CORRELATION THEORY OF THERMAL FLUCTUATIONS IN AN ISOTROPIC MEDIUM

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The so-called "fluctuation-dissipation" theorem of Callen and his coworkers is used to develop a spectral theory of thermal fluctuations in an isotropic visco-elastic continuous medium. The mechanical and thermal parameters of the medium in this case can possess any frequency dispersion consistent with the dissipation condition. Correlation functions of $\omega\mathbf{k}$-amplitudes (amplitudes in Fourier time-space expansions), have been found for stress, deformation, velocity, temperature, and entropy fluctuations. With the help of these functions, the spectral intensities (giving the spatial correlation at a frequency ω) and also the spatial correlation functions of quantities which are not decomposed spectrally have been calculated. The results are of interest in the spectral theory of Rayleigh light scattering.

1. INTRODUCTION

THE problem of the spectral description of thermal fluctuations of quantities which characterize dissipative systems, has undergone an important development in recent years. The introduction of certain fluctuating "external" forces, connected with the quantities under consideration in the same energy sense as in the connection between generalized coordinates and forces, has proved to be an extraordinarily effective means for the solution of this problem. The action of a "thermostat" on this system is replaced by the action of these random "external" forces, whose statistical properties are such that they produce just those fluctuations which are actually observed in the real system. In this case, inasmuch as the basic problem is the reproduction of the spectrum of the fluctuations, the necessary statistical characteristics of the "external" forces are reduced to their correlation functions.

The first such problem was solved in 1927 by Nyquist[1] and was applied to the current fluctuations in an electric circuit with lumped constants. Nyquist introduced a random external emf $\mathcal{E}(t)$ which produces thermal current fluctuations $I(t)$ in the circuit — a procedure which also appears in the researches of Langevin[2] and De Haas-Lorentz[3] on the theory of Brownian motion. In contrast to those authors, however, Nyquist transformed to the spectral representation of these random functions and, on the basis of thermodynamics and the equipartition theorem, obtained the spectral intensity of the emf, $\overline{\mathcal{E}_\omega^2}$.

Much later (1951-1952), the Nyquist theorem was greatly generalized by Callen and his coworkers.[4] They extended the whole approach to the case of an arbitrary dissipative system, the state of which is described by some number n of discrete parameters $\xi_j(t)$. Corresponding to this, they introduced n generalized "external" forces $f_j(t)$ and established the form of the correlation matrix $\overline{f_{j\omega} f_{k\omega}^*}$. The derivation of this so-called "fluctuation-dissipation" theorem was given in the papers referred to, both on the basis of classical statistical thermodynamics, and on the basis of quantum theory. By these means it was demonstrated that the correlation functions are proportional to the mean energy of the oscillator $\epsilon(\omega, T)$. Second, what determines the correlation matrix of the "external" forces (this is the impedance matrix of the system under consideration) is the natural extension of the concept of impedance to an arbitrary dissipative system.*

* It should be noted that in 1941 Leontovich[5] had come very close to the formulation of this general theorem, but he did not carry his derivation to the explicit expression of the correlation functions in terms of the impedance matrix. In the text of L. D. Landau and E. M. Lifshitz on macroscopic electrodynamics (which will appear shortly), the derivation of the theorem of Callen and his coworkers is given a more complete systematic-spectral form.

Sometimes we are not interested in lumped systems, described by the ordinary differential equations for the set of functions $\xi_j(t)$, but in distributed (in the general case, three dimensional) systems, the fluctuations in which are described by a set of random fields $\xi_j(t, \mathbf{r})$ which satisfy partial differential equations. For example, we may be dealing with the fluctuations of an electromagnetic field (here, we can also include thermal radiation), or with fluctuations of mechanical and thermal parameters which characterize the state of a continuous medium.

The problem of the spectral description of thermal fluctuations, as applicable to the electromagnetic field, was solved in 1952 — first, for a quasi-stationary region (by Leontovich and the author[6]), and then for the general case of a system of Maxwell's equations.[7] The Nyquist theorem served as the starting point of these researches, so that the introduction of "external" electromagnetic fields and the establishment of the form of the correlation function for its spectral amplitudes was carried out on the basis of a series of physical considerations, and not formulated by any regular method. Nonetheless, these basic elements were found to be valid. They allowed one to erect a consistent theory of thermal electric fluctuations[8] which includes as limiting cases the classical theory of thermal radiation (the approximation of geometric optics) and the theory of Nyquist (quasi-stationary region). Subsequently, this general theory was completed and extended in several directions by Levin[9] and Bunkin,[10] and was also applied to a series of more specialized problems,[11] among which was the construction of a macroscopic theory of molecular forces of cohesion (E. M. Lifshitz[12]).

Landau and Lifshitz (in the textbook of macroscopic electrodynamics cited above) have demonstrated that the results of Callen and his collaborators could be applied in a regular fashion to distributed systems. They made use of the division of a continuous system into small volumes and, correspondingly, of the substitution of difference spatial operators for differential ones. Applying this method to the Maxwell equations, they derived those correlation functions for the "external" electromagnetic fields, which were in fact guessed in Refs. 8 and 10.

Starting out from the work of Landau and Lifshitz just cited, I modified the application of the "fluctuation-dissipation" theorem for distributed systems (making use of the expansion of random fields in some complete set of functions) and obtained formulas[13] which are used below for the construction of the spectral theory of thermal fluctuations in a viscoelastic continuous medium.

Application of the "fluctuation-dissipation" theorem to the linearized equations of the theory of elasticity permits us to give a complete spectral description of the thermal fluctuations in the continuous medium for very general assumptions on the frequency dispersion parameters of this medium.* An arbitrary dependence is possible, compatible with the condition that the medium be a dissipative system. The theory developed below, which does not specify the mechanism of the dispersion, is purely phenomenological. Among the problems for which the results flowing therefrom are of interest and of direct application, we can name first of all the spectral theory of Rayleigh light scattering, an account of which I hope to give in a separate paper.

1. INITIAL EQUATIONS

We limit ourselves to the case of an isotropic medium, the mechanical properties of which can be characterized by two elastic moduli, while the thermal properties are characterized by the scalar coefficients of thermal conductivity and thermal expansion (in addition to the heat capacity).

All these parameters of the medium being functions of the frequency ω (or, more precisely, of $z = i\omega$) are, generally speaking, complex.

