ON THE THEORY OF THE SHUBNIKOV-DE HAAS EFFECT

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Quantum oscillations of the electrical conductivity $\sigma^{\alpha\beta}$ and specific resistance $\rho^{\alpha\beta}$ tensors are investigated on the basis of some general formulae presented in Ref. 1. It is shown that the oscillations of $\sigma^{\alpha\beta}$ and $\rho^{\alpha\beta}$ may be expressed in terms of the magnetic moment oscillations in the de Haas-van Alphen effect and in terms of the classical values of the mobility tensor. The asymptotic values of the oscillation amplitudes in strong magnetic fields are investigated and some simple cases are considered for which calculation of the oscillation amplitudes may be completely carried out.

In Ref. 1 I. M. Lifshitz developed a consistent quantum theory of the conductivity of metals in magnetic fields. The relation between the quantum kinetic equation and its classical analog derived there permits one to determine quantum corrections to the classical value of the electrical conductivity, and, in particular, to determine those corrections which account for the quantum oscillations of the conductivity. The present communication is devoted to a detailed study of these oscillations (Shubnikov-de Haas effect[2]).

It was shown in Ref. 1, that the simple (classical) part of the electrical conductivity tensor can be written in the form

$$\sigma_{\rm cl}^{\alpha\beta} = -2\iint \frac{\partial f_0}{\partial \varepsilon}\chi^{\alpha\beta}m^* d\varepsilon\, dp_z \quad (\alpha,\ \beta \neq z,\ z), \qquad \sigma_{\rm cl}^{zz} = -2\iint \frac{\partial f_0}{\partial \varepsilon}(\chi^{zz}+\chi_0^{zz})\, m^* d\varepsilon\, dp_z, \tag{A}$$

where $\chi(\epsilon, p_z)$ is constructed in a specific way from Green's function for the classical kinetic equation [see Eq. (59) in Ref. 1], and $f_0(\epsilon)$ is the Fermi distribution function.

Quantum oscillations of the conductivity tensor occur when $\chi(\epsilon, p_z)$ has the values determined by the following equations:[1]

$$\Delta\sigma^{\alpha\beta} = \sum_{k=1}^{\infty} I_k^{\alpha\beta}\ (\alpha,\ \beta\neq z,\ z), \quad \Delta\sigma^{zz} = \sum_{k=1}^{\infty}(I_k^{zz} - L_k^{zz}), \quad I_k^{\alpha\beta} = 2\iint f_0\, \frac{\partial(\chi^{\alpha\beta}m^*)}{\partial\varepsilon}\,\hbar\omega^* e^{2\pi ikn}\, dn\, dp_z,$$
$$L_k^{zz} = 2\iint \frac{\partial f_0}{\partial\varepsilon}\,\chi_0^{zz} m^*\hbar\omega^* e^{2\pi ikn}\, dn\, dp_z, \tag{B}$$

where $\omega^* = (eH/m^*c)$, $m^* = (1/2\pi)(\partial S/\partial\epsilon)$, and $S = S(\epsilon, p_z)$ is the area of intersection of the constant-energy surface for the electron having an arbitrary dispersion law $\epsilon = \epsilon(p)$ with the plane $p_z =$ const., perpendicular to the magnetic field.

The structure entering into the integrals in Eq. (B) permits one to determine easily from them the oscillatory factor in which we are interested and to determine in this way the oscillation of $\Delta\sigma^{\alpha\beta}$ from the oscillation of the magnetic moment and of the magnitude of the classical conductivity tensor.

1. CONDUCTIVITY OSCILLATIONS

Let us determine the oscillatory part of the expressions I_k and L_k in (B). To do this we shall use the concepts introduced in Ref. 3, and the same method, slightly modified and simplified.

First let us consider the contribution to the oscillatory part of the electrical conductivity of a single group of electrons with a given dispersion law.

In the integrals of I_k and L_k, let us transform from the variables n, p_z to n, ϵ changing the order and the limits of integration, and also using the relation

$$\hbar\omega^* \frac{D(n, p_z)}{D(n, \varepsilon)} = \left(\frac{\partial n}{\partial p_z}\right)_\varepsilon^{-1},$$

which results from the quasi-classical expression for the energy levels of an electron in a magnetic field:

$$S(\varepsilon, p_z) = (n + \gamma) ehH/c \quad (0 < \gamma < 1), \quad n \gg 1.$$

This gives for I_k

$$I_k = 2 \int f_0(\varepsilon) \sum \left\{ \int_{n_{\min}}^{n_{\max}} \frac{\partial(\chi^* m)}{\partial \varepsilon} \left| \frac{\partial n}{\partial p_z} \right|_\varepsilon^{-1} e^{2\pi i k n} dn \right\} d\varepsilon, \tag{1}$$

where the summation sign refers to summation over uniform intervals of change of $n(\epsilon, p_z)$ for a fixed value of ϵ.

The idea of the calculations which follow is based on the observation, that, with the exception of $f_0(\epsilon)$ and $e^{2\pi iKn}$, all the quantities entering the integrals for I_k and L_k change very slowly as functions of ϵ and n (in comparison with the range of variation of ϵ and n). Therefore, by virtue of the condition $n \gg 1$ the basic contribution to the internal integral (1) is made by integrating in the vicinity of the extreme points $n_m(\epsilon)$. Near these points

$$n - n_m = {}^1/_2 (p_z - p_z^m)^2 (\partial^2 n / \partial p_z^2)_m,$$

where $n = n(\epsilon, p_z)$ and $n_m = n(\epsilon, p_z^m)$ and consequently

$$|\partial n / \partial p_z| = \sqrt{2 (\partial^2 n / \partial p_z^2)_m (n - n_m)}.$$

This last relation permits one to determine the contribution to the inner integral of (1) very simply by evaluating the integral in the vicinity of the extreme points:

$$\int^{n_m} \frac{\partial(\chi m^*)}{\partial \varepsilon} \left| \frac{\partial n}{\partial p_z} \right|^{-1} e^{2\pi i k n} dn = \left| 2 \frac{\partial^2 n}{\partial p_z^2} \right|_m^{-1/2} \int^{n_m} \frac{\partial(\chi m^*)}{\partial \varepsilon} \frac{e^{2\pi i k n}}{\sqrt{|n - n_m|}} dn$$

$$\approx w_m(\varepsilon) e^{2\pi i k n_m} \int_{+\infty}^{0} e^{2\pi i k x} |x|^{-1/2} dx = w_m(\varepsilon) e^{2\pi i k n_m} e^{\pm i\pi/4} / \sqrt{2k},$$

where

$$w_m(\varepsilon) = \frac{\partial(\chi m^*)}{\partial \varepsilon} \left| 2 \frac{\partial^2 n}{\partial p_z^2} \right|_{p_z = p_z^m}^{-1/2}$$

The plus sign in the expression $e^{\pm i\pi/4}$ refers to $n_{\min}$, while the minus sign refers to $n_{\max}$. The term corresponding to $n_{\min} = 0$ does not contribute to the oscillatory part of I_k and may be omitted. All the remaining extremal points are repeated twice (for adjacent uniform intervals of change of $n(\epsilon, p_z)$ for a fixed value of ϵ), so that we may transform (1) into the form:

$$I_k \approx 2 \sqrt{\frac{2}{k}} \sum_m e^{\pm i\pi/4} \int f_0(\varepsilon) w_m(\varepsilon) e^{2\pi i k n_m(\varepsilon)} d\varepsilon, \tag{2}$$

where the summation is made over all extremal points. In what follows we shall omit the summation sign for summing over extremal points.

