ANALYSIS OF THE PROPERTIES OF THE ELECTRON-PHOTON COMPONENT OF EXTENSIVE AIR SHOWERS IN THE LOWER THIRD OF THE ATMOSPHERE

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It is concluded from an analysis of the experimental data that in the absence of large fluctuations in the longitudinal development of extensive air showers the energy spectrum in the region $E \lesssim 10^9$ ev and in the lower third of the atmosphere satisfies the condition $1.3 \geq s \geq 1.2$ (if electrons in equilibrium with μ-mesons are subtracted), s being the parameter of the electromagnetic cascade theory, which characterizes the electron energy spectrum. For a spectrum of the indicated type the lateral distribution of electrons at distances from 1.5 to 250 m from the shower axis can be explained completely by multiple Coulomb scattering.

THE lateral distribution of electrons in electronic showers was calculated by Nishimura and Kamata[1] for several values of the parameter s (0.6, 1.0, 1.4, and 2.0) in the approximation B of the cascade theory.[2] Greisen[3] proposed a simple interpolation formula which describes the curves of Nishimura and Kamata with an accuracy to a few percent. One can hope that this formula describes correctly the lateral distribution of electrons for intermediate values of s as well. This would permit a detailed comparison of the experimental data on the lateral distribution of electrons in extensive air showers (abbreviated as EAS in the following) with predictions of the cascade theory, which may yield some information on the singularities of the development of EAS. Such a comparison was already carried out by Greisen[3] who made use of the experimental data of Refs. 4-6 and by Khristiansen[7] who used data obtained in the Soviet Union and partially published in Refs. 8 and 9. It was shown that the lateral distribution of electrons in EAS coincides with the lateral distribution of electrons in electronic cascades with the parameter $s \approx 1.2-1.3$, in the distance interval 1.5 — 250 m at least. This conclusion follows more definitively from Ref. 7 and less so from Ref. 3, since less accurate experimental data have been used in the latter.

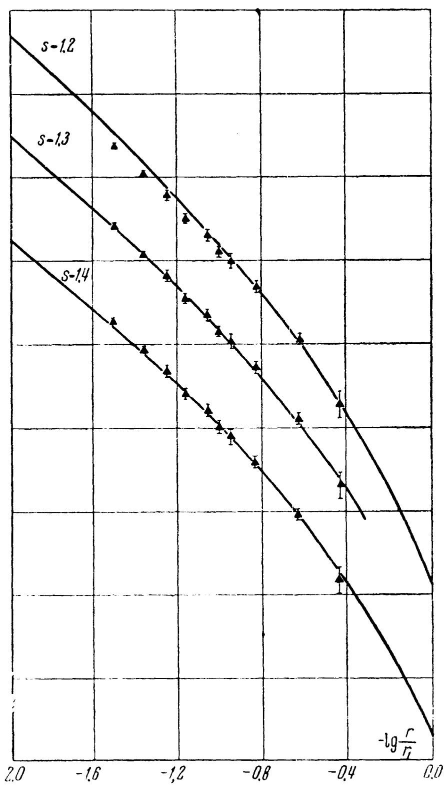

FIG. 3. Experimental dependence of the electron flux density on the distance from shower axis at sea level, shown together with the lateral distribution of electron flux density in electronic showers for s equal to 1.2, 1.3, and 1.4. r_1 = 80 m.

We have undertaken an additional analysis of the results* on the lateral distribution of electrons in the distance interval 1.5 — 30 m from the shower axis for two groups of showers, with the mean number of particles equal to 3.6×10^4 and 3.6×10^5 respectively. The lateral distribution of electrons was found to be identical† for both groups and we constructed therefore an average of the two distributions. This distribution is shown in Fig. 1 together with curves drawn according to the interpolation formula of Greisen.[3] The units of the logarithmic ordinate axis of Fig. 1 are proportional to electron flux density. The coefficients are arbitrary, for the sake

*These results were obtained by us in 1953 at sea level. They were partially analyzed before and published (Ref. 8).

†If the obtained distributions are approximated by the power law $f(r) \sim r^{-n}$, then for $\bar{N} = 3.6 \times 10^4$ we have $n = 0.99 \pm 0.06$ and for $\bar{N} = 3.6 \times 10^5$ $n = 1.02 \pm 0.04$ for the distances between 2.5 and 12 m from the shower axis.

of convenience different for various values of s. The abscissa represents logarithms (common) of the distance from the axis. The distances are measured in units of $r_1 = E_s t_0/\beta$, where $E_s \approx 21$ Mev, β is the critical energy of electrons in air equal to 72 Mev, and t_0 is the radiation length in air in meters at the level of observation (t_0 = 274 m at sea level).

