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SOME CHARACTERISTICS OF M1-CONVERSION NUCLEAR TRANSITIONS

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M1-conversion nuclear transitions are investigated within the framework of the single-particle model with LS coupling.

WITHIN the framework of the strictly single-particle model of the nucleus M1-conversion on nucleonic transitions of the types $n_1 s_{1/2} - n_2 s_{1/2}$ and $n_1 d_{1/2} - n_2 s_{3/2}$ are "forbidden". The contribution from the field of the external electronic current of the transition disappears in the case of a $n_1 s_{1/2} - n_2 s_{1/2}$ transition because of the orthogonality of the nucleonic radial functions and in the case of a $n_1 s_{1/2} - n_2 d_{3/2}$ transition because of the orthogonality of the nucleonic of the nucleonic angular functions. In these transitions the conversion probability is determined only by the contribution of the "internal" electronic transition current. However the spin-orbit interaction of the nucleons and the correlation of nucleonic motion which results from their interaction lead to the inclusion of an interaction with the external electronic current. The conversion probability in these cases is determined by entirely different matrix elements than for the usual "allowed" M1 nuclear transitions (such as $p_{1/2} - p_{3/2}$). The results of calculations of K-electron conversion probabilities for various M1 nuclear transitions are given below. It is assumed that the momentum of the final electronic state $p \gg Ze^2$; the K-electron transition to the state $j = 3/2, \lambda = +1/2, \ell = 2$ is neglected because the probability of this transition is smaller by the factor $\sim (Ze^2)^2/2$ than the transition probability to the state $j = 1/2, \lambda = -1/2, \ell = 0$. The relativistic units $\hbar = m_e = c = 1$ are used throughout.

In the model under consideration the nucleonic wave function is separated into radial and angular parts:

$$\Psi(x) = R_{l\Lambda}(x)\,\Omega_{lM}(\theta\varphi); \quad x = \xi / R_0,$$

where $R_0 = 1.2 \times 10^{-13} A^{1/3}$ cm and ξ is the coordinate of the transition nucleon. The wave functions of the electron are designated by $\psi_{j\mu\lambda}(\mathbf{r})$. The subscripts 1 and 2 denote the initial and final states, respectively. The electronic functions are normalized to $\delta_{jj'}\delta_{\lambda\lambda'}\delta_{\mu\mu'}\delta(\epsilon-\epsilon')$.

1. THE VECTOR POTENTIAL OF THE M1 TRANSITION

When a K-electron (1, j = 1/2, λ = −1/2, ℓ = 0) undergoes a transition to a continuum state (2, pε, j = = 1/2, λ = −1/2, ℓ = 0) a transition vector potential **A** is induced in the nucleus. The magnetic-dipole term $\mathbf{A}_1^0$ is

$$\mathbf{A}_1^0 = \sum_M B_{1M}^0(\xi)\,\mathbf{Y}_{1M}^0(\xi).$$

$$B_{1M}^0(\xi) = e\int (\langle \psi_{j\lambda\mu_2}^*(2) \,|\, \alpha \,|\, \psi_{j\lambda\mu_1}(1)\rangle\, \mathbf{Y}_{1M}^{0*}(\xi))\, \frac{dr\,d\omega_\xi}{|\mathbf{r}-\xi|}; \quad \alpha = \begin{Bmatrix} 0 & \sigma \\ \sigma & 0 \end{Bmatrix},$$

where σ denotes the Pauli matrices

$$\sigma_0 = \sigma_z = \begin{Bmatrix} 1 & 0 \\ 0 & -1 \end{Bmatrix}.$$

We divide B_{1M}^0 into parts corresponding to the contributions of the external and the internal electronic transition currents, for $r \geq R_0$ and $r \leq R_0$:

$$B_{1M}^0 = B_{1M>}^0 + B_{1M<}^0.$$

$B_{1M<}^0$ is calculated with electronic wave functions in the field of a uniformly charged sphere of radius R_0. In the calculation of $B_{1M>}^0$ it is necessary to take account of the distortion of the electronic wave functions which results from finite nuclear size, as was done by Sliv.[1] However, in order to compare the relative contributions of different terms in the interaction of the nucleon with the electromagnetic field of the external current $B_{1M>}^0$ can be obtained with electronic functions in the field of a point nucleus. Neglect of the finite nuclear size does not introduce an essential error; allowance for this effect changes $B_{1M>}^0$ by not more than ~20%. By neglecting the finite size of the nucleus we obtain

