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ENERGY SPECTRUM OF CASCADE ELECTRONS IN LIGHT MATERIALS

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The energy spectrum of cascade electrons in showers produced by a primary electron or photon in light substances has been computed. A brief analysis of the numerical results is given.

BELOW we present the results of a calculation of the energy spectrum of cascade electrons in light materials in electron-photon showers, carried out by the method of moments.[1] Recursion formulas have been obtained which make it possible to obtain all moments of the distribution functions for electrons with energies higher than E in a shower produced by a primary electron or photon with energy E_0. Explicit analytical expressions have been derived for the first two moments of the functions and numerical values

TABLE I. The Function $\{N_p(E_0, t, E)\}^P$

t \ ε	0.6; 0	0.6; 0.03	0.6; 0.06	0.6; 0.1	0.6; 0.2	0.6; 0.4	1.4; 0	1.4; 0.06	1.4; 0.2	1.4; 0.4	1.4; 0.8
0,1	0.0303	0.0277	0.0240	0.0203	0,0133	0.0035	0,063	0.056	0.043	0,030	0.012
0,3	0.0747	0.0681	0,0591	0.0498	0,0326	0.0085	0.156	0.137	0.106	0,073	0.029
0,5	0,102	0.0927	0,0805	0.0679	0,0444	0.0115	0,214	0.188	0,144	0,100	0.039
0,7	0,117	0,106	0.0918	0,0775	0.0505	0.0131	0,246	0,215	0,165	0,114	0,045
0,9	0,122	0,110	0,0958	0.0808	0,0526	0.0137	0.260	0,226	0,173	0.120	0.046
1,2	0,119	0,107	0.0927	0.0783	0,0508	0.0132	0.256	0,221	0,169	0,116	0,045
1,4	0,112	0.0998	0.0869	0.0734	0.0475	0.0123	0.243	0,209	0,159	0,109	0,042
1,6	0,102	0,0911	0,0793	0.0670	0,0432	0.0112	0.225	0.192	0,146	0,100	0,038
2.0	0.0812	0,0713	0.0622	0,0526	0,0337	0.0087	0,183	0.154	0,116	0,079	0,030
2.3	0,0652	0,0566	0.0494	0,0418	0,0266	0.0069	0,151	0.125	0,094	0,064	0,024
2.6	0,0505	0,0431	0.0377	0,0319	0,0201	0.0052	0.121	0,099	0,073	0,049	0,018
3.0	0,0340	0,0280	0.0245	0.0208	0,0128	0.0033	0.086	0,068	0,049	0,033	0,012
3.5	0.0180	0.0138	0.0122	0.0104	0,0061	0.0015	0.052	0,038	0,027	0,017	0,006
4.0	0.0073	0,0044	0.0040	0.0035	0,0016	—	0.028	0,018	0,012	0,007	0,002

t \ ε	3; 0	3; 0.06	3; 0.2	3; 0.4	3; 08	3; 2	5; 0	5; 0.03	5; 0.1	5; 0.4	5; 0.8	5; 2
0.1	0.110	0,095	0.085	0,071	0.057	0.020	0.147	0.142	0,132	0,105	0.094	0,057
0.3	0.274	0.245	0.212	0.176	0,140	0.049	0.374	0.360	0,373	0,264	0,232	0,139
0,5	0.381	0.340	0,292	0.241	0.191	0.065	0.527	0.505	0,466	0.366	0,319	0.189
0,7	0,444	0.395	0,338	0.278	0,218	0,073	0.623	0.594	0,546	0.426	0,367	0,214
0.9	0,475	0.420	0.358	0,294	0.227	0,074	0.676	0.642	0,587	0.455	0,386	0,223
1.2	0.479	0.421	0.355	0,291	0,220	0,068	0.698	0.658	0,598	0.458	0,380	0,216
1.4	0.462	0.405	0.340	0.277	0.206	0.060	0,686	0.644	0.582	0.442	0,361	0.199
1.6	0.437	0,381	0.317	0,257	0.189	0.052	0,661	0,616	0.554	0.417	0.335	0.180
2,0	0,374	0.321	0.262	0.211	0.148	0.036	0.585	0,539	0.479	0.353	0,272	0.138
2,3	0.318	0.272	0.219	0.174	0.118	0,024	0.518	0,473	0.416	0.301	0.224	0.108
2,6	0,266	0.225	0.178	0.140	0.090	0.015	0.451	0,407	0.355	0.251	0,179	0,080
3.0	0,204	0.170	0.130	0.101	0.059	0.004	0,366	0.326	0.280	0,192	0.126	0.047
3.5	0,141	0,114	0.083	0.062	0.030		0,275	0,239	0.201	0,131	0.076	0.021
4.0	0.093	0.072	0,049	0.035	0.010		0,201	0,171	0.140	0,085	0.040	0.008
5,0	0.036	0.024	0.012				0,103	0,082	0.063	0.031	0.010	
6.0							0,050	0.038	0.027	0.009		
7.0							0,025	0.018	0.011			
8,0												