Thus, for example, the elastic moduli, for which we may take the bulk modulus $\overline{K}$ and the shear modulus $\overline{\mu}$, have the form

$$\overline{K} = K(\omega^2) + i\omega\zeta(\omega^2), \quad \overline{\mu} = \mu(\omega^2) + i\omega\eta(\omega^2), \tag{1.1}$$

where K and μ are the ordinary bulk and shear moduli, and ζ and η are the volume and shear viscosities. The medium can be either a solid (amorphous) body, for which $\mu \neq 0$ at $\omega = 0$, or a liquid [$\mu(0) = 0$]. Since, in the presence of dispersion, the shear modulus cannot be set identically equal to zero, we must even in the case of a liquid start out, not from the hydrodynamic equations, but from the general

*Much earlier, E. M. Lifshitz drew my attention to the analogous theory developed by him and Landau for the case of a viscous liquid in the absence of dispersion (by the method set forth in Ref. 14, Secs. 117-120; see Ref. 20).

equations of the theory of elasticity. The presence in liquids at high frequencies of fluctuating deformations of this same type as in solids brings about an added optical anisotropy and therefore is important for the so-called "wing" in the theory of Rayleigh light scattering.[5]

Deviations from thermodynamic equation of state of the medium under consideration, along with the random fields of such quantities, as well as the displacement **s** and the velocity **v** of the particles of the medium, the deformation tensor $u_{\alpha\beta}$, the stress tensor $\sigma_{\alpha\beta}$, the temperature T and the specific (per unit mass) entropy S can all be described (under known limitations, see Ref. 14. Secs. 117 and 118) by certain parameters ξ_j, the number of which it is not necessary to limit and which characterize all possible internal processes.

For thermal fluctuations in a medium which is thermostated, the random fields of all these quantities (understood as fluctuating deviations from the corresponding equilibrium values

$$s_0 = 0, \quad v_0 = 0, \quad u_{\alpha\beta 0} = 0, \quad \sigma_{\alpha\beta} = 0, \quad T_0, \quad S_0, \; \xi_{j0} = 0,$$

are statistically stationary in time and homogeneous and isotropic in space. These small deviations satisfy a system of linear differential equations which consists of the linearized equations of the dynamical theory of elasticity, the linearized equation of heat transfer, and the kinetic equations for the parameters ξ_j. We assume that these latter equations are purely temporal; consequently, for the transition to the spectral description ($d/dt \to i\omega$), they are converted into the linear algebraic equations with complex coefficients. With their help, eliminating all the parameters ξ_j, we retain for the description of the behavior of the medium only the mechanical and thermal quantities mentioned above. As a result, we get for the spectral amplitudes of these quantities, the usual equations of elasticity theory and heat transport theory, but with the coefficients depending on $z = i\omega$. Thus the dispersion of these coefficients expresses in terms of the phenomenological theory all the internal kinetics of the medium under consideration.*

For the fields $u_{\alpha\beta}$, $\sigma_{\alpha\beta}$, $\vartheta = T/T_0$, and S, we have, consequently, the linearized equations of motion:†

$$i\omega\rho_0 v_\alpha = \partial\sigma_{\alpha\beta}/\partial x_\beta, \tag{1.2}$$

$$v_\alpha = i\omega s_\alpha, \tag{1.3}$$

$$\sigma_{\alpha\beta} = 2\bar{\mu}\, u'_{\alpha\beta} + \bar{K}(u - C\vartheta)\,\delta_{\alpha\beta}, \tag{1.4}$$

$$u_{\alpha\beta} = {}^1/_2\,(\partial s_\alpha/\partial x_\beta + \partial s_\beta/\partial x_\alpha), \tag{1.5}$$

$$u \equiv u_{\alpha\alpha} = \operatorname{div} s, \quad u'_{\alpha\beta} = u_{\alpha\beta} - \frac{u}{3}\delta_{\alpha\beta}, \tag{1.6}$$

and the linearized heat-transfer equation

$$i\omega\rho_0 S = \varkappa\Delta\vartheta. \tag{1.7}$$

In addition to the elastic moduli $\bar{K}$ and $\bar{\mu}$, these equations also contain the (spectral) coefficient of thermal conductivity κ and the coefficient C which determines the dependence of the deformation on the temperature. The deformation tensor $u_{\alpha\beta}$ divides into: (1) the trace $u = u_{\alpha\alpha}$, which is equal, with opposite sign, to the relative compression, and (2) the deformation of pure shear $u'_{\alpha\beta}$, the trace of which vanishes. Such a division is especially convenient in the theory of light scattering.

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*The considerations set forth are only the continuation of those which were introduced by Leontovich[15] and developed by the researches of him and L. I. Mandel'shtam,[16,17] which touched on the relaxation mechanism of the dispersion and absorption of sound waves in liquids. In Ref. 17, in addition to the linearity of the equation for ξ_j and the condition that $\xi_j \neq$ const. only for deviations from thermodynamic equilibrium, an inertia-free (relaxational) character is assumed for the change of each of the ξ_j. This latter condition, according to which the differential equations for the separated variables ξ_j have a first order, is not generally obligatory. The most general requirement of the dissipativity of the system is sufficient. On the other hand, the assumption that the equation for ξ_j does not contain spatial derivatives of the mechanical and thermal quantities is essential, since only under these conditions can the equations for the spectral amplitudes $u_{\alpha\beta}$, $\sigma_{\alpha\beta}$, T, S be represented in the usual form under the allowance of only frequency dispersion of the parameters of the system.

†In order not to encumber the formulas we shall not use the index ω to indicate that all the equations are written for the spectral amplitudes at the frequency ω (v_ω and so forth).

Since the number of variables in the system just described is greater by one than the number of equations, it is necessary to introduce an additional relation which, naturally, ought to be linear. We write it in the form

$$\rho_0 T_0 S = \frac{C}{3}\sigma + D\vartheta \quad (\sigma \equiv \sigma_{\alpha\alpha}), \tag{1.8}$$

taking here as the coefficient for σ the same C as enters into Eq. (1.4). This is dictated by the condition of symmetry of the spectral kinetic coefficients (see below, Sec. 3).