$$\mathbf{A}_{1>}^0 = i\frac{\sqrt{2}}{3} eC_jN_jJR_0x \sum_M \sqrt{4\pi}\, C_{1Mj\mu_1}^{j\mu_2}\, \mathbf{Y}_{1M}^0(\mathbf{x});$$

$$\mathbf{A}_{1<}^0 = i\frac{\sqrt{2}}{18} eC_jN_jS\Sigma R_0^{2\gamma}(3Ze^2 + \Delta R_0)\, K(x) \sum_M \sqrt{4\pi}\, C_{1Mj\mu_1}^{j\mu_2}\, \mathbf{Y}_{1M}^0(\mathbf{x}).$$

Here

$$C_j = (2p)^\gamma \exp(\pi Ze^2\varepsilon / 2p)\,|\Gamma(\gamma + iZe^2\varepsilon/p)|\,/\sqrt{\pi p}\,\Gamma(2\gamma+1). \qquad N_j = 2^\gamma (Ze^2)^{\gamma+1/2}/\sqrt{\Gamma(2\gamma+1)}; \quad \gamma = +\sqrt{1-(Ze^2)^2};$$

$$\varepsilon_1 = 1 - (Ze^2)^2/2, \qquad S = -\frac{2(1-\varepsilon_1^2)^{1/2}}{(1-\varepsilon_1)^{1/2}(\gamma+1)(1-\omega_1^2/6) - (1+\varepsilon_1)^{1/2}(\omega_1/3)(1-\gamma)};$$

$$\Sigma = \frac{(\varepsilon^2-1)^{1/2}[\operatorname{Im} Q(-\gamma)\operatorname{Re} Q(+\gamma) - \operatorname{Re} Q(-\gamma)\operatorname{Im} Q(+\gamma)]}{(\varepsilon-1)^{1/2}(1-\omega_2^2/6)\operatorname{Im} Q(-\gamma) - (1+\varepsilon)^{1/2}(\omega_2/3)\operatorname{Re} Q(-\gamma)}; \quad J \approx {}^1/_2\{[(\varepsilon+1)(1-\varepsilon_1)]^{1/2}(I+I^*) - i(I-I^*)[(\varepsilon-1)(1+\varepsilon_1)]^{1/2}\};$$

$$I \approx \frac{e^{i\eta}(\gamma + iZe^2\varepsilon/p)}{(Ze^2+ip)^{2\gamma-1}}\Big\{\Gamma(2\gamma-1)\,{}_2F_1\Big(\gamma+1+iZe^2\varepsilon/p;\ 2\gamma-1;\ 2\gamma+1;\ \frac{2p}{p-iZe^2}\Big) - \frac{[(Ze^2+ip)R_0]^{2\gamma-1}}{(2\gamma-1)}\exp(-Ze^2R_0)\,{}_1F_1(\gamma+1$$

$$+ iZe^2\varepsilon/p;\ 2\gamma+1;\ 2ipR_0)\Big\}; \quad K(x) = \{[1 - {}^3/_8(Ze^2)^2 - {}^1/_4\,\Delta R_0]\,x - 0{,}6\,x^3\};$$

$$Q(\hat\gamma) = e^{i\hat\eta}(\hat\gamma + iZe^2\varepsilon/p)\,{}_1F_1(\hat\gamma+1+iZe^2\varepsilon/p;\ 2\hat\gamma+1;\ 2ipR_0);$$

$$\hat\gamma = \pm\gamma; \quad e^{2i\hat\eta} = \frac{1+iZe^2/p}{\hat\gamma + iZe^2\varepsilon/p}; \quad \omega_1 = 3Ze^2/2; \quad \omega_2 = 3Ze^2/2 + \Delta R_0;$$

Δ is the nuclear transition energy.