Considering the behavior of $f_0(\epsilon)$ in the vicinity $\epsilon = \zeta$ and assuming that $(\partial n_m / \partial \epsilon)_{\epsilon = \zeta} \neq 0$, we find from (2)

$$I_k \approx 2 \sqrt{\frac{2}{k}} w_m(\zeta) \exp\left\{2\pi i k n_m(\zeta) \pm i \frac{\pi}{4}\right\} \int_0^\infty f_0(\varepsilon) \exp\left\{2\pi i k \frac{dn_m}{d\zeta}(\varepsilon - \zeta)\right\} d\varepsilon = \frac{\sqrt{2}}{\pi k^{3/2}} \frac{w_m(\zeta)}{(dn_m/d\zeta)} \frac{k\lambda}{\operatorname{sh} k\lambda}$$

$$\times \exp\left\{2\pi i k n_m(\zeta) - i\frac{\pi}{2} \pm i\frac{\pi}{4}\right\}, \tag{3}$$

where $\lambda = 2\pi^2 \theta / \hbar\omega^*$, and ζ is the chemical potential.

Thus, the oscillatory part of $\sigma^{\alpha\beta}(\alpha, \beta \neq z, z)$ is given by the expression

$$\Delta\sigma^{\alpha\beta} = \sum_{k=1}^{\infty} \frac{\sqrt{2}}{\pi k^{3/2}} \frac{w_m^{\alpha\beta}(\zeta)}{(dn_m/d\zeta)} \left(\frac{k\lambda}{\operatorname{sh} k\lambda}\right) \exp\left\{2\pi i k n_m(\zeta) - i\left(\frac{\pi}{2} \mp \frac{\pi}{4}\right)\right\}. \tag{4}$$

Equation (4) for $\Delta\sigma^{\alpha\beta}$ can be rewritten in a much more compact form, if we introduce the quantity

$$F_k = 2\iint f_0 e^{2\pi ikn} 2\pi m^* d\varepsilon\, dp_z. \tag{5}$$

It can be easily shown by calculation, analogous to that performed above, that integration in the vicinity of the extremal points (excluding the trivial case $n_{min} = 0$) makes the following contribution to F_k:

$$F_k = A_k(\zeta)\exp\left\{2\pi ikn_m(\zeta) - i\left(\frac{\pi}{2} \mp \frac{\pi}{4}\right)\right\},$$

$$A_k(\zeta) = \frac{2m^*(\zeta, p_z^m)(k\lambda / \operatorname{sh} k\lambda)}{k^{3/2}(dn_m / d\zeta)\left|\partial^2 n / \partial p_z^2\right|_{m,\zeta}^{1/2}} = 2\left(\frac{ehH}{kc}\right)^{3/2} \frac{m^*(\zeta, p_z^m)(k\lambda / \operatorname{sh} k\lambda)}{(dS_m / d\zeta)\left|\partial^2 S(\zeta, p_z) / \partial p_z^2\right|_m^{1/2}}. \tag{6}$$

From this last equation we can see the relation between the quantum numbers $n(\epsilon, p_z)$, and the area $S(\epsilon, p_z)$. As has been shown, all the quantities, which enter into Eqs. (4) and (6), are to be evaluated at the points $\epsilon = \zeta$ and $p_z = p_z^m$ which correspond to extremal areas of cross sections of the boundary Fermi surface. (This problem is examined in greater detail in Ref. 3 and integrals similar to I_k and F_k are evaluated there.)

From Eqs. (3) and (4), it can be seen that

$$I_k = \frac{1}{2\pi m^*}\left(\frac{\partial}{\partial\varepsilon}\chi^{\alpha\beta}m^*\right)_{\zeta, p_z^m} \cdot F_k. \tag{7}$$

Finally, Eq. (4) can be written thus:

$$\Delta\sigma^{\alpha\beta} = \frac{1}{2\pi m^*}\left(\frac{\partial}{\partial\varepsilon}\chi^{\alpha\beta}m^*\right)_{\zeta, p_z^m} \cdot F,\ F = \sum_k F_k. \tag{8}$$

Note, that the oscillatory part of the magnetic moment of the electron gas in the de Haas-van Alphen effect can also be written in terms of the quantity F:

$$\Delta M^z = -h^{-3}\sum_k \iint f_0 \frac{\partial\varepsilon}{\partial H} e^{2\pi ikn} 2\pi m^*\, d\varepsilon p_z \approx -\frac{S_m(\zeta)F}{h^3 H(dS_m/d\zeta)}.$$

Hence, we find that

$$F = -h^3 H(d\ln S_m / d\zeta)\Delta M^z. \tag{9}$$

Substitution of (9) into (8), permits one to express the oscillatory part of $\Delta\sigma^{\alpha\beta}$ in terms of the oscillatory part of the magnetic moment ΔM^z, thus;

$$\Delta\sigma^{\alpha\beta} = -\frac{h^3 H}{(dS_m / d\zeta)}\frac{\partial}{\partial\zeta}(\chi^{\alpha\beta}m^*)_m \frac{d\ln S_m}{d\zeta}\Delta M^z. \tag{10}$$

Let us introduce the "classical mobility tensor" $q^{\alpha\beta}$, which is related to the classical conductivity tensor in the following manner:

$$\sigma_{\text{кл}}^{\alpha\beta} = 2h^{-3}\iint f_0 q^{\alpha\beta} 2\pi m^* d\varepsilon\, dp_z = N_0\overline{q^{\alpha\beta}}, \tag{11}$$

where N_0 is the number of electrons in the conduction band, and $\overline{q}$ is the mean value of the mobility, weighted by f_0.

Since, on the other hand, the classical Eq. (A) can be transformed into

$$\sigma_{\text{кл}}^{\alpha\beta} = 2\iint f_0\left(\frac{\partial}{\partial\varepsilon}\chi^{\alpha\beta}m^*\right) d\varepsilon\, dp_z \quad (\alpha, \beta \neq z, z),$$

one obtains the following expression for $q^{\alpha\beta}$:

$$q^{\alpha\beta} = \frac{h^3}{2\pi m^*}\frac{\partial}{\partial\varepsilon}(\chi^{\alpha\beta}m^*) \quad (\alpha, \beta \neq z, z). \tag{12}$$

Substituting (12) into (10), one finds the following expression for the oscillatory part of $\Delta\sigma^{\alpha\beta}$ (for $\alpha, \beta \neq z, z$) in terms of the independent variables H, ζ:

$$(\Delta\sigma^{\alpha\beta})_{H,\zeta} = -q_m^{\alpha\beta} H(d\ln S_m / d\zeta)\Delta M^z. \tag{13}$$

If there are several groups of electrons which determine the electrical conductivity of the metal, i.e., if there are several overlapping, partially filled energy bands, then every group of electrons makes its own contribution to $\Delta\sigma^{\alpha\beta}$, so that $\Delta\sigma^{\alpha\beta}$ assumes the form of a summation of terms like Eq. (13).