If dispersion is absent, i.e., if $\overline{K}$, $\overline{\mu}$, C and D do not depend on frequency (and consequently are real), then all the given equations are valid for spectrally non-decomposed quantities and the laws of thermodynamics become applicable, i.e., the state of the system is uniquely defined by six "mechanical" and one "thermal" variables, for example, $\sigma_{\alpha\beta}$ and ϑ. From the group of equations (1.4), and from (1.8), it follows that in this case the coefficients have the following thermodynamic meaning:

$$\frac{1}{K} = 3\left(\frac{\partial u}{\partial \sigma}\right)_\vartheta = \frac{1}{\rho_0}\left(\frac{\partial \rho}{\partial p}\right)_T = \beta_T, \qquad C = \left(\frac{\partial u}{\partial \vartheta}\right)_\sigma = -\frac{T_0}{\rho_0}\left(\frac{\partial \rho}{\partial T}\right)_p = \alpha T_0, \quad D = \rho_0 T_0\left(\frac{\partial S}{\partial \vartheta}\right)_\sigma = \rho_0 T_0^2\left(\frac{\partial S}{\partial T}\right)_p = \rho_0 T_0 c_p, \tag{1.9}$$

where β_T is the isothermal compressibility, α is the coefficient of thermal expansion, and c_p is the heat capacity at constant pressure.

Making use of the well known thermodynamic relations

$$\alpha^2 T_0 = \rho_0 c_p (\beta_T - \beta_S), \quad c_p / c_v = \beta_T / \beta_S \equiv \gamma,$$

where β_S is the adiabatic compressibility and c_v is the heat capacity at constant volume, it is easy to obtain the value of one combination of parameters which enters into what follows, namely:

$$D_1 = D - C^2 K = \rho_0 T_0 c_v. \tag{1.10}$$

2. GENERAL FORMULAS FOR THE CORRELATION AND SPECTRAL INTENSITY FUNCTIONS

Let $\xi(t, \mathbf{r})$ be the random field of any of the quantities considered (scalar quantities or components of a vector or tensor) and let $\xi_\omega(\mathbf{r})$ be the corresponding spectral amplitude:

$$\xi(t, \mathbf{r}) = \int_{-\infty}^{+\infty} \xi_\omega(\mathbf{r})\, e^{i\omega t}\, d\omega. \tag{2.1}$$

In turn, we can represent (as is frequently convenient) $\xi_\omega(\mathbf{r})$ in the form of a three-dimensional Fourier integral

$$\xi_\omega(\mathbf{r}) = \int_{-\infty}^{+\infty} \xi_{\omega\mathbf{k}}\, e^{i\mathbf{k}\mathbf{r}}\, d\mathbf{k}, \tag{2.2}$$

so that

$$\xi(t, \mathbf{r}) = \iint_{-\infty}^{+\infty} \xi_{\omega\mathbf{k}}\, e^{i(\omega t + \mathbf{k}\mathbf{r})}\, d\omega\, d\mathbf{k}. \tag{2.3}$$

As a consequence of the statistical stationarity of these fields in time and their spatial homogeneity, the correlation functions of the $\omega\mathbf{k}$-amplitudes of any two quantities $\xi(t, \mathbf{r})$ and $\eta(t, \mathbf{r})$ (in particular, $\eta = \xi$) have the form

$$\overline{\xi_{\omega\mathbf{k}}\eta^*_{\omega'\mathbf{k}'}} = \overline{\xi_{\omega\mathbf{k}}\eta^*_{\omega\mathbf{k}}}\, \delta(\omega - \omega')\, \delta(\mathbf{k} - \mathbf{k}'). \tag{2.4}$$

In view of the isotropy of the fluctuations, the spectral $\omega\mathbf{k}$-intensity, which we denote by $\overline{\xi_{\omega\mathbf{k}}\eta^*_{\omega\mathbf{k}}}$, in the case of scalar quantities ξ and η, depends (in addition to ω) only on the absolute value of the vector $\mathbf{k}$.

It then follows from (2.4) that the correlation functions of the ω-amplitudes that are not spatially decomposed is

$$\overline{\xi_\omega(\mathbf{r} + \boldsymbol{\rho})\, \eta^*_{\omega'}(\mathbf{r})} = \overline{\xi_\omega(\mathbf{r} + \boldsymbol{\rho})\, \eta^*_\omega(\mathbf{r})}\, \delta(\omega - \omega'), \tag{2.5}$$

where the ω-intensity which multiplies $\delta(\omega - \omega')$ is equal to

$$\overline{\xi_\omega(\mathbf{r}+\rho)\,\eta^*_\omega(\mathbf{r})} = \int\limits_{-\infty}^{+\infty} \overline{\xi_{\omega\mathbf{k}}\,\eta^*_{\omega\mathbf{k}}}\, e^{i\mathbf{k}\rho}\, d\mathbf{k}. \tag{2.6}$$

For <u>scalar</u> quantities, this intensity depends (in addition to ω) only on the absolute magnitude of the vector ρ, i.e., on the <u>distance</u> between the two points under consideration.

Finally, according to the same general rules, we define the space-time correlation function of the fields $\xi(t, \mathbf{r})$ and $\eta(t, \mathbf{r})$:

$$\Psi_{\xi\eta}(\tau, \rho) \equiv \overline{\xi(t+\tau, \mathbf{r}+\rho)\,\eta(t, \mathbf{r})} = \int\limits_{-\infty}^{+\infty} \overline{\xi_\omega(\mathbf{r}+\rho)\,\eta^*_\omega(\mathbf{r})}\, e^{i\omega\tau}\, d\omega = \iint\limits_{-\infty}^{+\infty} \overline{\xi_{\omega\mathbf{k}}\,\eta^*_{\omega\mathbf{k}}}\, e^{i(\omega\tau+\mathbf{k}\rho)}\, d\omega\, d\mathbf{k}. \tag{2.7}$$

According to (2.6) and (2.7), one needs to know only the $\omega\mathbf{k}$-intensities for the calculation of any correlation functions. We note that for the theory of Rayleigh scattering, the ω-intensities (2.6) are of immediate interest. These give the spatial correlation at the frequency ω. Let us now set down the formulas giving expressions for the spectral intensities of the "outside" fields[13] which are necessary for what follows.