2. M1 NEUTRON TRANSITIONS

The transition probability W_{M1} is determined by the matrix element of the interaction operator: $\hat{\mathscr{H}} = (\mu_n e/2M)(\sigma \mathbf{H})$, where μ_n is the magnetic moment in nuclear Bohr magnetons: $\mu_n = -1.97$.

It is easily seen that the potential $\mathbf{A}^0_{1>}$ makes no contribution. Indeed,

$$\mathbf{A}^0_{1>} \sim \xi \mathbf{Y}^0_{1M}(\xi);\ \mathrm{curl}\,\mathbf{A}^0_{1>} \sim \mathbf{Y}^{-1}_{1M}(\xi)$$

and does not contain the radial variable ξ. It can thus be seen that in an $M1(s_{1/2} - s_{1/2})$ transition the matrix element of the operator $\mathscr{H}$ vanishes as a result of the orthogonality of the radial functions and in a $d_{3/2} - s_{1/2}$ transition because of the orthogonality of the angular nucleonic functions. A nonvanishing contribution is made only by the term $\mathbf{A}^0_{1<}$ this is proportional to x^3.

We obtain

$$\mathscr{H}_< = 0.1\frac{\sqrt{2}}{3} eC_jN_jS\Sigma R_0^{2\gamma}(3Ze^2 + \Delta R_0)\left\{5\sqrt{\frac{2}{3}}\left(\sum_M \sqrt{4\pi}C^{j\mu_1}_{1Mj\mu_2}\mathbf{Y}^{-1}_{1M}(\mathbf{x})\right)x^2 + \frac{2}{\sqrt{3}}\left(\sum_M \sqrt{4\pi}C^{j\mu_1}_{1Mj\mu_2}\mathbf{Y}^{+1}_{1M}\right)x^2\right\}.$$

The first term makes a nonvanishing contribution in an $s_{1/2} - s_{1/2}$ transition and the second term in an $s_{1/2} - d_{3/2}$ transition:

$$W^e_{M1\,s-s} = \frac{2^{4\gamma}}{54}\frac{(Ze^2)^{2\gamma+1}}{[\Gamma(2\gamma+1)]^3}\left(\frac{e^2R_0}{M}\right)^2 \mu_n^2(3Ze^2 + \Delta R_0)^2 R_0^{4\gamma-4}\, p^{2\gamma-1}S^2\Sigma^2 \exp(\pi Ze^2\varepsilon/p)\,|\Gamma(\gamma + iZe^2\varepsilon/p)|^2\,|\langle R^*_{I_2\Lambda_2}|x^2|R_{I_1\Lambda_1}\rangle|^2 P_{ss};$$

$$P_{ss} = \frac{1}{(2j+1)(2I_1+1)}\sum_{\substack{\mu_1\mu_2\\ M_1M_2}}\left|\sum_M \sqrt{4\pi}\,C^{j\mu_1}_{1Mj\mu_2}\langle\Omega^*_{I_2M_2}|\sigma\mathbf{Y}^{-1}_{1M}|\Omega_{I_1M_1}\rangle\right|^2,\quad W^e_{M1s-d} = \frac{2^{4\gamma}}{9\cdot 75}\frac{(Ze^2)^{2\gamma+1}}{[\Gamma(2\gamma+1)]^3}\left(\frac{e^2R_0}{M}\right)^2\mu_n^2\,(3Ze^2$$

$$+ \Delta R_0)^2\, R_0^{4\gamma-4}\, p^{2\gamma-1}S^2\Sigma^2 \exp(\pi Ze^2\varepsilon/p)\,|\Gamma(\gamma + iZe^2\varepsilon/p)|^2\,|\langle R^*_{I_2\Lambda_2}|x^2|R_{I_1\Lambda_1}\rangle|^2 P_{sd},$$

$$P_{sd} = \frac{1}{(2\gamma+1)(2I_1+1)}\sum_{\substack{\mu_1\mu_2\\ M_1M_2}}\left|\sum_M \sqrt{4\pi}\,C^{j\mu_1}_{1Mj\mu_2}\langle\Omega^*_{I_2M_2}|\sigma\mathbf{Y}^{+1}_{1M}|\Omega_{I_1M_1}\rangle\right|^2.$$

