the electron gas is justified.

Hence, the electron gas in the conduction band can apparently be assumed to be non-degenerate at the low T and high H under consideration for not too large n_1 and s and for large enough ΔE.

The above conclusion is in agreement with results of Refs. 9 and 10, where a detailed investigation of the conditions of electron gas degeneracy is made for H = 0. However, large H contribute to the absence of degeneracy because the field quantization of the energy level leads to a large increase, in comparison with kT, in the energy gap between the impurity level and the bottom of the band: $\hbar\omega_0/2 \gg kT$.

Let us take the opportunity to note that the expression for n in Ref. 5 is not completely exact, but let us emphasize that the results of Ref. 5 are true not only for $\mu_B H \gg kT$ but for any $\mu_B H$ if $\exp(\mu_B H/kT)$ in the formulas of Ref. 5 are everywhere replaced by $\cosh(\mu_B H/kT)$.

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Translated by M. D. Friedman
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