Let the fluctuations be described by the set of random fields $\xi^{(m)}(t, \mathbf{r})$, $m = 1, 2, \ldots$, to which correspond the fluctuating "external" forces with volume densities $f^{(m)}(t, \mathbf{r})$ such that the changes in free energy in the volume V associated with their operation take place with velocity

$$\frac{d\bar{E}}{dt} = \sum_m \int\limits_V \overline{f^{(m)} \frac{\partial \xi^{(m)}}{\partial t}}\, dV = -\sum_m \int\limits_V \overline{\xi^{(m)} \frac{\partial f^{(m)}}{\partial t}}\, \Delta V. \tag{2.8}$$

Further, let the spectral amplitudes of the fields $\xi^{(m)}$ and $f^{(m)}$ be connected by the equations

$$f^{(m)}_\omega(\mathbf{r}) = \sum_n A_{mn}(\nabla)\, \xi^{(n)}_\omega(\mathbf{r}), \tag{2.9}$$

where the $A_{mn}(\nabla)$ are linear spatial differential operators. Then we have for the matrices of the spatial correlation functions (ω-intensities) of the forces $f^{(m)}$

$$\overline{f^{(m)}_\omega(\mathbf{r}+\rho)\, f^{(n)*}_\omega(\mathbf{r})} = iH(\omega)\,\{A^*_{nm}(-\nabla_\rho) - A_{mn}(\nabla_\rho)\}\,\delta(\rho). \tag{2.10}$$

Here

$$H(\omega) = \frac{\hbar}{4\pi} \coth \frac{\hbar\omega}{2\Theta}, \tag{2.11}$$

$\hbar$ is Planck's constant divided by 2π, $\Theta = kT_0$ is the temperature of the system in energy units. In the classical region ($\hbar\omega \ll \Theta$),

$$H(\omega) = \Theta / 2\pi\omega. \tag{2.12}$$

The following expression for the $\omega\mathbf{k}$-intensities of the "external" forces follows from Eq. (2.1):

$$\overline{f^{(m)}_{\omega\mathbf{k}}\, f^{(n)*}_{\omega\mathbf{k}}} = \frac{iH(\omega)}{(2\pi)^3}\,\{A^*_{nm}(-i\mathbf{k}) - A_{mn}(i\mathbf{k})\}. \tag{2.13}$$

We now return to the problem of interest — the thermal fluctuations in a visco-elastic isotropic medium.

3. SPECTRAL INTENSITIES OF THE EXTERNAL FORCES

For the description of the fluctuations in a visco-elastic medium, it is appropriate to take as "passive" variables $\xi^{(m)}(t, \mathbf{r})$, the velocity $\mathbf{v}$ of flow of the medium, the stress $\sigma_{\alpha\beta}$ and the reduced temperature $\vartheta = T/T_0$. The generalized forces $f^{(m)}(t, \mathbf{r})$ which should be introduced in correspondence with (2.8), will now be the external momentum with volume density $\mathbf{P}$, the external deformation

$U_{\alpha\beta}$* and the external sources of heat with volume densities Q. Thus the power developed by these forces is

$$\frac{d\overline{E}}{dt} = -\int_V \{\overline{\mathbf{v}\dot{\mathbf{P}}} + \overline{\sigma_{\alpha\beta}\dot{U}_{\alpha\beta}} + \overline{\vartheta\dot{Q}}\}\, dV.$$

The spectral equations (1.2), (1.3), (1.5), and (1.7), if we eliminate the displacement **s** from them, change to the following upon introduction of the external forces:

$$i\omega\rho_0 v_\alpha = \frac{\partial\sigma_{\alpha\beta}}{\partial x_\beta} - i\omega P_\alpha, \qquad i\omega u_{\alpha\beta} = \frac{1}{2}\left(\frac{\partial v_\alpha}{\partial x_\beta} + \frac{\partial v_\beta}{\partial x_\alpha}\right) - i\omega U_{\alpha\beta}, \qquad i\omega\rho_0 S = \varkappa\Delta\vartheta - i\omega\frac{Q}{T_0}. \tag{3.1}$$

Solving (1.4) for the deformation, we obtain

$$u_{\alpha\beta} = \sigma'_{\alpha\beta}/2\overline{\mu} + (\sigma/9\overline{K})\,\delta_{\alpha\beta} + (C\vartheta/3)\delta_{\alpha\beta}, \tag{3.2}$$

where, in accord with (1.6), we use the notation

$$\sigma \equiv \sigma_{\alpha\alpha}, \quad \sigma'_{\alpha\beta} = \sigma_{\alpha\beta} - \frac{\sigma}{3}\delta_{\alpha\beta}. \tag{3.3}$$

Equations (1.8) and (3.2) permit us to eliminate the deformations $u_{\alpha\beta}$ and the entropy S from (3.1). As a result we obtain ten equations for the variables v_α, $\sigma_{\alpha\beta}$, and ϑ, which we now write in the form (2.9), i.e., solving them for the ω-amplitudes of the external forces. Moreover, making use of the symmetric tensor $\sigma_{\alpha\beta}$, we rewrite the latter in the form $\frac{1}{2}(\sigma_{\alpha\beta} + \sigma_{\beta\alpha})$. Thus,

$$P_\alpha = -\rho_0 v_\alpha + \frac{1}{2i\omega}\nabla_\beta(\sigma_{\alpha\beta} + \sigma_{\beta\alpha}), \qquad U_{\alpha\beta} = \frac{\nabla_\alpha v_\beta + \nabla_\beta v_\alpha}{2i\omega} - \frac{\sigma_{\alpha\beta} + \sigma_{\beta\alpha}}{4\overline{\mu}} + \left(\frac{1}{2\overline{\mu}} - \frac{1}{3\overline{K}}\right)\frac{\sigma}{3}\delta_{\alpha\beta} - \frac{C\vartheta}{3}\delta_{\alpha\beta},$$

$$Q = -\frac{C\sigma}{3} - D\vartheta + \frac{\varkappa T_0}{i\omega}\nabla^2\vartheta. \tag{3.4}$$

In such a description, a comparison of (3.4) with (2.9) easily allows us to establish the form of the matrices of the operators $A_{mn}(\nabla)$, which ought to be symmetric in the given case (absence of fields of the Coriolis type), and which actually satisfies this condition, as already specified in (1.8). Not writing out the elements of this matrix (although 30 of the 55 elements are equal to zero), we at once obtain the ωk-intensities of the external forces, made up from Eq. (2.13). We limit ourselves in this case to the classical frequencies, i.e., we use the expression (2.12) for $H(\omega)$. The intensities are then

$$\overline{P_\alpha P_\beta^*} = 0, \quad \overline{P_\alpha U_{\beta\gamma}^*} = 0, \quad \overline{P_\alpha Q^*} = 0, \tag{3.5}$$

$$\overline{U_{\alpha\beta}U_{\mu\nu}^*} = -\frac{\Theta}{(2\pi)^4 i\omega}\left\{\frac{1}{4}\left(\frac{1}{\overline{\mu}} - \frac{1}{\overline{\mu}^*}\right)\left(\delta_{\alpha\mu}\delta_{\beta\nu} + \delta_{\beta\mu}\delta_{\alpha\nu} - \frac{2}{3}\delta_{\alpha\beta}\delta_{\mu\nu}\right) + \frac{1}{9}\left(\frac{1}{\overline{K}} - \frac{1}{\overline{K}^*}\right)\delta_{\alpha\beta}\delta_{\mu\nu}\right\}, \tag{3.6}$$

$$\overline{U_{\alpha\beta}Q^*} = -\frac{\Theta}{(2\pi)^4 i\omega}(C - C^*)\frac{\delta_{\alpha\beta}}{3}, \tag{3.7}$$

$$\overline{QQ^*} = -\frac{\Theta}{(2\pi)^4 i\omega}(d - d^*), \tag{3.8}$$

where the notation

$$d = D + \varkappa T_0 k^2/i\omega. \tag{3.9}$$

has been introduced.