It is assumed that the nucleon is in the field of the nuclear core with angular momentum J so that

$$\Omega_{IM} = \sum_{m\nu} C^{IM}_{JmF\nu}U_{Jm}V_{F\nu}(\xi).$$

For P_{ss} we obtain

$$P_{ss} = 2(2I_2+1)\,w^2(I_1J1{}^1/_2;\ {}^1/_2I_2).$$

For P_{sd} in the special case $J = 0$

$$P_{sd} = (2I_2+1)/4,$$

and in the general case $J \neq 0$

$P_{sd} = 2(2I_2+1)\,w^2(I_1J1{}^3/_2;\ {}^1/_2I_2)$ for an $s_{1/2} \to d_{3/2}$ transition;

$P_{sd} = 2(2I_2+1)\,w^2(I_1J1{}^1/_2;\ {}^3/_2I_2)$ for a $d_{3/2} \to s_{1/2}$ transition,

where $w(abcd;\ ef)$ are Racah coefficients.

The probability of γ radiation is obtained in these cases by employing the potential of magnetic dipole radiation (a light quantum with polarization vector ϵ_λ and wave vector Δ)

$$\mathbf{A}_{10} = i4\pi\sqrt{2\pi/\Delta}\sum_M(\varepsilon_\lambda \mathbf{Y}^0_{1M}(\Delta))\,f_1(\Delta\xi)\,\mathbf{Y}^0_{1M}(\xi),$$

where f_1 is a spherical Bessel function. We obtain

$$W^\gamma_{M1s-s} = \frac{1}{108}\frac{R_0^4e^2}{M^2}\mu_n^2\Delta^7\,|\langle R^*_{I_2\Lambda_2}|x^2|R_{I_1\Lambda_1}\rangle|^2 L_{ss};\quad L_{ss} = \frac{1}{(2I_1+1)}\sum_{M_1M_2\lambda}\left|4\pi\sum_M(\varepsilon_\lambda \mathbf{Y}^{0*}_{1M}(\Delta))\langle\Omega^*_{I_2M_2}|\sigma\mathbf{Y}^{-1}_{1M}|\Omega_{I_1M_1}\rangle\right|^2;$$

$$W^\gamma_{M1sd} = \frac{1}{1350}\frac{R_0^4e^2}{M^2}\mu_n^2\Delta^7\,|\langle R^*_{I_2\Lambda_2}|x^2|R_{I_1\Lambda_1}\rangle|^2 L_{sd};\quad L_{sd} = \frac{1}{(2I_1+1)}\sum_{M_1M_2\lambda}\left|4\pi\sum_M(\varepsilon_\lambda \mathbf{Y}^{0*}_{1M}\Delta())\langle\Omega^*_{I_2M_2}|\sigma\mathbf{Y}^{+1}_{1M}|\Omega_{I_1M_1}\rangle\right|^2.$$

In the same special cases as for P_{ss} and P_{sd} we obtain

$$L_{ss} = 6\,(2I_2 + 1)\,w^2\,(I_1 J 1 {}^1\!/_2;\ {}^1\!/_2 I_2); \quad L_{sd} = 3\,(2I_2 + 1)/4 \text{ for } J = 0,$$

and for $J \neq 0$:

$$L_{sd} = 6\,(2I_2 + 1)\,w^2\,(I_1 J 1 {}^3\!/_2;\ {}^1\!/_2 I_2), \text{ for an } s_{1/2} \to d_{3/2} \text{ transition;}$$
$$L_{sd} = 6\,(2I_2 + 1)\,w^2\,(I_1 J 1 {}^1\!/_2;\ {}^3\!/_2 I_2), \text{ for a } d_{3/2} \to s_{1/2} \text{ transition.}$$

3. M1 PROTON TRANSITIONS. EFFECT OF THE LS INTERACTION

LS proton coupling leads to an additional proton-electron interaction term; the interaction operator for proton transitions is [2,3]

$$\mathcal{H} = (\mu_p e/2M)\,(\sigma \mathbf{H}_<) + eD\,(x)\,(\sigma\,[\xi \mathbf{A}_>]).$$

We neglect the contribution of $\mathbf{A}_<$, which is one order smaller than that of the first term $\sim (\sigma \mathbf{H}_<)$.