t \ ε	10; 0	10; 0,03	10; 0.1	10; 0.4	10; 0.8	10; 2	15; 0.03	15; 0.1	15; 0.2	15; 0.4	15; 0.8	15; 2
0.1	0.207	0.197	0,182	0.154	0.142	0.108	0.206	0.202	0,193	0.18	0.179	0.145
0.3	0.537	0.510	0.469	0.396	0.563	0.275	0.551	0.535	0.506	0.490	0,457	0.363
0.5	0.742	0.703	0.644	0,557	0,502	0.371	0.785	0.761	0.695	0.681	0,620	0.487
0,7	0.939	0.887	0.811	0.664	0,590	0.429	1,023	0,970	0.908	0.815	0,771	0.588
0.9	1.046	0.984	0.897	0,726	0.637	0.454	1,171	1,103	1,022	0,910	0.842	0,629
1.2	1.123	1,051	0,955	0.759	0.653	0,451	1.310	1.216	1.116	0,982	0.878	0.635
1.4	1.134	1.059	0,958	0.753	0,638	0.430	1,358	1.250	1.140	0,995	0.870	0.614
1.6	1.123	1,044	0,951	0.731	0.610	0.401	1,378	1.258	1.140	0.987	0.844	0.580
2.0	1.054	0.973	0.872	0.659	0.533	0,331	1.356	1.218	1,092	0.929	0.759	0,494
2.3	0.977	0,897	0.800	0.593	0.466	0.276	1.301	1.155	1.027	0,864	0,681	0.424
2,6	0.892	0.814	0.722	0.524	0.400	0.224	1.225	1.077	0,949	0.789	0.601	0,356
3.0	0.774	0.701	0.616	0.435	0,317	0.164	1.106	0.960	0,837	0,685	0.498	0,274
3.5	0.633	0,568	0.494	0,336	0.231	0.104	0,946	0.809	0,698	0.560	0,384	0.191
4,0	0,508	0.451	0.388	0,254	0.163	0,061	0.790	0,667	0.583	0.448	0,289	0,128
5,0	0.315	0.274	0.229	0.138	0.075	0,014	0.521	0.431	0.360	0.274	0,158	0,051
6.0	0.189	0.161	0,130	0,072	0.031		0,326	0.266	0.217	0.160	0,084	0,018
7.0	0.111	0,093	0,073	0.037	0.012		0.195	0.158	0.127	0,091	0,045	0,006
8.0	0,065	0.053	0.040	0,019	0,004		0.114	0,091	0.072	0,050	0,024	0,003

TABLE II. The Function $\{N_p(E_0, t, E)\}^{\Gamma}$

t \ ε	0.6; 0	0.6; 0.06	0.6; 0,1	0.6; 0.4	1.4; 0	1.4; 0,06	1.4; 02	1.4; 0.4	1.4; 0.8	3; 0	3; 0.06	3; 0.2	3; 0.4	3; 0.8	3; 2	5; 0	5; 0.03	5; 0.06
0.1	0.814	0.806	0,803	0.778	0.871	0.860	0.844	0.829	0.805	0.954	0.934	0.912	0.904	0.876	0.826	1.028	1.015	1.006
0.3	0.517	0,498	0.488	0.430	0.655	0.627	0.590	0.553	0.495	0,861	0.810	0,757	0.709	0,666	0,544	1,046	1.011	0,990
0,5	0.300	0,276	0.263	0.186	0.487	0,449	0.399	0.349	0.272	0.767	0.698	0.625	0.559	0,500	0.337	1,025	0.975	0,946
0.7	0.147	0.120	0.105	0.019	0,356	0,313	0.257	0.201	0.115	0.677	0,587	0.513	0.432	0.370	0.187	0,976	0.918	0,883
0,9	0.041	0.013			0.256	0,211	0,153	0,096	0.008	0,591	0,507	0.419	0.340	0.268	0,081	0.911	0.848	0.809
1,2					0.148	0.105	0,050			0.473	0.392	0.304	0.229	0.156		0,796	0.731	0.691
1.6					0,062	0.025				0,341	0.270	0.193	0.129	0.063		0.636	0.574	0.536
2.0					0,016					0,236	0.179	0.117	0.066	0,011		0.486	0.432	0.397
2.3										0.174	0.127	0,077	0.037			0.389	0.340	0.310
2,6										0,124	0.088	0.047	0.017			0.304	0.262	0,235
3.0										0.074	0.049	0.021	0.002			0,212	0.178	0.156
3,5										0.033	0.018	0,002				0.127	0.102	0.086
4,0										0,008	0.001					0.070	0.052	0,040