It follows from (3.5) that the external momentum $\mathbf{P} \equiv 0$, i.e., it would have been possible to do without it from the very beginning. One could have foreseen this, since the center of mass of the system should remain at rest in the given arrangement of the problem, and such can be guaranteed only in the absence of the field **P**.

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*At first glance, it would appear more natural to introduce the external stresses $\Sigma_{\alpha\beta}$ rather than the external deformations $U_{\alpha\beta}$. In essence, both methods are equivalent. As substitution of the second equation of (3.1) in (1.4) shows, the introduction of $U_{\alpha\beta}$ is equivalent to the action of the external stresses $\Sigma_{\alpha\beta} = 2\overline{\mu}U'_{\alpha\beta} + \overline{K}U\delta_{\alpha\beta}$. However, our choice of "passive" variables and generalized forces turns out to be much more convenient from the computational viewpoint.

There is no necessity of our using Eqs. (2.6) and (2.7) to calculate the ω-intensities and space-time correlation functions of the external forces $U_{\alpha\beta}$ and Q. These functions are necessary only to obtain the spectral statistical characteristics of "passive" variables — the velocity **v**, the stress $\sigma_{\alpha\beta}$, and the temperature ϑ, and in addition the others — the deformation $u_{\alpha\beta}$ and the entropy S, which are expressed in terms of $\sigma_{\alpha\beta}$ and ϑ by Eqs. (3.2) and (1.8). We shall now proceed to do so.

4. SPECTRAL ωk-INTENSITIES OF THE STRESS AND TEMPERATURE

In order to express the ω**k**-amplitudes of the quantities v_α, $\sigma_{\alpha\beta}$, and ϑ in terms of the ω**k**-amplitudes of the external forces $U_{\alpha\beta}$ and Q, it is necessary to solve (3.4), rewriting it for ω**k**-amplitudes ($\nabla \rightarrow i\mathbf{k}$) and considering that **P** = 0:

$$\rho_0\omega v_\alpha - k_\beta\sigma'_{\alpha\beta} - k_\alpha\sigma/3 = 0, \tag{4.1}$$

$$(k_\alpha v_\beta + k_\beta v_\alpha)/2\omega - \sigma'_{\alpha\beta}/2\bar{\mu} - (\sigma/3\bar{K} + C\vartheta)\,\delta_{\alpha\beta}/3 = U_{\alpha\beta}, \tag{4.2}$$

$$C\sigma/3 + d\vartheta = -Q. \tag{4.3}$$

The notation of (3.3) and (3.9) is used here. Using the symbols

$$d_1 = d - C^2\bar{K},\quad \Delta = A_1 d - A_3 C^2 \bar{K},$$

$$A_1 = \rho_0\omega^2 - (\bar{K} + {}^4\!/_3\bar{\mu})\,k^2,\quad A_2 = \rho_0\omega^2 - 2\bar{\mu}k^2,\quad A_3 = \rho_0\omega^2 - {}^4\!/_3\bar{\mu}k^2,\qquad A_4 = \rho_0\omega^2 - \bar{\mu}k^2 \tag{4.4}$$

(the last of these is necessary somewhat later) we obtain the following result:

$$\sigma = -\frac{3\bar{K}}{\Delta}\{2\bar{\mu}dV + A_2 dU - A_3 CQ\}, \tag{4.5}$$

$$\vartheta = \frac{1}{\Delta}\{2\bar{\mu}\bar{K}CV + A_2 C\bar{K}U - A_1 Q\}. \tag{4.6}$$

$$v_\alpha = -\frac{2\omega\bar{\mu}}{A_4}U_{\beta\alpha}k_\beta - \frac{\omega k_\alpha}{\Delta}\left\{\frac{2\bar{\mu}}{3A_4}(3\bar{K}d + \bar{\mu}d_1)V + (\bar{K}d - \frac{2}{3}\bar{\mu}d_1)U - C\bar{K}Q\right\}. \tag{4.7}$$

$$\frac{\sigma'_{\alpha\beta}}{2\bar{\mu}} = -U_{\alpha\beta} - \frac{\bar{\mu}}{A_4}(U_{\gamma\beta}k_\alpha + U_{\gamma\alpha}k_\beta)\,k_\gamma - \frac{k_\alpha k_\beta}{\Delta}\left\{\frac{2\bar{\mu}}{3A_4}(3\bar{K}d + \bar{\mu}d_1)V + \left(\bar{K}d - \frac{2}{3}\bar{\mu}d_1\right)U - C\bar{K}Q\right\}$$
$$+\frac{\delta_{\alpha\beta}}{3\Delta}\{(2\bar{\mu}V + A_2 U)\,d_1 - C\bar{K}Q\}, \tag{4.8}$$

where $U = U_{\alpha\alpha}$, $V = U_{\alpha\beta}k_\alpha k_\beta$. Expanding the expressions for the ω**k**-amplitudes of the quantities v_α, $\sigma_{\alpha\beta}$, and ϑ in terms of the ω**k**-amplitudes of the external forces $U_{\alpha\beta}$ and Q, for which the ω**k**-intensities are known [Eq. (3.6) — (3.8)], we can now compute the ω**k**-intensities of these quantities. We omit the rather cumbersome intermediate equations and immediately write down the final results, pertaining to $\sigma'_{\alpha\beta}$, σ, and ϑ. This suffices, since all the intensities which contain v_α, $u_{\alpha\beta}$, and S can then be obtained with the aid of Eqs. (4.1), (3.2), and (1.8), respectively (see Sec. 5).