For the probability of M1 proton transitions with K-electron conversion we obtain:

In an s — s transition

$$W^e_{M1ss} = \frac{2^{4\gamma+3}}{27}\,(eR_0)^4 \frac{(Ze^2)^{2\gamma+1}}{[\Gamma\,(2\gamma+1)]^3}\,p^{2\gamma-1} \exp\,(\pi Ze^2\varepsilon/p)\,|\,\Gamma\,(\gamma + iZe^2\varepsilon/p)\,|^2 J^2 \times$$
$$\times \left|\,\langle R^*_{l_2\Lambda_2}\,|\,x^2 D\,(x)\,|\,R_{l_1\Lambda_1}\rangle - 0{,}25\,\frac{\mu_p}{MR_0}\,\frac{S\Sigma}{J}\,R_0^{2\gamma-2}\,(3Ze^2 + \Delta R_0)\langle R^*_{l_2\Lambda_2}|x^2|R_{l_1\Lambda_1}\rangle\right|^2 P_{ss};$$

In an s — d proton transition

$$W^e_{M1sd} = \frac{2^{4\gamma+2}}{27}\,(eR_0)^4\,\frac{(Ze^2)^{2\gamma+1}}{[\Gamma\,(2\gamma+1)]^3}\,p^{2\gamma-1} \exp\,(\pi Ze^2\varepsilon/p)\,|\,\Gamma\,(\gamma + iZe^2\varepsilon/p)\,|^2 J^2 \Big|\,\langle R^*_{l_2\Lambda_2}\,|\,x^2 D\,(x)\,|\,R_{l_1\Lambda_1}\rangle$$
$$- 0{,}1\,\frac{\mu_p}{MR_0}\,\frac{S\Sigma}{J}\,R_0^{2\gamma-2}\,(3Ze^2 + \Delta R_0)\,\langle R^*_{l_2\Lambda_2}\,|\,x^2\,|\,R_{l_1\Lambda_1}\rangle\,\Big|^2 P_{sd}.$$

Here M is the nucleon mass $\approx 2 \times 10^3$.

The value of J should be calculated with the correct electronic wave functions outside the nucleus, as was done by Sliv,[1] but for comparison of the relative contributions of the two matrix elements we use the approximate value of J with $p \gg Ze^2$:

$$\approx (2p)^{-1}(\varepsilon - 1)^{1/2}(1 + \varepsilon_1)^{1/2}\,\gamma \cos \eta, \quad S \approx (1 + \varepsilon_1)^{1/2}, \qquad \varepsilon \approx (1 + \varepsilon)^{1/2}.$$

Then for the ratio δ of the matrix element $\sim \langle 2|\,x^2|\,1\rangle$ to the matrix element $\sim \langle 2|\,D\,(x)\,x^2|\,1\rangle$, using the rough estimate (see Ref. 3)

$$\langle 2\,|\,D\,(x)\,x^2\,|\,1\rangle \approx \langle D\,(x)\rangle_{cp}\,\langle 2\,|\,x^2\,|\,1\rangle, \quad \langle D\,(x)\rangle_{cp} \approx -9{,}6A^{-2/3},$$

we obtain for s — s transitions

$$\delta_{ss} = \frac{0{,}5}{9{,}6}\,A^{2/3} R_0^{2\gamma-2}\Delta\,(3Ze^2 + \Delta R_0)\,\frac{\mu_p}{MR_0}\,\frac{1}{\gamma \cos \eta};$$

and for s — d transitions

$$\delta_{sd} = 0{,}4\,\delta_{ss}.$$

We present a brief table of estimated values of δ_{ss} for $\Delta \approx 1$ as a function of Z:

$Z =$	10	20	30	40	50	60	70	80	90
$\delta_{ss} =$	0.015	0.037	0.067	0.096	0.14	0.195	0.24	0.31	0.39