In a similar way one finds the expression for the oscillatory part of $\Delta\sigma^{zz}$. Integrating L_k^{zz} in the way described above one finds

$$L_k^{zz} = \frac{2}{\sqrt{k}}\left(\chi_0^{zz} m^* \left|\partial^2 n/\partial p_z^2\right|^{-1/2}\right)_{\zeta,\, p_z^m}\left(\frac{k\lambda}{\operatorname{sh} k\lambda}\right)\exp\left\{2\pi i k n_m(\zeta) + i\left(\pi \pm \frac{\pi}{4}\right)\right\},$$

which can be written in terms of F_k:

$$L_k^{zz} = -ik\chi_{0m}^{zz}(\zeta)\,(dn_m/d\zeta)\,F_k. \tag{14}$$

Because of the additional coefficient of $\partial n_m/\partial\zeta$, the integral L_k^{zz} [Eq. (14)] is considerably greater than I_k^{zz} (in the ratio ζ/μ^*H where $\mu^*H \equiv \hbar\omega^*$). Therefore, the fundamental contribution to $\Delta\sigma^{zz}$ from those extremal sections, (of the boundary Fermi surface) on which χ_0^{zz} does not vanish, is made by such terms, namely:

$$\Delta\sigma^{zz} = i\chi_{0m}^{zz}(\zeta)\frac{dn_m}{d\zeta}\sum_k kF_k.$$

Using the relation between F and ΔM^z, it is easy to show for this case to this degree of approximation

$$(\Delta\sigma^{zz})_{H,\zeta} = \frac{h^3}{2\pi}H^2\chi_{0m}^{zz}(\zeta)\left(\frac{d\ln S_m}{d\zeta}\right)^2\frac{\partial\Delta M^z}{\partial H}. \tag{15}$$

If χ_0^{zz} vanishes on the extremal cross-section (of the boundary Fermi surface) [in particular, as is evident from the form of the general Eq. (59) of Ref. 1 for χ_0^{zz}, and from symmetry considerations, χ_0^{zz} vanishes on central cross-sections of the Fermi surface], one must take account in the integrals

$$Q_k(\varepsilon) = \int^{n_m}\chi_0^{zz} m^*\left|\frac{\partial n}{\partial p_z}\right|^{-1} e^{2\pi i k n}\,dn$$

of the subsequent (non-vanishing) terms of the expansion of χ_0^{zz} in powers of $(p_z - p_z^m)$.

Assume that near the extremal points

$$\chi_0^{zz} = (p_z - p_z^m)^s\frac{1}{s!}\frac{\partial^s}{\partial p_z^s}\chi_0^{zz}. \tag{16}$$

Then

$$Q_k(\varepsilon) \approx (\pm 1)^s\frac{2^{(s-1)/2}}{s!}\frac{\partial^s\chi_0^{zz}}{\partial p_z^s}m^*\left|\frac{\partial^2 n}{\partial p_z^2}\right|_m^{-(s+1)/2}\int^{n_m}|n - n_m|^{(s-1)/2}e^{2\pi i k n}\,dn, \tag{17}$$

where as before the plus sign goes with n_{min} and the minus sign with n_{max}.

In evaluating the integral in Eq. (17), one can make use of the asymptotic equation for integrals of this form, which is given in Ref. 3.

Note that if $\chi(\epsilon, p_z)$ is an even function of p_z, then from the equality $\chi_0(\epsilon, p_z) = \chi_0(\epsilon, -p_z)$ it follows that $\partial\chi_0/\partial p_z = 0$ for $p_z = 0$, and consequently for central cross-sections, the expansion in Eq. (16) commences with a quadratic term. This means that for central cross-sections

$$Q_k(\varepsilon) = \frac{m^*}{8\pi k^{3/2}}\frac{\partial^2\chi_0^{zz}}{\partial p_z^2}\left|\frac{\partial^2 n}{\partial p_z^2}\right|_0^{-3/2}\exp\left\{2\pi i k n_m \pm i\frac{3}{4}\pi\right\},$$

and the basic contribution to the oscillatory part is given by

$$\Delta\sigma^{zz} = \frac{1}{2\pi}\left\{\frac{1}{m^*}\frac{\partial(\chi^{zz}m^*)}{\partial\varepsilon} \mp \frac{1}{2}\frac{dS_m/d\zeta}{|\partial^2 S/\partial p_z^2|}\frac{\partial^2\chi_0^{zz}}{\partial p_z^2}\right\}_{\zeta,\,p_z=0}\cdot F. \tag{18}$$

By introducing the component q^{zz} of the "classical conductivity tensor" into this equation by analogy with Eq. (11), viz:

$$q^{zz} = \left(\frac{h^3}{2\pi m^*}\right)\frac{\partial}{\partial\varepsilon}(\chi^{zz} + \chi_0^{zz})m^* - \frac{h^3}{m^*}\frac{\partial}{\partial p_z}\left(\frac{\chi_0^{zz}m^{*2}}{\partial S/\partial p_z}\right), \tag{19}$$

it is easy to verify that for central cross-sections

$$q_m^{zz} = \frac{h^3}{2\pi}\left\{\frac{1}{m^*}\frac{\partial(\chi^{zz}m^*)}{\partial\varepsilon} \mp \frac{dS_m/d\varepsilon}{2|\partial^2 S/\partial p_z^2|}\frac{\partial^2\chi_0^{zz}}{\partial p_z^2}\right\}_{p_z=0},$$

and therefore, Eq. (18) can be written in the form

$$(\Delta\sigma^{zz})_{H,\zeta} = -q_m^{zz} H\left(\frac{d\ln S_m}{d\zeta}\right)\Delta M^z. \tag{20}$$

If the boundary Fermi surface is convex, there is a unique extremal cross-section, i.e., the central one, and consequently the oscillatory part of the electrical conductivity of the given group of electrons is determined from Eqs. (13) and (20). Thus all the components of $\Delta\sigma^{\alpha\beta}$ have one single order of magnitude of oscillation. If there are other extremal cross-sections in addition to a central cross-section, $\Delta\sigma^{zz}$ is determined from Eq. (15), and the amplitude of oscillation of $\Delta\sigma^{zz}$ will be considerably greater than the amplitude of oscillation of $\Delta\sigma^{\alpha\beta}$ $(\alpha,\beta \neq z,z)$ for these groups of electrons.