Making use of the symbols of (4.4), and further introducing

$$\tilde{\delta}_{\alpha\beta} = k_\alpha k_\beta / k^2 - \delta_{\alpha\beta}/3 \qquad (\tilde{\delta}_{\alpha\alpha} = 0), \tag{4.9}$$

we have

$$\overline{\sigma'_{\alpha\beta}\sigma'^{*}_{\mu\nu}} = -\frac{\Theta}{(2\pi)^4 i\omega}\Big\{(\bar{\mu}^* - \bar{\mu})\Big[\delta_{\alpha\mu}\delta_{\beta\nu} + \delta_{\beta\mu}\delta_{\alpha\nu} - \frac{2}{3}\delta_{\alpha\beta}\delta_{\mu\nu} + 3\tilde{\delta}_{\alpha\beta}\tilde{\delta}_{\mu\nu}\left(\left|\frac{d_1\rho_0\omega^2 - d\bar{K}k^2}{\Delta}\right|^2 - 1\right) - \frac{1}{k^2}\left(\frac{\rho_0^2\,\omega^4}{|A_4|^2} - 1\right)(\delta_{\alpha\mu}k_\beta k_\nu$$
$$+ \delta_{\alpha\nu}k_\beta k_\mu + \delta_{\beta\mu}k_\alpha k_\nu + \delta_{\beta\nu}k_\alpha k_\mu - 4\frac{k_\alpha k_\beta k_\mu k_\nu}{k^2})\Big] + 4\tilde{\delta}_{\alpha\beta}\tilde{\delta}_{\mu\nu}\left|\frac{\bar{\mu}k^2}{\Delta}\right|^2(d_1 d^*\bar{K}^* - d_1^* d\bar{K})\Big\}, \tag{4.10}$$

$$\overline{\sigma'_{\alpha\beta}\sigma^*} = \frac{6\Theta k^2\tilde{\delta}_{\alpha\beta}}{(2\pi)^4 i\omega}\left(\frac{\bar{\mu}\bar{K}d}{\Delta} - \frac{\bar{\mu}^*\bar{K}^*d^*}{\Delta^*}\right), \tag{4.11}$$

$$\overline{\sigma\sigma^*} = \frac{9\Theta}{(2\pi)^4 i\omega}\left(\frac{A_3\bar{K}d}{\Delta} - \frac{A_3^*\bar{K}^*d^*}{\Delta^*}\right), \tag{4.12}$$

$$\overline{\sigma'_{\alpha\beta}\vartheta^*} = -\frac{2\Theta k^2\tilde{\delta}_{\alpha\beta}}{(2\pi)^4 i\omega}\left(\frac{\bar{\mu}\bar{K}C}{\Delta} - \frac{\bar{\mu}^*\bar{K}^*C^*}{\Delta^*}\right), \tag{4.13}$$

$$\overline{\sigma\vartheta^*} = -\frac{3\Theta}{(2\pi)^4 i\omega}\left\{\frac{A_3C\bar{K}}{\Delta} - \frac{A_3^*C^*\bar{K}^*}{\Delta^*} - (C - C^*)\bar{K}\left(\frac{A_1}{\Delta} - \frac{A_1^*}{\Delta^*}\right)\right\}, \tag{4.14}$$

$$\overline{\vartheta\vartheta^*} = \frac{\Theta}{(2\pi)^4 i\omega}\left(\frac{A_1}{\Delta} - \frac{A_1^*}{\Delta^*}\right). \tag{4.15}$$

5. SPECTRAL ωk-INTENSITIES OF THE VELOCITY AND THE DEFORMATION

As has already been noted, the ωk-intensities of all the remaining quantities can now be found in the form of a linear combination of the intensities (4.10) – (4.15). Thus Eq. (4.1):

$$\rho_0\omega v_\alpha = k_\beta\sigma'_{\alpha\beta} + k_\alpha\frac{\sigma}{3}$$

permits us to obtain any intensities containing the velocity. For example, it follows from this equation that

$$\overline{v_\alpha v_\beta^*} = \frac{1}{\rho_0^2\omega^2}\left\{k_\mu k_\nu\overline{\sigma'_{\alpha\mu}\sigma'^*_{\beta\nu}} + \frac{k_\alpha k_\nu}{3}\overline{\sigma\sigma'^*_{\beta\nu}} + \frac{k_\beta k_\mu}{3}\overline{\sigma'_{\alpha\mu}\sigma^*} + \frac{k_\alpha k_\beta}{9}\overline{\sigma\sigma^*}\right\}$$

– an expression in which it remains to substitute Eqs. (4.10) – (4.12). In this manner, we find

$$\overline{v_\alpha v_\beta^*} = \frac{\Theta}{(2\pi)^4 i\omega\rho_0}\left\{\left(\frac{\bar{\mu}}{A_4} - \frac{\bar{\mu}^*}{A_4^*}\right)(\delta_{\alpha\beta}k^2 - k_\alpha k_\beta) + k_\alpha k_\beta\left[\frac{1}{\Delta}\left(\bar{K}d + \frac{4}{3}\bar{\mu}d_1\right) - \frac{1}{\Delta^*}\left(\bar{K}^*d^* + \frac{4}{3}\bar{\mu}^*d_1^*\right)\right]\right\}, \tag{5.1}$$

$$\overline{v_\alpha\sigma'^*_{\mu\nu}} = \frac{\Theta}{(2\pi)^4 i}\left\{\left(\frac{\bar{\mu}}{A_4} - \frac{\bar{\mu}^*}{A_4^*}\right)\left(\delta_{\alpha\mu}k_\nu + \delta_{\alpha\nu}k_\mu - 2\frac{k_\alpha k_\mu k_\nu}{k^2}\right) + 2k_\alpha\tilde{\delta}_{\mu\nu}\left(\frac{\bar{\mu}d_1}{\Delta} - \frac{\bar{\mu}^*d_1^*}{\Delta^*}\right)\right\}, \tag{5.2}$$

$$\overline{v_\alpha\sigma^*} = \frac{3\Theta k_\alpha}{(2\pi)^4 i}\left(\frac{\bar{K}d}{\Delta} - \frac{\bar{K}^*d^*}{\Delta^*}\right) \tag{5.3}$$

$$\overline{v_\alpha\vartheta^*} = -\frac{\Theta k_\alpha}{(2\pi)^4 i}\left\{\frac{C\bar{K}}{\Delta} - \frac{C^*\bar{K}^*}{\Delta^*} - \frac{(C - C^*)\bar{K}}{\rho_0\omega^2}\left(\frac{A_1}{\Delta} - \frac{A_1^*}{\Delta^*}\right)\right\}. \tag{5.4}$$