It can be seen from Fig. 1 that the experimental distribution for the distances 1.5—30 meters from the axis is in a better agreement with the theoretical curve for s = 1.3. This curve, as it can be seen from Fig. 2 taken from Ref. 7, evidently fits better the experimental data and over a wider distance interval as well. It also does not contradict the experimental data used by Greisen.[3]

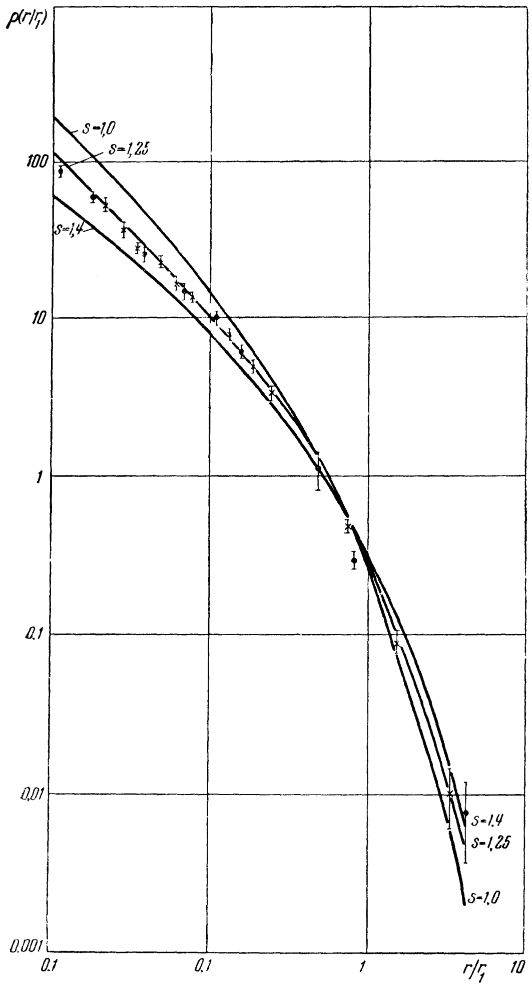

FIG. 2. Dependence of the electron flux density distribution in EAS (without the electrons in equilibrium with μ-mesons) on the distance from shower axis. Solid curves correspond to calculations of Nishimura and Kamata for s equal to 1.0, 1.25 and 1.4. Measurements: x — Moscow, • — Pamir.

The lateral distribution of electrons in electronic showers is basically due to multiple Coulomb scattering and depends on the energy spectrum of shower electrons near the observation level. In EAS, besides multiple scattering, a certain role can be played by the emission angles of π^0-mesons produced in nuclear interactions (nuclear scattering) and which transfer the energy from the nuclear-active into the soft component of the shower, and also by the angles of emission of photons originating in the decay of π^0-mesons. The distribution of electrons in EAS should therefore, for the same energy spectrum of electrons near the observation level, be not less extensive than in purely electronic cascades. Consequently, the agreement between the experimental data on the lateral distribution of electrons in EAS and the electron distribution calculated for electronic cascades with s = 1.3 signifies that the energy spectrum of electrons in EAS at energies $E \lesssim 10^9$ ev* should satisfy the condition $s \leq 1.3$.

Additional information about the energy spectrum of electrons can be obtained from the analysis of their longitudinal development in a single shower. There are no direct data of this kind. Some information can be obtained, however, from the altitude dependence of shower intensity. Numerous experimental data† show that the decreasing intensity $\Phi(>N)$ of showers with number of particles >N in the lower third of the atmosphere can be satisfactorily approximated by an exponential law with the absorption coefficient $\mu = -\partial \ln \Phi(>N)\,\partial t$ which is independent of N and equal to 0.25 per unit radiation length (absorption mean free path equal to 135 g/cm^2). This is true both for the total number of showers with number of particles >N at the observation level and for vertical showers.[3] If we assume now that the longitudinal development of showers is not accompanied by large fluctuations, i.e., if the number of particles in showers of identical initial energy and traversed path does not vary greatly from one shower to another, then the change of the intensity of vertical showers with the depth can be expressed in the following way:[10]

$$\Phi_{\text{vert}}(>N, t) \sim \int_0^t \frac{1}{k(t-t_0, E_0)} \exp\left\{\frac{\gamma}{k(t-t_0, E_0)}\left[\ln\frac{N}{A(t-t_0, E_0)}\right]\right\} \exp\left\{\frac{\gamma\lambda(t-t_0)\,t}{k(t-t_0, E_0)} - \left[\frac{\gamma\lambda(t-t_0)}{k(t-t_0, E_0)} + \mu_0\right] t_0\right\} dt_0. \quad (1)$$

*If the lateral distribution of electrons is determined by the multiple scattering, then the bulk of electrons with energies of the order of a few Bev is found within the distance of 1.5 m from the shower axis and, in consequence, it is impossible to draw any conclusions about such electrons from the lateral distribution for distances > 1.5 m.