Eqs. (13), (15) and (20) for the oscillatory part of $\Delta\sigma^{\alpha\beta}$ derived above are given in terms of the independent variables ζ and H. It was shown in Ref. 1 that for a concrete application of these equations and for comparison with experiment it is necessary to examine the oscillation of the chemical potential $\zeta = \zeta(H)$, resulting from the constancy of the number of electrons in all bands:

$$2h^{-3}\sum_j\sum_n\int f_0 2\pi m_j\, dp_z\Delta\varepsilon_n \equiv \sum_j N_j = N = \text{const}$$

(summation over j is extended over all bands).

In this case, in contrast to the de Haas-van Alphen effect, in which they can be neglected, these oscillations play a fundamental role, because of the larger magnitude of $\sigma^{\alpha\beta}$.

If we symbolize by $N_0(\zeta)$ the classical relation between the electron concentration and the chemical potential, namely

$$N^0(\zeta) = 2h^{-3}\sum_j\iint f_0 2\pi m_j^*\, dp_z d\varepsilon = 2h^{-3}\sum_j\int f_0\,(d\,\mathbf{p}),$$

we have, using Poisson's equation and considering Eq. (5),

$$N(\zeta) = N^0(\zeta) + h^{-3}\sum_j\sum_{k=1}^{\infty} F_k^j = N^0(\zeta) + h^{-3}\sum_j F^j(\zeta). \tag{21}$$

Next, putting $\zeta = \zeta_0 + \Delta\zeta$, where ζ_0 is the chemical potential for $H = 0$, we can write

$$N^0(\zeta) = N^0(\zeta_0) + \sum_j (\partial N_j^0/\partial\zeta)\,\Delta\zeta,$$

and then it follows from Eq. (21) that

$$\Delta\zeta h^3 \sum_j (\partial N_j^0/\partial\zeta) = -\sum_j F^j(\zeta_0). \tag{22}$$

Taking into consideration that the oscillatory pertubation on $\sigma^{\alpha\beta}$, as a function of the relation $\zeta = \zeta(H)$, has the form

$$\Delta\sigma_1^{\alpha\beta} = (\partial\sigma^{\alpha\beta}/\partial\zeta_0)\,\Delta\zeta, \tag{23}$$

we can substitute into Eq. (23) the expression for $\Delta\zeta$ from Eq. (22):

$$\Delta\sigma_1^{\alpha\beta} = -(\partial\sigma^{\alpha\beta}/\partial\zeta_0)\sum_j F^j(\zeta_0) \Big/ h^3\sum_j(\partial N_j^0/\partial\zeta_0).$$

If we now use (9), the $\Delta\sigma_1^{\alpha\beta}$ are expressed in terms of:

$$\Delta\sigma_1^{\alpha\beta} = H\frac{\partial\sigma^{\alpha\beta}}{\partial\zeta_0}\sum_j\frac{d\ln S_{mj}}{d\zeta_0}\Delta M_j^z \Big/ \sum_k\frac{\partial N_k^0}{\partial\zeta_0}. \tag{24}$$

Finally in Eq. (24), one can express $\sigma^{\alpha\beta}$ in terms of the mean value of the "classical mobility tensor",

$$\Delta\sigma_1^{\alpha\beta} = H\sum_k\frac{\partial}{\partial\zeta_0}(N_k^0\overline{q_k^{\alpha\beta}})\sum_j\frac{d\ln S_{mj}}{d\zeta}\Delta M_j^z \Big/ \sum_k\frac{\partial N_k^0}{\partial\zeta_0}. \tag{25}$$

The experimentally observed oscillations of the conductivity as a function of the magnetic field are described by the sum $\Delta\sigma^{\alpha\beta} + \Delta_1\sigma^{\alpha\beta}$ so that in Eqs. (13), (15), and (20) (or in equations corresponding to them) one should substitute $\zeta = \zeta_0$.

For $\alpha, \beta \neq z,z$ this sum becomes

$$\Delta\sigma^{\alpha\beta} + \Delta\sigma_1^{\alpha\beta} = H \sum_j \left[\sum_k \frac{\partial}{\partial\zeta_0} (N_k^0 \overline{q_k^{\alpha\beta}}) \Big/ \sum_k \frac{\partial N_k^0}{\partial\zeta_0} - q_{mj}^{\alpha\beta}(\zeta_0) \right] \frac{d \ln S_{mj}}{d\zeta_0} \Delta M_j^z . \tag{26}$$

If all the Fermi surfaces are convex, then $\Delta\sigma^{zz} + \Delta\sigma_1^{zz}$ are determined by the analogous expression:

$$\Delta\sigma^{zz} + \Delta\sigma_1^{zz} = H \sum_j \left[\sum_k \frac{\partial}{\partial\zeta_0} (N_k^0 \overline{q_k^{zz}}) \Big/ \sum \frac{\partial N_k^0}{\partial\zeta_0} - q_{mj}^{zz}(\zeta_0) \right] \frac{d \ln S_{mj}}{d\zeta_0} \Delta M_j^z . \tag{27}$$

If there are Fermi surfaces with non-central extremal cross-sections, then

$$\Delta\sigma^{zz} + \Delta\sigma_1^{zz} = \frac{\hbar^3}{2\pi} H^2 \sum_i \chi_{0mi}^{zz}(\zeta_0) \left(\frac{d \ln S_{mi}}{d\zeta_0} \right)^2 \frac{\partial \Delta M_i^z}{\partial H} , \tag{28}$$

where the summation extends over all Fermi surfaces with non-central cross-sections.

From the formulae for the oscillatory part of the conductivity $\Delta\sigma^{\alpha\beta}$ [in what follows $\Delta\sigma^{\alpha\beta}$ will be used to symbolize all the terms of $\sigma^{\alpha\beta}$ which oscillate as the magnetic field changes, i.e., sums of the form of Eqs. (26) and (27)] it is evident that each group of electrons makes its own contribution to $\Delta\sigma^{\alpha\beta}$. It turns out, that the contribution of each band is related to ΔM^z only for similar groups of electrons. Therefore, the period of oscillation of $\Delta\sigma^{\alpha\beta}$ is always determined by the same coefficient as the period of oscillation of ΔM^z, and, of course, it coincides with that of the de Haas-van Alphen effect.[3]

$$\Delta(1/H) = eh/cS_m(\zeta_0).$$

Principal emphasis in what follows will be on the amplitude of the oscillations of $\Delta\sigma^{\alpha\beta}$. First, in contrast to the oscillations of ΔM^z, the amplitude of the summands in $\Delta\sigma^{\alpha\beta}$ corresponding to given "anomalously narrow" bands are determined by all the "normal" groups of electrons. Secondly, the undetermined quantities $\chi(\epsilon, p_z)$ enter into a calculation of the amplitude. These quantities are also involved in the classical expression for the conductivity.

In certain concrete cases, which are introduced in Sec. 4, the quantities $\chi(\epsilon, p_z)$ can be obtained from the solution of the corresponding kinetic equations, and the magnitudes of the oscillations can be calculated exactly.