Similarly, with the help of Eq. (3.2), from which it follows that

$$u'_{\alpha\beta} = \sigma'_{\alpha\beta}/2\bar{\mu}, \quad u = \sigma/3\bar{K} + C\vartheta,$$

it is not difficult to compute the ωk-intensities of the deformation. We give the corresponding expressions, since they are necessary for the theory of Rayleigh scattering:

$$\overline{u'_{\alpha\beta}u'^*_{\mu\nu}} = -\frac{\Theta}{(2\pi)^4 i\omega}\left\{\frac{1}{4}\left(\frac{1}{\bar{\mu}} - \frac{1}{\bar{\mu}^*}\right)\left[\delta_{\alpha\mu}\delta_{\beta\nu} + \delta_{\beta\mu}\delta_{\alpha\nu} - \frac{2}{3}\delta_{\alpha\beta}\delta_{\mu\nu} + 3\tilde{\delta}_{\alpha\beta}\tilde{\delta}_{\mu\nu}\left(\left|\frac{d_1\rho_0\omega^2 - d\bar{K}k^2}{\Delta}\right|^2 - 1\right)\right.\right.$$
$$\left.\left. - \frac{1}{k^2}\left(\frac{\rho_0^2\omega^2}{|A_4|^2} - 1\right)\left(\delta_{\alpha\mu}k_\beta k_\nu + \delta_{\alpha\nu}k_\beta k_\mu + \delta_{\beta\mu}k_\alpha k_\nu + \delta_{\beta\nu}k_\alpha k_\mu - 4\frac{k_\alpha k_\beta k_\mu k_\nu}{k^2}\right)\right] + \tilde{\delta}_{\alpha\beta}\tilde{\delta}_{\mu\nu}\frac{k^4}{|\Delta|^2}(d_1d^*\bar{K}^* - d_1^*d\bar{K})\right\}, \tag{5.5}$$

$$\overline{u'_{\alpha\beta}u^*} = \frac{\Theta k^2\tilde{\delta}_{\alpha\beta}}{(2\pi)^4 i\omega}\left\{\frac{\bar{K}(d - CC^*\bar{K}^*)}{\bar{K}^*\Delta} - \frac{\bar{\mu}^*d_1^*}{\bar{\mu}\Delta^*}\right\}, \tag{5.6}$$

$$\overline{uu^*} = \frac{\Theta}{(2\pi)^4 i\omega}\left\{\frac{A_3d_1}{\bar{K}^*\Delta} - \frac{A_3^*d_1^*}{\bar{K}\Delta^*} - CC^*k^2\left(\frac{\bar{K}}{\Delta} - \frac{\bar{K}^*}{\Delta^*}\right)\right\}, \tag{5.7}$$

$$\overline{u'_{\alpha\beta}\vartheta^*} = -\frac{\Theta k^2\tilde{\delta}_{\alpha\beta}}{(2\pi)^4 i\omega}\left(\frac{C\bar{K}}{\Delta} - \frac{\bar{\mu}^*C^*\bar{K}^*}{\bar{\mu}\Delta^*}\right), \tag{5.8}$$

$$\overline{u\vartheta^*} = -\frac{\Theta}{(2\pi)^4 i\omega}\left(\frac{C\bar{K}k^2}{\Delta} - \frac{C^*\bar{K}^*A_3^* - C\bar{K}A_1^*}{\bar{K}\Delta^*}\right). \tag{5.9}$$

By the same method, we can obtain the mutual intensities of deformation and stress. We set down only one of them – for the scalar quantities u and σ (the relative density and pressure):

$$\overline{u\sigma^*} = \frac{\Theta}{(2\pi)^4 i\omega}\left\{\frac{A_3d_1}{\Delta} - \frac{\bar{K}^*A_3^*(d^* - CC^*\bar{K})}{\bar{K}\Delta^*} - C(C - C^*)\bar{K}^*\left(\frac{A_1}{\Delta} - \frac{A_1^*}{\Delta^*}\right)\right\}. \tag{5.10}$$

So far as the ωk-intensities containing the entropy S are concerned, they are easily computed by means of (1.8). However, we shall not carry out the corresponding derivations since they are not of much interest.

6. SPATIAL CORRELATION

Making use of the ωk-intensities, we can compute, by Eq. (2.6), the corresponding ω-intensities which give the spatial correlation of the quantities under consideration at the frequency ω, while from (2.7) we find the correlation functions of the spectrally-nonexpanded quantities.

For the calculation of the integrals (2.6), it is not difficult to carry out integration in **k** space over the angles, after which there are left single integrals over k, which are taken with residues at the poles which determine the roots k_j of the dispersion equation. It is not difficult to see [from (4.5) to (4.8)] that the dispersion equation divides into two parts:

$$\Delta = 0, \quad A_4 = 0,$$

where the first defines the roots k_1 and k_2, corresponding to the compressional waves and heat waves:

$$k_{1,2}^2 = \frac{i\omega\rho_0}{2\varkappa_1(\overline{K}+\overline{M})}(-P \pm R), \qquad P = i\omega\varkappa_1 + D\overline{K} + D_1\overline{M}, \; R = +\sqrt{P^2 - 4i\omega\varkappa_1 D_1(\overline{K}+\overline{M})}, \tag{6.1}$$

while the second defines the root k_3, corresponding to shear waves:

$$k_3^2 = \rho_0\omega^2/\overline{\mu}. \tag{6.2}$$

In (6.1) the following notation was introduced:

$$\varkappa_1 = \varkappa\rho_0 T_0, \; D_1 = D - C^2\overline{K}, \; \overline{M} = {}^4/_3\overline{\mu}. \tag{6.3}$$

It should be emphasized that the determination of the roots k_j and, it seems, the computation of the ω-intensities also, does not require a concrete form for the dispersion law, i.e., the correlation at a frequency ω can be found for an arbitrary frequency dependence of the parameters of the medium. The root k_3 plays a role only for velocities v_α and tensor quantities $\sigma'_{\alpha\beta}$, $u'_{\alpha\beta}$; the scalar quantities σ, ϑ, u and S naturally are not connected with shear waves.