†For bibliography see Ref. 3.

where t_0 is the depth of the shower initiation level, t is the depth of the observation level, μ_0 is the interaction coefficient of primary cosmic particles in air, γ is the exponent of the energy spectrum of primary particles, $\lambda = \partial \ln N(t-t_0)/\partial t$ is the absorption coefficient of particles in a shower, $k(t-t_0, E_0)$ is a parameter of value close to that of the age parameter s, A is a slowly varying function of E_0 and $t-t_0$, the form of which depends on the mechanism of shower development.

The constancy of the shower absorption coefficient in the lower third of the atmosphere can be interpreted at least in two ways:

1. One can imagine that the showers observed in the lower third of the atmosphere are the tails of cascades originating near the top of the atmosphere. This can take place if μ_0 corresponds to the geometrical cross-section for the interaction of primary particles with nuclei of air atoms, and

$$-\gamma\lambda (t-t_0)/k(t-t_0, E_0) < \mu_0.$$

At observation levels 650–1000 g/cm² deep the values of $\gamma\lambda/k$, ln (N/A) and λ will vary slowly with t_0 and formula (1) can be written as

$$\Phi_{\text{vert}}(>N, t) \sim N^{-\varkappa} \frac{\exp[\varkappa\lambda t] - \exp[-\mu_0 t]}{\varkappa\lambda + \mu_0}, \tag{1a}$$

where $\kappa = \gamma/k(t-t_{0\,\text{eff}}, E_0)$ and $t_{0\,\text{eff}}$ is the effective depth of shower initiation.

For the depths fulfilling the condition $\exp[-(\kappa\lambda + \mu_0)t] \ll 1$, we have

$$\Phi_{\text{vert}}(>N, t) \sim e^{\varkappa\lambda t}, \text{ i.e., } \mu = -\partial \ln \Phi_{\text{vert}}(>N, t)/\partial t = -\varkappa\lambda.$$

Since μ and κ are known we can find $\lambda(t-t_{0\,\text{eff}})$. According to numerous experimental data[3] $\kappa \approx 1.5$ for showers with $N \sim 5\times10^4 - 5\times10^5$ in the lower third of the atmosphere. Consequently, $-\lambda(t-t_{0\,\text{eff}}) = 0.17$ per radiation length.

This value of the absorption coefficient is of course an average for all recorded showers. In the considered case, however, deviations of the particle absorption coefficient in a particular shower from this average value, due to variations in the initiation level, should not be large. In the first approximation we can therefore regard the mean absorption coefficient as corresponding in the mean to each particular recorded shower.

We can explain the fact that the value $-\lambda(t-t_{0\,\text{eff}})$ remains constant in the lower third of the atmosphere by the assumption that the electrons of the shower are in equilibrium with the nuclear-active component.*

There is a basis to assume that high-energy electrons ($E \gg \beta$) are in equilibrium with the nuclear-active component and low-energy electrons are produced as the result of cascade multiplication of the high-energy ones. In fact, the measurements of Nikol'skii and his collaborators[11] have shown that, in showers with the initial energy $E_0 \gg 5\times10^{14}$ ev detected in mountains, there are more than 100 electrons per nuclear-active particle, accounting already for the fact that electrons observed at a given level are produced by nuclear-active particles at a higher altitude, where the number of the latter is larger than at the observation level. We can conclude that the mean energy of photons originating in the π^0-meson decay and initiating the electronic cascades in EAS should be $\gtrsim 5\times10^9$ ev.† In reality the mean energy of these photons can be still higher, since not all the nuclear-active particles generate the soft component of EAS with the same efficiency.

It is known from the theory of electronic cascades that the coefficient of absorption of the shower particles is determined by the energy spectrum of particles in the high-energy region ($E \gg \beta$). In this theory the energy spectrum for $E \gg \beta$ is

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*If the constancy of the electron absorption coefficient in the lower third of the atmosphere were caused by a purely electromagnetic mechanism of shower development, then the value of the coefficient would be equal to the minimum value of the photon absorption coefficient which equals 0.585 for air, while the observed value of the coefficient is 0.17.

†In fact, such photons produce about 10 electrons near the cascade maximum. In order to have 100 and more electrons for one generating particle it is necessary to assume that not less than 10 photons are produced in the mean in every interaction. The effective energy of generating particles which may yield such a multiplicity should be equal to several hundred Bev.[12] The effective multiplicity of meson production is in general less than the mean.[14] The effective energy of primary photons should, therefore, be larger than the mean energy.

$$f(E)\,dE \sim dE\,/\,E^{1+s}, \tag{2}$$

s increasing slowly with increasing energy for showers of finite initial energy. The effective part of the spectrum which contributes to the absorption coefficient is characterized by the value of s determined from the equation

$$\lambda_1(s) = \lambda, \tag{3}$$

where $\lambda_1(s)$ is a tabulated function evolved in the theory, and $(-\lambda)$ is the absorption coefficient.