2. ASYMPTOTIC VALUES OF THE CONDUCTIVITY OSCILLATIONS IN STRONG MAGNETIC FIELDS

Let us now examine the behavior of the amplitude of oscillation of $\Delta\sigma^{\alpha\beta}$ in strong magnetic fields, when $\gamma_0 \ll 1$, where, in accordance with Ref. 4, we denote by γ_0 the relation $\gamma_0 = 1/(\epsilon H t_0/m_0 c)$ where m_0 and t_0 are the characteristic mass and relaxation time, respectively. In this case we can make an asymptotic expansion of the amplitude in powers of γ_0 making use of the asymptotic value of $\sigma_0^{\alpha\beta} \equiv \sigma_{KL}^{\alpha}$ obtained in Refs. 1 and 4. If the boundary Fermi surface is split into several closed surfaces, the asymptotes, $\sigma_0^{\alpha\beta}$, have the form

$$\sigma_0^{\alpha\beta}(\mathbf{H}) = \begin{pmatrix} \gamma_0^2 a_{11} & \gamma_0 a_{12} + \gamma_0^2 a_{12}' & \gamma_0 a_{13} \\ \gamma_0 a_{21} + \gamma_0^2 a_{21}' & \gamma_0^2 a_{22} & \gamma_0 a_{23} \\ \gamma_0 a_{31} & \gamma_0 a_{32} & a_{33} \end{pmatrix}. \tag{29}$$

Here $a_{\alpha\beta}(\zeta_0)$ is a matrix whose elements can be expanded in powers of γ_0 beginning with a zero order term, such that

$$\gamma_0 a_{12} = \frac{ec}{H} \left(\sum_k N_k^+ - \sum_i N_i^- \right), \tag{30}$$

where N^+ is the electron density, and N^- is the hole density for corresponding groups.

Since for every group of electrons $\sigma^{\alpha\beta} = N_0 \overline{q^{\alpha\beta}}$ the asymptotic expansion of the elements of $\overline{q^{\alpha\beta}}$ begins with terms of the same order in γ_0, as $\sigma_0^{\alpha\beta}$, namely

$$\overline{q^{\alpha\beta}} = \begin{pmatrix} \gamma_0^2 \overline{u_{11}} & ec/H + \gamma_0^2 \overline{u_{12}} & \gamma_0 \overline{u_{13}} \\ -ec/H + \gamma_0^2 \overline{u_{21}} & \gamma_0^2 \overline{u_{22}} & \gamma_0 \overline{u_{23}} \\ \gamma_0 \overline{u_{31}} & \gamma_0 \overline{u_{32}} & \overline{u_{33}} \end{pmatrix}. \tag{31}$$

The expansion of the elements of the matrix $\overline{u_{\alpha\beta}}$ in powers of γ_0 begins, generally speaking, with a zero order term (in some cases, the expansion of $a_{\alpha\beta}$ is the same as $\overline{u_{\alpha\beta}}$, cf. Ref. 4). All the elements of $u^{\alpha\beta}$ depend on the form of the collision integral, and generally speaking, they turn out to be functions of ζ_0.

Since $q_m^{\alpha\beta}$ is generally of the same order of magnitude as $q^{\alpha\beta}$, the asymptotes of $q_m^{\alpha\beta}$, in strong magnetic fields, should have the form of Eq. (31). In this case it is interesting to examine the possibility of an exact calculation of the first term in the asymptotic expansion of q_m^{xy} as follows.

For perpendicular magnetic field H (applied along the axis OZ) and electric field E (along the axis OY) the electron finds itself in a steady state with a constant velocity (depending on its state) whose mean value differs from zero. This velocity is directed along the axis OX and it is given by cE/H.* Note that the indicated velocity does not depend on the dispersion law for the electrons nor on the state of the electrons, i.e., it does not change because of collisions experienced by the electron. The steady state mobility of the electron $q_0^{xy} = ec/H$ is related to this velocity.

In strong magnetic fields when the mean time between collisions of the electrons is much greater than the time required for one revolution of the electron around its classical orbit, q_0^{xy} makes the principal contribution to the elements q^{xy} of the mobility tensor, and consequently

$$q_m^{xy} = ec/H + \gamma_0^2 u_m^{12}, \tag{32}$$

where $\gamma_0^2 u_m^{12}$ is the mobility, which is a function of the collision integrals. Thus it is seen by equating (32) and (31), that the first terms in the expansion of $\overline{q^{xy}}$ and $\overline{q_m^{xy}}$ in powers of γ_0 are identical.

From Eqs. (26) and (27), and also from Eqs. (31) and (32), it follows that the amplitudes of all the elements of $\Delta\sigma^{\alpha\beta}$, with the exception of $\Delta\sigma^{xy}$, have the same order of magnitude in terms of γ_0, as do the corresponding elements of $\sigma_0^{\alpha\beta}$. As for $\Delta\sigma^{xy}$, its ratio to γ_0 has a higher order of magnitude than that of σ_0^{xy}, if $\sum_k N_k^+ \neq \sum_i N_i^-$. One can write for the relative magnitude of the oscillations in the case of convex Fermi surfaces and for $\sum_k N_k^+ \neq \sum_i N_i^-$

$$\Delta\sigma^{\alpha\beta}/\sigma_0^{\alpha\beta} = H \sum_j \Psi_j^{\alpha\beta} \frac{d \ln S_m}{d\zeta_0} \Delta M_j^z, \tag{33}$$

where

$$\Psi^{\alpha\beta} = \begin{pmatrix} \psi_{11} & \gamma_0\psi_{12} & \psi_{13} \\ \gamma_0\psi_{21} & \psi_{22} & \psi_{23} \\ \psi_{31} & \psi_{32} & \psi_{33} \end{pmatrix}, \tag{34}$$

and the matrix $\psi_{\alpha\beta}$

$$\psi_{\alpha\beta}^j = \frac{\partial \ln a_{\alpha\beta}}{\partial\zeta_0} \Big/ \sum_k \frac{\partial N_k^0}{\partial\zeta_0} - u_{mj}^{\alpha\beta} \Big/ a_{\alpha\beta}, \quad (\alpha, \beta \neq x, y),$$

$$\psi_{xy}^j = \frac{\partial a'_{12}}{\partial\zeta_0} \Big/ a_{12} \sum_k \frac{\partial N_k^0}{\partial\zeta_0} - u_{mj}^{12} \Big/ a_{12},$$

and, consequently, the expansion of the terms of the $\psi_{\alpha\beta}$ matrix begins, generally speaking, with a zero order term in γ_0.

It is characteristic that the relative order of magnitude of the oscillations of σ^{xy} (in terms of γ_0) is less than the relative magnitude of oscillation of the remaining elements of the tensor $\sigma^{\alpha\beta}$.† There is a simple physical reason for this conclusion, namely that the asymptotic value σ_0^{xy} in strong fields for

*It is easier to verify that this is so by noting that in a steady state the mean value of the Lorentz force acting on the electron is zero. Since the expression for the Lorentz force does not depend on the form of the dispersion law, the steady-state velocity acquired by the electron in crossed magnetic and electric fields is independent of the dispersion law.