The spectral correlation lengths are naturally determined by the path length (reduced by a factor e) of waves of different types. As an example, let us introduce the ω-intensity of the temperature ϑ:

$$\overline{\vartheta_\omega(\mathbf{r}+\boldsymbol{\rho})\,\vartheta^*_\omega(\mathbf{r})} = \frac{\Theta}{16\pi^2\rho}\left\{\frac{1}{\varkappa}\left[\left(1 - \frac{P - 2i\omega\varkappa_1}{R}\right)e^{-ik_1\rho} + \left(1 + \frac{P - 2i\omega\varkappa_1}{R}\right)e^{-ik_2\rho}\right] + \text{compl. conj.}\right\}$$

If the thermal conductivity $\kappa \to 0$, then, in view of the fact that $k_2 \sim 1/\sqrt{\kappa_1}$, the correlation determined by thermal waves remains (for each ρ) suitable small in comparison with the correlations determined by the compressional waves.

For spectrally non-decomposed quantities, if we limit ourselves to the case $\tau = 0$, we have, in accord with (2.7),

$$\Psi_{\xi\eta}(0, \rho) = \overline{\xi(t, \mathbf{r}+\boldsymbol{\rho})\,\eta(t, \mathbf{r})} = \iint_{-8}^{+\infty} \overline{\xi_{\omega\mathbf{k}}\eta^*_{\omega\mathbf{k}}}\, e^{i\mathbf{k}\boldsymbol{\rho}}\, d\omega d\mathbf{k}. \tag{6.4}$$

For the calculation of $\Psi_{\xi\eta}$ it turns out to be much more effective to carry out the integration initially over ω. The fact is that the integral

$$I_{\xi\eta}(\mathbf{k}) = \int_{-\infty}^{+\infty} \overline{\xi_{\omega\mathbf{k}}\eta^*_{\omega\mathbf{k}}}\, d\omega \tag{6.5}$$

is evaluated in a number of cases with the help of certain general theorems from the theory of residues, which permit us either to do without a concrete model of the dispersion laws, or to limit ourselves to a precise statement of these laws only for certain ranges of the parameters. We set forth these theorems, omitting their proofs (although they are very simple), and not pursuing the most general form.

Let the path of integration Γ run along the imaginary axis $z = i\omega$ and close up at infinity to include the left half plane (in which all the roots of the dispersion equation of our dissipative system are found for real k). Further, let f(z) be the Hurwitz polynomial of degree N, and g(z) a polynomial of degree $\leq N$ which does not have common roots with f(z). Finally, let a and α be real positive numbers. Then

$$\frac{1}{2\pi i}\oint_{\Gamma}\left\{\frac{g(z)(z-\alpha)}{f(z)(z-a)}-\frac{g(-z)(z+\alpha)}{f(-z)(z+a)}\right\}\frac{dz}{z}=\frac{g(\infty)}{f(\infty)}-2\frac{g(a)}{f(a)}\left(1-\frac{\alpha}{a}\right)-\frac{g(0)\alpha}{f(0)a}. \tag{6.6}$$

In particular, for $a = \alpha$,

$$\frac{1}{2\pi i}\oint_{\Gamma}\left\{\frac{g(z)}{f(z)}-\frac{g(-z)}{f(-z)}\right\}\frac{dz}{z}=\frac{g(\infty)}{f(\infty)}-\frac{g(0)}{f(0)}. \tag{6.7}$$

For the same conditions,

$$\frac{1}{2\pi i}\oint_{\Gamma}\left\{\frac{g(z)}{f(z)}-\frac{g(-z)}{f(-z)}\right\}dz=\lim_{z\to\infty}\frac{z}{2}\left\{\frac{g(z)}{f(z)}-\frac{g(-z)}{f(-z)}\right\}. \tag{6.8}$$

It is not difficult to see that most of the ωk-intensities either themselves have the form of the half-integral expressions entering here, or to reduce to such a form for particular assumptions on the character of the dispersion. Thus, for example, the intensities (4.11) – (4.13), (4.15), and (5.1) have the form of the integrand in (6.7), while the intensity (4.14) takes on such a form for real C (absence of dispersion in the coefficient of thermal expansion). The intensities (5.2) and (5.3) [but for real C and (5.4)] have the form of the integrand in (6.8). So far as the integrals (6.5) are concerned, [integrals of the intensities which contain the deformations $u'_{\alpha\beta}$ and u], we must for their computation rely on the laws of dispersion for the compressional modulus $\bar{K}$ and (or) the shear modulus $\bar{\mu}$. Equation (6.6) is obtained for just that case in which use is made of a very simple law with a single relaxation time

$$\bar{K}=\frac{K_\infty z+K_0/\tau'}{z+1/\tau'},\qquad \bar{\mu}=\frac{\mu_\infty z+\mu_0/\tau}{z+1/\tau}.$$

The expressions for $I_{\xi\eta}(\mathbf{k})$ are so simple that the subsequent calculation of the correlation functions (6.4) presents no difficulties. As a result we obtain formulas in a number of cases which depend materially on the dispersion of the parameters (containing values of the parameters for $z = 0$ and $z = \infty$). Because of lack of space we limit ourselves for illustration only to autocorrelation functions for the relative density u:

$$\Psi_{uu}(0,\rho)=\Theta\left(\frac{1}{K_\infty}-\frac{\beta_T^2M_0}{1+\beta_TM_0}\right)\delta(\rho)+\frac{\Theta(K_\infty-K_0)\psi(\rho,a)}{2\pi K_0K_\infty\varkappa_{1a}(K_a+M_a)}, \tag{6.9}$$

where

$$\psi(\rho,a)=\frac{F(k_{1a}^2)\exp(-|k_{1a}|\rho)-F(k_{2a}^2)\exp(-|k_{2a}|\rho)}{(k_{1a}^2-k_{2a}^2)\,p},\qquad F(k^2)=(\rho_0a^2+M_ak^2)(D_{1a}+\varkappa_{1a}k^2).$$

The index a here denotes parameters at $z = a = K_0/K_\infty\tau'$.

In the absence of dispersion in the bulk modulus ($K_\infty = K_0 = 1/\beta_T$) the second term of Eq. (6.9) vanishes and the first yields the thermodynamic expression*

$$\Psi_{uu}=\frac{\Theta\beta_T}{1+{}^4/_3\,\beta_T\,\mu_0}\,\delta(\rho).$$

In the case of a liquid ($\mu_0 = 0$), if we consider that $u = -\rho/\rho_0$, where ρ = fluctuations in the density, we then obtain the usual formula

$$\Psi_{\rho\rho}=\Theta\beta_T\rho_0^2\,\delta(\rho).$$

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*Calculated with account of the correlation of the displacement of the medium at different points (and, consequently, fluctuations of the form also of nonoverlapping volumes), which takes place in a solid, in contrast to a liquid. (See Sec. 3 in Ref. 18, and also Ref. 19, in particular p. 56.)

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