These considerations make it possible to find the shape of the energy spectrum of the primary photons of EAS in the region determining the electron absorption coefficient. Since $-\lambda = 0.17$, it follows from Eq. (3) that $s = 1.2$. The energy spectrum of electrons produced by these photons should not be flatter, i.e., the condition $s \geq 1.2$ should be satisfied.

The fact that the electron absorption coefficient of EAS in the lower third of the atmosphere is constant signifies that the shape of the energy spectrum of primary photons (and, consequently, electrons) remains constant (or almost constant) in the region which determines the properties of the soft component of EAS.* The energy spectrum of electrons in the region of lower energies ($E \lesssim \beta$) should then, however, be similar to the one of high-energy electrons, i.e., in the low-energy region we should have $s \geq 1.2$ as well.

It follows therefore from the variation of shower intensity with altitude in the lower third of the atmosphere that the energy spectrum of electrons in EAS is characterized by the condition $s \geq 1.2$ both in the high ($E \gg \beta$) and low ($E \lesssim \beta$) energy region. On the other hand, from the analysis of the lateral distribution of electrons it follows that $s \leq 1.3$ for $E \leq 10^9$ ev. Consequently, in the energy region $E \lesssim 10^9$ ev the energy spectrum of electrons in EAS, in the absence of large fluctuations in the longitudinal development of showers, should satisfy the condition $1.3 \geq s \geq 1.2$.

For this energy spectrum the lateral distribution of electrons (at least at the distances between 1.5 and 250 m from the shower axis) is fully explained by multiple scattering and, in consequence, the contribution of nuclear scattering to the lateral distribution of electrons in EAS in the considered case should be small. An analogous conclusion was already reached by Khristiansen.[7]

2. We considered the case which should occur if the shower tails were absorbed more slowly than the primary radiation. The constancy of the shower absorption coefficient in the lower third of the atmosphere (as well as a number of other properties of showers) can be understood also under the assumption that the tails of showers are absorbed faster than the primary radiation, i.e., when $(-\kappa\lambda_{\text{lim}}) > \mu_0$. In that case the longitudinal development of showers under large thicknesses of matter will be determined by the absorption of the primary radiation, i.e., $\mu_0 = 1/135$ g/cm^2.

Although it is known that in light elements the mean free path for the interaction of protons and neutrons of $\sim 10^9 - 10^{10}$ ev is not larger than 80 g/cm^2 (see Ref. 15) and there are no indications of its increase with the energy, such possibility, nevertheless, is not ruled out for ultra-high energies.

Among the showers registered at any level in the lower layers of the atmosphere there should be some which did not attain their maximum, as well as others far beyond the maximum of their development. It follows from formula (1) that showers for which $-\kappa\lambda = \mu_0$ will be recorded with the greatest probability. The same equation should, in the first approximation, determine the mean age of showers as well. The energy spectrum of electrons in showers of the mean age should, in consequence, fulfill the condition $s \geq 1.2$.

The experimentally determined lateral distribution of electrons should also characterize in the mean all showers, since it is necessary to average over a large number of showers in order to obtain an accurate form of the distribution function. Since it follows from the lateral distribution that $s \leq 1.3$, then the conclusion that the main contribution to the lateral spread of the electrons is due to multiple Coulomb scattering remains valid. However, in contrast to the former case when we were able to draw certain conclusions the details of development of a particular shower, in the present case it is possible to state with respect to a given shower only that the upper limit of the particle absorption length should be smaller than 200 g/cm^2.

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*This of course cannot be said for the spectrum of primary photons in general, since these photons are in equilibrium with the absorbed component. It is evidently necessary to assume that the above region is gradually shifted towards lower energies.

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HYDRODYNAMICAL INTERPRETATION OF ONE CHARACTERISTIC OF LARGE SHOWERS RECORDED IN PHOTOGRAPHIC EMULSIONS

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The experimental distribution of the transverse momentum components of secondary particles is compared with the predictions of the hydrodynamical theory of multiple particle production. It is found that the predictions of the one-dimensional theory for a final temperature of $T_{fin} = mc^2/k$ (where m is the π-meson mass) agree satisfactorily with the experimental data. This permits us to draw some conclusions concerning the nature of $\pi - \pi$ interaction.

THE comparison of predictions of the various phenomenological theories of multiple particle production with experimental results has a decisive value for the confirmation of their validity. Such a comparison made at first for energies of $\gtrsim 10^{12}$ ev (see Ref. 1) indicated that the hydrodynamical theory pro-