†In some concrete cases, in particular for an isotropic dispersion law, the expansion of Ψ^{xy} can begin with a term of the order γ_0^2 (for $\sum_k N_k^+ \neq \sum_i N_i^-$)

$\sum_k N_k^+ \neq \sum_i N_i^-$ in Eq. (30) has a fixed value for a given metal, and it does not experience any quantum oscillations.

If $\sum_k N_k^+ = \sum_i N_i^-$ the expansion of σ_0^{xy} begins with γ_0^2 and therefore the expansion of Ψ^{xy} begins with a a zero order term $\Psi^{xy} = \psi'_{12}$ where

$$\psi'_{12} = \frac{\partial \ln a'_{12}}{\partial \zeta_0} \Big/ \sum_k \frac{\partial N_k^0}{\partial \zeta_0} - u_m^{12} \Big/ a'_{12}.$$

The expansion of the remaining elements in $\Psi^{\alpha\beta}$ results as before in equations of the form of Eq. (34).

3. OSCILLATIONS OF THE RESISTIVITY

In experiments one usually measures not the electrical conductivity tensor $\delta^{\alpha\beta}$ but rather the specific resistivity tensor $\rho^{\alpha\beta} = \sigma_{\alpha\beta}^{-1}$. Consequently it is necessary to determine the oscillatory part of $\rho^{\alpha\beta}$.

The relation between the elements of the tensor $\rho^{\alpha\beta}$ and those of $\sigma^{\alpha\beta}$ is defined by the well known expression

$$\rho^{\alpha\beta} = D_{\beta\alpha}/ \parallel \sigma \parallel, \quad \parallel \sigma \parallel = \det \mid \sigma^{\alpha\beta} \mid, \tag{35}$$

where $D_{\alpha\beta}$ are the algebraic complement of the elements of $\sigma^{\alpha\beta}$ in the determinant $\parallel \sigma \parallel$.

Let us write $\sigma^{\alpha\beta} = \sigma_0^{\alpha\beta} + \Delta\sigma^{\alpha\beta}$ designating by $\Delta\sigma^{\alpha\beta}$ the oscillatory part of $\sigma^{\alpha\beta}$, which, as is known, represents a small quantum perturbation to $\sigma^{\alpha\beta}$. Then, leaving only linear terms in $\Delta\sigma^{\alpha\beta}$, it can be easily shown.

$$\parallel \sigma \parallel = \parallel \sigma_0 \parallel + D_{\alpha\beta}^0 \Delta\sigma^{\alpha\beta} = \parallel \sigma_0 \parallel (1 + \rho_0^{\alpha\beta} \Delta\sigma^{\alpha\beta}) \tag{36}$$

(we omit the sign for summation over indices which are to be taken in pairs from 1 to 3).

Similarly we find to the same degree of approximation

$$D_{\alpha\beta} = D_{\alpha\beta}^0 + \varepsilon_{\alpha kl}\varepsilon_{\beta pq}\sigma_0^{kp}\Delta\sigma^{lq}, \tag{37}$$

where ϵ_{ikl} is an antisymmetric unit tensor of the third rank.

By using Eqs. (36) and (37), there follows from Eq. (35)

$$\Delta\rho^{\alpha\beta} = (\parallel \rho_0 \parallel \varepsilon_{\beta kl}\varepsilon_{\alpha pq}\sigma_0^{kp} - \rho_0^{\alpha\beta}\rho_0^{lq})\Delta\sigma^{lq}, \quad \Delta\rho^{\alpha\beta}(\mathbf{H}) = \Delta\rho^{\beta\alpha}(-\mathbf{H}), \tag{38}$$

where

$$\parallel \rho_0 \parallel = \det \mid \rho_0^{\alpha\beta} \mid = \parallel \sigma_0 \parallel^{-1}.$$

It is evident from Eq. (38), that the equation for the oscillatory part of the electrical resistivity tensor in the general case has an exceedingly cumbersome form. The expression for $\Delta\rho^{\alpha\beta}$ retains the classical value and the oscillatory parts, generally speaking, of all the components of $\sigma^{\alpha\beta}$. Even in the simplest cases $\Delta\rho^{\alpha\beta}$ retains some terms, which can have a single order of magnitude of amplitude and various periods of oscillation. Simplification of Eq. (38) occurs only for certain special cases.

In particular, if there are boundary Fermi surfaces with non central cross-sections (it is obvious that these always occur in even numbers, so that the cross-section which are placed symmetrically around $p_z = 0$ make equal contributions to the oscillation $\Delta\sigma^{\alpha\beta}$), on which $\chi_0^{zz} \neq 0$, then, in magnetic fields that satisfy the relation $\gamma_0 \sim 1$, one need retain in Eq. (38) only those terms which contain $\Delta\sigma^{zz}$.

The components of $\Delta\rho^{\alpha\beta}$ in this case have the form

$$\Delta\rho^{xx} = (\parallel \rho_0 \parallel \sigma_0^{yy} - \rho_0^{xx}\rho_0^{zz})\Delta\sigma^{zz}, \Delta\rho^{yy} = (\parallel \rho_0 \parallel \sigma_0^{xx} - \rho_0^{yy}\rho_0^{zz})\Delta\sigma^{zz},$$
$$\Delta\rho^{xy} = -(\parallel \rho_0 \parallel \sigma_0^{xy} + \rho_0^{xy}\rho_0^{zz})\Delta\sigma^{zz}, \quad \Delta\rho^{\alpha z} = -\rho_0^{\alpha z}\rho_0^{zz}\Delta\sigma^{zz} \quad (\alpha = x, y, z),$$

where $\Delta\sigma^{zz}$ is determined by Eq. (28).

If the Fermi surfaces are all convex, one can calculate the asymptotic values of $\Delta\rho^{\alpha\beta}$ in strong magnetic fields ($\gamma_0 \ll 1$). To do this, just as was done in Sec. 2, one uses the asymptotes of the tensors $\sigma_0^{\alpha\beta}$

[Eq. (29)] and $\rho^{\alpha\beta}$ (Ref. 4), which allow one to express the relative value of the oscillations of the specific resistance tensor in the following way:

$$\Delta\rho^{\alpha\beta}/\rho_0^{\alpha\beta} = H \sum_j \Phi_j^{\alpha\beta} \frac{d \ln S_{mj}}{d\zeta_0} \Delta M_j^z. \tag{39}$$

If $\sum_k N_k^+ \neq \sum_i N_i^-$, the matrix $\Phi^{\alpha\beta}$ has the form:

$$\Phi^{\alpha\beta} = \begin{pmatrix} \varphi_{11} & \gamma_0\varphi_{12} & \varphi_{13} \\ \gamma_0\varphi_{21} & \varphi_{22} & \varphi_{13} \\ \varphi_{31} & \varphi_{32} & \varphi_{33} \end{pmatrix}. \tag{40}$$

Expansion of the elements of the $\phi_{\alpha\beta}$ matrix in powers of γ_0 begins with zero order terms.