Unlike the conversion transition, M 1 γ radiation is completely determined in both s — s and s — d transition by the spin-orbit term. For an s — s transition

$$W^\gamma_{M1ss} = {}^4\!/_{27} e^2 R_0^4 \Delta^3\,|\,\langle R^*_{l_2\Lambda_2}\,|\,x^2 D\,(x)\,|\,R_{l_1\Lambda_1}\rangle\,|^2 L_{ss};$$

and for an s — d transition

$$W^\gamma_{M1sd} = {}^2\!/_{27} e^2 R_0^4 \Delta^3\,|\,\langle R^*_{l_2\Lambda_2}\,|\,x^2 D\,(x)\,|\,R_{l_1\Lambda_1}\rangle\,|^2 L_{sd}.$$

Since we cannot neglect the interaction between the magnetic moment of the nucleon and the field in $W^e_{M1\,ss}$ and $W^e_{M1\,sd}$, in these M1 proton transitions the conversion coefficient depends on the nuclear matrix elements.

4. EFFECT OF NUCLEONIC INTERACTIONS

Thus far we have been considering the ideal case of a strictly single-particle model with **LS** interaction; this does not exist in reality and one can only speak of a certain degree of approximation. The interactions between core nucleons and the nucleon undergoing the transition result in a departure from the model, since the single-nucleon functions become nonorthogonal. They also result in the inclusion of the interaction between the nucleon and the external electronic transition current, thus strongly increasing the probability of a conversion transition. This interaction also sharply increases the probability of a radiative nuclear transition. We shall roughly estimate the effect of nucleonic interactions for a neutron interaction ($\nu s_{1/2} - \mu s_{1/2}$) when a single $ns_{1/2}$ proton is situated outside of the closed shells. It is assumed that the independent-particle model is a good zero-order approximation and that the interaction between nucleons can be taken in the form of the δ function potential

$$\hat{V}(|\xi_n - \xi_p|) = \hat{C}\delta(\xi_n - \xi_p),$$

where $\hat{C}$ is the spin operator with average value of the order 10^{-36} Mev-cm^3. The nucleonic interaction also affects other cases in which the proton is in an $s_{1/2}$ state. But the calculation then requires a specific potential and a finite range of nuclear forces as well as the inclusion of tensor forces, thus making it much more difficult to obtain even a rough estimate.

The zero-order neutron functions will be denoted by $\varphi_\nu(x)$, $\varphi_\mu(x)$, $\int|\varphi|^2 dx = 1$, and the proton functions by $f_n(y)$, $0 \le x \le 1$ and $0 \le y \le 1$ ($x = \xi_n/R_0$, $y = \xi_p/R_0$). The two-particle wave functions will be obtained by perturbation theory:

$$\psi_{\nu n}(x, y) = \chi_{JM}(\varphi_\nu(x) f_n(y) + \psi'_{\nu n}(x, y))$$

with conservation of the spin of the system; χ_{IM} is the spin wave function of the system.

Assuming that contributions are obtained only from terms with identical radial wave functions we obtain for the probability of K-electron conversion

$$W^e_{M1ss} = \frac{2^{4\gamma+1}}{3}\frac{e^4}{M^2}\frac{(Ze^2)^{2\gamma+1}}{[\Gamma(2\gamma+1)]^3} p^{2\gamma-1} \exp(\pi Ze^2\varepsilon/p)\,|\,\Gamma(\gamma + iZe^2\varepsilon/p)|^2 J^2\alpha^2 A_{ss},$$

where

$$\alpha^2 = \left|\frac{\langle\chi^*_{I_1M_1}|\hat{C}|\chi_{I_1M_1}\rangle - \langle\chi^*_{I_2M_2}|\hat{C}|\chi_{I_2M_2}\rangle}{R_0^3\varepsilon_0}\right|^2 \left|\frac{\int f_n^2(x)\varphi_\nu(x)\varphi_\mu(y)\,dx}{\pi^2(\nu^2-\mu^2)}\right|^2;$$