t \ ε	5; 0.1	5; 0,2	5; 04	5; 08	5; 2	10; 0	10; 0.06	10; 0,2	10; 0.4	10; 0.8	10; 2	15; 0.03	15; 0.06	15; 0.2	15; 0.4	15; 08	15; 2
0.1	0,994	0.973	0.949	0.929	0,884	1,156	1.118	1.069	1.041	1.013	0.955	1,193	1.182	1,140	1,102	1,078	1.006
0,3	0.960	0.907	0.847	0,797	0.687	1,376	1,276	1,152	1,081	1,007	0,861	1,480	1.449	1.340	1.240	1.171	0,988
0.5	0.905	0,832	0.749	0,677	0,527	1,493	1,352	1.180	1.077	0.970	0.765	1.658	1,612	1,452	1,307	1,201	0.942
0.7	0.836	0.752	0.656	0,571	0,399	1,535	1.368	1.167	1.042	0.912	0.671	1,753	1,694	1,498	1.322	1,186	0.878
0.9	0.760	0,671	0.570	0,476	0,297	1,522	1.341	1.126	0,987	0,842	0,582	1.785	1,717	1,495	1.300	1,139	0,804
1.2	0,643	0.555	0.453	0.355	0.181	1.436	1.249	1.030	0.882	0,726	0.460	1.749	1,672	1.429	1,220	1.032	0,684
1.6	0.493	0.416	0.324	0.231	0.081	1,254	1,075	0.874	0,727	0,570	0.322	1.607	1,513	1,274	1.066	0.861	0,527
2,0	0.363	0.300	0.224	0,140	0.021	1,045	0.886	0.712	0.576	0,429	0,214	1.412	1,329	1.090	0.895	0.686	0.388
2,3	0.281	0,230	0.165	0,091		0,893	0.750	0.600	0.474	0.338	0.150	1.257	1,176	0.950	0.770	0.566	0.300
2.6	0.212	0.171	0.118	0,053		0,751	0,626	0.497	0,384	0.260	0.100	1.103	1,026	0,818	0.654	0,459	0,226
3.0	0.140	0,111	0,071	0,018		0.585	0.482	0.380	0.283	0.177	0,051	0.912	0,842	0.658	0,517	0,339	0.147
3,5	0,076	0,058	0,032			0,437	0.338	0,264	0,187	0.101	0,011	0.704	0.645	0,491	0.376	0,223	0,077
4.0	0,035	0,025				0,293	0.231	0.178	0.119	0.050		0,535	0.485	0.360	0.269	0,141	0.033
5.0						0.138	0.103	0.075	0.041			0,297	0.264	0.185	0,130	0.048	
6.0						0.064	0.043	0.029	0.011			0,159	0,140	0.112	0.061	0.012	
7,0						0,032	0.019	0,010				0,084	0.073	0.046	0.028		

have been obtained for primary-particle energies E_0 from 0.6 to 15 and values of E from 0.03 to 2.*

The distribution functions for electrons with energies higher than E in a shower, produced by a primary electron $\{N_p(E_0, t, E)\}^{P}$ or photon $\{N_p(E_0, t, E)\}^{\Gamma}$, as in Ref. 2, has been approximated by a sum of Laguerre polynomials L_n^0 and $L_n^1(x)$ respectively.

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*The energy of shower particles in cascade theory is measured in units of $q\beta$ where β is the critical energy for the given material and q is a constant equal to 2.29.

The results of the calculations of these functions are shown in Tables I and II.* An investigation of the computed functions (cf. for example, Refs. 2 and 3, in which this type of analysis was first applied) shows that in the present case the error in the electron energy spectrum is approximately 10 percent. We may note that the method of moments is so far the only method which makes it possible to determine with reasonable accuracy the average behavior of an electron-photon shower in the energy region being considered. In calculating the moments, the photon absorption coefficient σ has been assumed constant and equal to its asymptotic value $\sigma = \sigma_0$. However, in computing the distribution functions the photon absorption coefficient has been assumed equal to $\sigma_{\text{min}} = 0.65$. As follows from the results of Ref. 1, this method makes it possible to take account of the approximate dependence of $\sigma(E)$ on energy, leading to better agreement of theory and experiment. The value 0.65 corresponds to the value of σ_{min} in air; the computed curves, however, apply to the development of showers in materials with atomic number Z approximately up to 30 with good accuracy.[4] The numerical results which have been presented may be useful in the analysis of certain cosmic-ray experiments.

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CONTRIBUTION TO THE THEORY OF SCATTERING OF LIGHT NEAR POINTS OF SECOND-ORDER PHASE TRANSITIONS

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The scattering of light by ferroelectric and ferromagnetic crystals in the vicinity of second-order transition points is considered. Calculations are performed both for single component crystals and for solid solutions. By way of illustration light scattering by $BaTiO_3$ type crystals as well as by Rochelle salt or KH_2PO_4 type crystals is examined. The effect of an external field on the scattering is investigated.

I.

Ginzburg[1] has shown that at temperatures close to the temperature of a second-order transition there occurs additional scattering of light which is particularly intense near the critical point, that is, near the point where the curve for second-order transitions merges into the curve for transitions of the first order. This effect has been experimentally detected by Iakovlev, Mikheeva, and Velichkina[2] who investigated the dispersion of light in quartz near the point of the $\alpha \rightleftharpoons \beta$-transformation. Herein we shall carry out a further theoretical investigation of the problem, primarily where ferroelectric materials are con-

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* In the first line of each column of the tables the first number denotes the energy of the initial electron or photon and the second the energy of the secondary electron.