It can be shown, just as in the oscillations of the conductivity, that the relative magnitudes of the oscillations of the elements, ρ^{xy} is smaller by orders of magnitude (in terms of γ_0) than the relative magnitude of the oscillation of the remaining elements in the $\rho^{\alpha\beta}$ tensor.

If $\sum_k N_k^+ = \sum_i N_i^-$, the expansion of the element ϕ^{xy} again begins with a zero order term in γ_0, and the expansions of the other terms in the $\Phi^{\alpha\beta}$ matrix have the form shown in Eq. (40).

4. CALCULATION OF THE OSCILLATIONS IN SOME CONCRETE CASES.

In this section we will examine a series of simple cases, which permit simplification of the general formulae for the oscillatory parts of the tensors $\sigma^{\alpha\beta}$ and $\rho^{\alpha\beta}$.

The relations for $\Delta\sigma^{\alpha\beta}$ are greatly simplified in the presence of a single conduction band with convex Fermi surfaces. In fact, in Eqs. (26) and (27), we retain only one term and find

$$\Delta\sigma^{\alpha\beta} = H \left\{ \overline{q^{\alpha\beta}} - q_m^{\alpha\beta}(\zeta_0) + \frac{\partial \overline{q^{\alpha\beta}}}{\partial\zeta_0} \Big/ \frac{\partial \ln N_0}{\partial \zeta_0} \right\} \frac{d \ln S_m}{d\zeta_0} \Delta M^z. \tag{41}$$

It is interesting to note that if $q^{\alpha\beta}$ does not depend on ϵ and p_z the sum in Eq. (41) vanishes. This means, that the oscillations of $\sigma^{\alpha\beta}$ and $\rho^{\alpha\beta}$ for the case of a single conduction band with convex Fermi surfaces depend on the functional relation between the mobility and ϵ and p_z.

To calculate the amplitude of the oscillations of $\Delta\sigma^{\alpha\beta}$ and $\Delta\rho^{\alpha\beta}$, whether there exist one or several groups of electrons, it is necessary to know the functions $\chi(\epsilon, p_z)$, which can be determined only on the basis of certain assumptions concerning the collision integrals.

If the collision integrals in the kinetic equation can be replaced by a "relaxation time" t_0, which generally depends on ϵ and p_z, then

$$\chi^{\alpha\beta} = 2\pi e^2 t_0 h^{-3} 2\,\mathrm{Re} \sum_k \frac{\gamma}{\gamma + ik} v_{-k}^{\alpha} v_k^{\beta}, \quad \chi_0^{zz} = 2\pi e^2 t_0 h^{-3} (v_0^z)^2, \ \gamma = 1/t_0\omega^* = m^*c/et_0H; \tag{42}$$

where v_k are the Fourier components of the velocity of the electron.

In the isotropic case Eq. (42) becomes even simpler. Noting that for any isotropic dispersion law $\epsilon = \epsilon(p)$ the relations $p = m^* v$ and $v = \nabla_p \epsilon$* are satisfied we have

$$\chi^{xx} = \chi^{yy} = \frac{e^2 t_0}{h^3} \frac{\gamma^2}{1+\gamma^2} \frac{S(\varepsilon, p_z)}{m^{*2}}, \qquad \chi^{xy} = \frac{e^2 t_0}{h^3} \frac{\gamma}{1+\gamma^2} \frac{S(\varepsilon, p_z)}{m^{*2}},$$
$$\chi^{\alpha z} = 0 (\alpha = x, y, z), \quad \chi_0^{zz} = 2\pi \frac{e^2 t_0}{h^3} \frac{p_z^2}{m^{*2}}, \ S(\varepsilon, p_z) = \pi(p^2(\varepsilon) - p_z^2), \tag{43}$$

where $p = p(\epsilon)$ is an inverse function of $\epsilon = \chi(p)$.

From Eq. (43) and from the definition of the mean value of the "classical mobility tensor [Eq. (11)], with the assumption that $f_0'(\epsilon)$ can be approximated by a δ-function, it follows that:

$$\overline{q^{xx}} = \overline{q^{yy}} = q_0/(1 + \omega^{*2} t_0^2), \ \overline{q^{xy}} = \omega^* t_0 q_0/(1 + \omega^{*2} t_0^2), \quad \overline{q^{zz}} = q_0 e^2 t_0/m^*, \quad \overline{q^{xz}} = \overline{q^{yz}} = 0, \ t_0 = t_0(\zeta_0), \tag{44}$$

*In the isotropic case $v = (\partial\epsilon/\partial p) p/p$ and from the expression $S(\epsilon, p_z) = \pi(p^2(\epsilon) - p_z^2)$ it follows that $m^* = p(\partial p/\partial\epsilon) = p(\partial\epsilon/\partial p)$ so that $p = m^* v$.

where, as above, $\omega^* = eH/m^*c$.

Furthermore, from Eqs. (12) and (19), using Eq. (43), we find that

$$q_m^{\alpha\beta}(\zeta_0) = \overline{q^{\alpha\beta}} + \frac{p^2(\zeta_0)}{2m^*}\frac{\partial \overline{q^{\alpha\beta}}}{\partial \zeta_0} \quad (\alpha,\beta \neq z,z); \; q_m^{zz} = \overline{q^{zz}}. \tag{45}$$

We will now apply Eqs. (44) and (45) to an examination of two models.

(1) One Conduction Band

If there is only one conduction band, when

$$\partial \ln N^0/\partial \zeta_0 = 3m^*/p^2(\zeta_0),$$

the oscillating terms of $\Delta\sigma^{\alpha\beta}$ have the form

$$\Delta\sigma^{\alpha\beta} = -\frac{1}{3}H\frac{\partial \overline{q^{\alpha\beta}}}{\partial \zeta_0}\Delta M^z \quad (\alpha,\beta \neq z,z); \; \Delta\sigma^{zz} = \frac{2}{3}H\frac{\partial \overline{q^{zz}}}{\partial \zeta_0}\Delta M^z, \tag{46}$$

where $\overline{q^{\alpha\beta}}$ is determined from Eq. (44).