$$A_{ss} = \frac{1}{(2j+1)(2I_1+1)}\sum_{\substack{\mu_1\mu_2\\ M_2M_2}}\Big|\sum_M \sqrt{4\pi}C^{j\mu_1}_{1Mj\mu_2}\langle\chi^*_{I_2M_2}|(\mu_p\sigma_p + \mu_n\sigma_n)\mathbf{Y}^{-1}_{1M}|\chi_{I_1M_1}\rangle\Big|^2;$$

or

$$A_{ss} = (\mu_p - \mu_n)^2\cdot 2(2I_2+1)\,w^2(I_1{}^1/{}_2 1{}^1/{}_2;\ {}^1/{}_2 I_2);$$

and for the probability of gamma radiation

$$W^\gamma_{M1ss} = (e^2/3M^2)\,\Delta^3\alpha^2 B_{ss},$$

where

$$B_{ss} = \frac{1}{(2I_1+1)}\sum_{M_1M_2\lambda}\Big|4\pi\sum_M(\mathbf{e}_\lambda\mathbf{Y}^0_{1M}(\Delta))\langle\chi^*_{I_2M_2}|(\sigma_p\mu_p + \sigma_n\mu_n)\mathbf{Y}^{-1}_{1M}|\chi_{I_1M_1}\rangle\Big|^2$$

or

$$B_{ss} = (\mu_p - \mu_n)^2\cdot 6(2I_2+1)\,w^2(I_1{}^1/{}_2 1{}^1/{}_2;\ {}^1/{}_2 I_2).$$

Here $\epsilon_0 \approx \hbar^2/2MR_0^2$ in ordinary units; ν, μ, n are the principal (radial) quantum numbers of the nucleonic states. The integral $\int f_n^2\varphi_\nu\varphi_\mu\,dx$ can be calculated with single-particle nucleonic wave functions and a simple potential field.

We have neglected interaction terms which do not vanish in a strict single-particle model.

The ratio of conversion probabilities for a proton transition is

$$\frac{(W)_{LS}}{(W)_{np}} = \frac{4}{9}\,\frac{M^2 R_0^4}{(\mu_p - \mu_n)^2}\,\alpha^{-2}\,|\langle 2 \,|\, x^2 D(x) \,|\, 1\rangle|^2;$$

Here $(\mu_p - \mu_n)^2 = (4.7)^2$ and the subscripts LS and np refer to the LS and n—p interactions. We again make use of the rough estimate

$$|\langle 2 \,|\, x^2 D(x) \,|\, 1\rangle \sim 9{,}6\, A^{-2/3} \langle 2 \,|\, x^2 \,|\, 1\rangle$$

and obtain with $A \approx 100\, R_0 \approx 1.4 \times 10^{-4}\,\alpha^{-2}$

$$(W_{M1}^e)_{LS} \,/\, (W_{M1}^e)_{np} \approx 4 \cdot 10^{-4} \alpha^{-2},$$

and for the ratio of the matrix elements

$$((W_{M1}^e)_{LS}/(W_{M1}^e)_{np})^{1/2} \approx 0{,}02 \,/\, \alpha.$$

Since the independent-particle model does not describe reality, the participation of other configurations in a real nuclear state must be of the order $\alpha \sim 0.02$. Therefore even for proton transitions we may neglect all terms except that representing a two-nucleon interaction. The conversion coefficient then is independent of the nuclear structure and agrees with the usual value for "allowed" transitions.[1] The conversion coefficient may be expected to differ from its usual value only for nuclei where the nucleon undergoing transition is in the field of closed shells or where there are other nucleons identical with it outside of the closed shells.

The proton-neutron interaction in M1 transitions considerably alters the probability of γ radiation. Data obtained by Groshev and Demidov* from a study of γ rays emitted in the capture of thermal neutrons by nuclei, in the range $11 \leq Z \leq 20$, show that transitions occur mainly in a $p_{3/2}$ neutron state by emission of an E1 quantum. When the target nucleus is of the $N \cdot He_2^4$ type, the $(s_{1/2}, s_{1/2})$ M1 transition is relatively weak. For target nuclei with an additional odd proton the intensity of M1 transitions increases by a factor of ~ 30, evidently as the result of the neutron-proton interaction.

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* Private communication from L. V. Groshev.