In strong magnetic fields ($\gamma \ll 1$) all the elements of $\overline{q^{\alpha\beta}}$ and their derivatives can be readily expressed in terms of q_0, and we find*

$$\frac{\Delta\sigma^{xx}}{\sigma_0^{xx}} = \frac{\Delta\sigma^{yy}}{\sigma_0^{yy}} = \frac{1}{3}H\frac{\partial \ln q_0}{\partial \zeta_0}\frac{\Delta M^z}{N_0}, \quad \frac{\Delta\sigma^{xy}}{\sigma_0^{xy}} = -\frac{2}{3}\gamma^2 H\frac{\partial \ln q_0}{\partial \zeta_0}\frac{\Delta M^z}{N_0}, \quad \frac{\Delta\sigma^{zz}}{\sigma_0^{zz}} = \frac{2}{3}H\frac{\partial \ln q_0}{\partial \zeta_0}\frac{\Delta M^z}{N_0}. \tag{47}$$

The expression for the oscillatory parts of the magnetic moment ΔM^z in the case of an isotropic dispersion law can be found from the general equations,[3] in which one should substitute

$$S_m(\zeta_0) = \pi p^2(\zeta_0), \quad dS_m/d\zeta_0 = 2\pi m^*, \quad |\partial^2 S/\partial p_z^2| = 2\pi.$$

Using $\sigma_0^{\alpha\beta} = N_0\overline{q^{\alpha\beta}}$, and also Eqs. (38) and (46), we can calculate the oscillations of the specific resistivity tensor

$$\Delta\rho^{xx} = \Delta\rho^{yy} = \frac{H}{3\sigma_0^2}\left\{(1-\omega^{*2}t_0^2)\frac{\partial \overline{q^{xx}}}{\partial \zeta_0} - 2\omega^* t_0\frac{d\overline{q^{xy}}}{\partial \zeta_0}\right\}\Delta M^z, \quad \Delta\rho^{xy} = \frac{H}{3\sigma_1^2}\left\{(1+3\omega^{*2}t_0^2)\frac{\partial \overline{q^{xy}}}{\partial \zeta_0} - 2\omega^* t_0\frac{\partial \overline{q^{xx}}}{\partial \zeta_0}\right\}\Delta M^z,$$
$$\Delta\rho^{zz} = -\frac{2H}{3\sigma_0^2}\left\{2\omega^* t_0\frac{\partial \overline{q^{xy}}}{\partial \zeta_0} + \frac{\partial \overline{q^{zz}}}{\partial \zeta_0}\right\}\Delta M^z, \tag{48}$$

where $\sigma_0 = N_0 e^2 t_0/m^*$ is the conductivity of the metal in the absence of a magnetic field.

In strong magnetic fields one can substitute $\omega^* t_0 \gg 1$, which transforms Eq. (48) into

$$\frac{\Delta\rho^{xx}}{\rho_0^{xx}} = \frac{\Delta\rho^{yy}}{\rho_0^{yy}} = \frac{1}{3}H\frac{\partial \ln q_0}{\partial \zeta_0}\frac{\Delta M^z}{N_0}, \quad \frac{\Delta\rho^{xy}}{\rho_0^{xy}} = -\frac{8}{3}\gamma^2 H\frac{\partial \ln q_0}{\partial \zeta_0}\frac{\Delta M^z}{N_0}, \quad \frac{\Delta\rho^{zz}}{\rho_0^{zz}} = -\frac{2}{3}\frac{\partial \ln q_0}{\partial \zeta_0}\frac{\Delta M^z}{N_0}. \tag{49}$$

It should be observed that the relative magnitudes of the oscillations of $\Delta\sigma^{xy}/\sigma_0^{xy}$, and also of $\Delta\rho^{xy}/\rho_0^{xy}$ in strong magnetic fields in our case are smaller by two orders of magnitude of γ_0 than the relative magnitudes of the oscillations of the remaining terms in the $\sigma^{\alpha\beta}$ and $\rho^{\alpha\beta}$ tensors.

(2) Two Bands with $N^+ = N^-$

Let us assume, that there are two groups of electrons having a quadratic dispersion law, for one of which $(1/2\pi)\, dS_{m1}/d\zeta_0 = m_1 > 0$ (electrons), and for the other $(1/2\pi)\, dS_{m2}/d\zeta_0 = -m_2 < 0$ (holes), and that $N^+ = N^-$.

Symbolizing by q_1 the mobility of the electrons, and by q_2 that of the holes, it can be easily shown from the general Eqs. (26) and (27)

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*If $m^* = m = \text{const}$, then $\partial \ln q_0/\partial\zeta = \partial \ln t_0/\partial\zeta_0$ and the expression for $\Delta\sigma^{xy}/\sigma_0^{xy}$ in Eq. (47) coincides with that given elsewhere,[5] where, however, there is a misprint in the numerical coefficient.

$$\Delta\sigma^{\alpha\beta} = -\frac{H}{(1+\mu)\zeta_0}\left\{\left[\mu\,(\overline{q_1^{\alpha\beta}} - \overline{q_2^{\alpha\beta}}) + \left(\mu + \frac{1}{3}\right)\zeta_0\,\overline{\frac{\partial q_1^{\alpha\beta}}{\partial\zeta_0}} + \frac{2}{3}\zeta_0\,\overline{\frac{\partial q_2^{\beta\alpha}}{\partial\zeta_0}}\right]\Delta M_1^z\right.$$
$$\left. + \left[\mu\,(\overline{q_1^{\alpha\beta}} - \overline{q_2^{\alpha\beta}}) + \frac{2}{3}\mu\zeta_0\,\overline{\frac{\partial q_1^{\alpha\beta}}{\partial\zeta_0}} - \left(\frac{5}{3}\mu + 1\right)\zeta_0\,\overline{\frac{\partial q_2^{\alpha\beta}}{\partial\zeta_0}}\right]\Delta M_2^z\right\}, \tag{50}$$

for α, $\beta \neq z,z$ and

$$\Delta\sigma^{zz} = -\frac{H}{(1+\mu)\zeta_0}\left\{\left[\mu\,(\overline{q_1^{zz}} - \overline{q_2^{zz}}) - \frac{2}{3}\zeta_0\left(\overline{\frac{\partial q_1^{zz}}{\partial\zeta_0}} - \overline{\frac{\partial q_2^{zz}}{\partial\zeta_0}}\right)\right]\Delta M_1^z + \left[\mu\,(\overline{q_1^{zz}} - \overline{q_2{}^z}) + \frac{2}{3}\mu\zeta_0\left(\overline{\frac{\partial q_1^{zz}}{\partial\zeta_0}} - \overline{\frac{\partial q_2^{zz}}{\partial\zeta_0}}\right)\right]\Delta M_2^z\right\}, \tag{51}$$

where $\mu = m_2/m_1$, and the mobilities are determined from Eqs. (44) in which one substitutes in the case of the electrons $m^* = m_1$, and for the holes, $m^* = -m_2$.

For the extreme cases $\mu \gg 1$ or $\mu \ll 1$, one can leave out some of the terms in Eqs. (50) and (51); however, the general character of the expressions does not change.

Eqs. (50) and (51) show that the assumption $N^+ = N^-$ does not introduce any fundamental simplification of the general expressions for $\Delta\sigma^{\alpha\beta}$.

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ON THE BRIGHTNESS OF STRONG SHOCK WAVES IN AIR

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The emission and absorption of light at high temperatures in a shock wave front in air are considered. The dependence of the brightness of the shock wave front on its amplitude is derived.

IN our preceding article[1]* we considered in a general form the problem of the internal structure of the front of strong shock waves in gases, taking account of radiation. We operated throughout with integral characteristics of the radiation — the total energy flux and density. Also, passing from the geometrical coordinate to the optical thickness, we excluded from consideration the actual distribution of quantities in space, which is determined by the coefficient of absorption of light in the gas. This approach is in-

*Hereafter referred to as I.