ELASTIC COLLISION CROSS SECTION FOR HYDROGEN ATOMS*

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The method of perturbed stationary states is used to calculate the cross sections for elastic scattering of unexcited hydrogen atoms with each other. These cross sections are encountered in the kinetic theory of gases. A graph is given for the dependence of the cross sections on the energy of relative motion in the energy interval from 0.05 to 3 kev.

1. INTRODUCTION

THE system consisting of two hydrogen atoms moving past each other at a certain distance is examined. If the relative velocity of the atoms is small compared to the velocity of the electrons inside the atoms, then both atoms are in an almost adiabatic condition, and the wave function can be first approximated by the wave function of the stationary states of the hydrogen molecule for given positions of the nuclei. In the next approximation the kinetic energy of the nucleus appears as a small perturbation causing transition.[1] Atomic units are used henceforth.

The Schrödinger equation for the system under consideration has the form (the center of mass is at rest):

$$\left\{ -\frac{1}{M} \Delta_r + \hat{H}(\mathbf{r}, \mathbf{r}_1, \mathbf{r}_2) \right\} \Psi(\mathbf{r}, \mathbf{r}_1, \mathbf{r}_2) = E\Psi(\mathbf{r}, \mathbf{r}_1, \mathbf{r}_2), \tag{1}$$

where H is the Hamiltonian of the system for stationary nuclei, i.e.,

$$\hat{H}(\mathbf{r}, \mathbf{r}_1, \mathbf{r}_2) = -\frac{1}{2}\Delta_{r_1} - \frac{1}{2}\Delta_{r_2} - \frac{1}{r_1} - \frac{1}{r_2} - \frac{1}{|\mathbf{r}+\mathbf{r}_1|} - \frac{1}{|\mathbf{r}-\mathbf{r}_2|} + \frac{1}{|\mathbf{r}+\mathbf{r}_1-\mathbf{r}_2|} + \frac{1}{r}. \tag{2}$$

In equations (1) and (2) $\mathbf{r}$ is the radius vector of the nucleus of the first atom relative to the nucleus of the second; $\mathbf{r}_1$ and $\mathbf{r}_2$ are the radius vectors of the electrons in the first and second atoms relative to the respective nuclei. The mass of the electron is unity and the mass of the nucleus is M. Since $M \gg 1$, the center of mass of each atom may be considered as coincident with that of the nucleus. Then the center of mass of the atoms bisects the line joining the nuclei. The reduced mass of the atom in relation to the motion, equal to $(M + 1)/2$, is replaced by $M/2$ in Eq. (1), and in Eq. (2) the reduced mass of the electron and nucleus, equal to $M/M + 1$, is replaced by unity. The eigenfunctions $\varphi_n(\mathbf{r}, \mathbf{r}_1, \mathbf{r}_2)$ of the Hamiltonian (2) and the eigenvalues of the energy $\epsilon_n(r)$ prove to be electron functions and electron terms of the hydrogen molecule for a given position of the nuclei, determined by the radius vector r

$$\hat{H}(\mathbf{r}, \mathbf{r}_1, \mathbf{r}_2)\varphi_n(\mathbf{r}, \mathbf{r}_1, \mathbf{r}_2) = \varepsilon_n(r)\varphi_n(\mathbf{r}, \mathbf{r}_1, \mathbf{r}_2). \tag{3}$$

The functions φ_n represent an orthonormalized system (for any $\mathbf{r}$):

$$\iint \varphi_m^* \varphi_n \, d\mathbf{r}_1 d\mathbf{r}_2 = \delta_{mn}. \tag{4}$$

We expand the wave function of the system $\Psi(\mathbf{r}, \mathbf{r}_1, \mathbf{r}_2)$ in terms of the eigenfunctions $\varphi_n(\mathbf{r}, \mathbf{r}_1, \mathbf{r}_2)$, where the coefficients of expansion χ_n are functions of $\mathbf{r}$

$$\Psi(\mathbf{r}, \mathbf{r}_1, \mathbf{r}_2) = \sum_n \chi_n(\mathbf{r}) \varphi_n(\mathbf{r}, \mathbf{r}_1, \mathbf{r}_2). \tag{5}$$

Substituting the expansion (5) in (1), multiplying both sides by φ_m^*, and integrating over $\mathbf{r}_1$ and $\mathbf{r}_2$ using (3) and (4), we obtain

$$(1/M)\Delta_r\chi_m(\mathbf{r}) + [E - \varepsilon_m(r)]\chi_m(\mathbf{r}) = -\frac{1}{M}\left\{ \sum_{n \neq m}' \nabla_r\chi_n \iint \varphi_m^* \nabla_r\varphi_n \, d\mathbf{r}_1 d\mathbf{r}_2 + \sum_n \chi_n \iint \varphi_m^* \Delta_r\varphi_n \, d\mathbf{r}_1 d\mathbf{r}_2 \right\}, \tag{6}$$

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*This work was performed in 1953.

which is exact. On the right side of Eq. (6) the integrand contains derivatives of the electron function φ_n with respect to the nuclear coordinate r. When the relative velocity of the atom is small compared to the electron velocities in the atom, this derivative is small compared to the derivative of the nuclear function with respect to the nuclear coordinates, and since on the right side of Eq. (6) there is still a small factor $1/M$, the terms on the right side may, in general, be disregarded and we obtain the equation

$$(1 / M) \Delta_r \chi_m (r) + [E - \varepsilon_m (r)] \chi_m (r) = 0, \tag{7}$$

which describes the motion of one particle with mass $M/2$ in a field with potential energy $\epsilon_m(r)$. Thus the problem of the elastic scattering of atoms on each other leads to the problem of the scattering of a particle in a field $\epsilon_m(r)$. The solution of Eq. (7) should have an aysmptotic form

$$\chi (r) \sim e^{ikz} + f (\theta) e^{ikr} / r, \tag{8}$$

which corresponds to elastic scattering. The wave function of the system for $r \to \infty$ corresponds to two unexcited hydrogen atoms moving with respect to one another, and consists of the product $\chi(r) \psi(r_1) \psi(r_2)$ [$\psi(r_1) = \pi^{-1/2} e^{-r_1}$ is the wave function of the unexcited hydrogen atom], which is symmetric or anti-symmetric with respect to the electron coordinates depending on whether the total electron spin of the system is zero or one. (The spins of the nuclei are not essential for the given problem since they have a small influence on the electron motion which determines the atomic interaction.) For a finite r, the electronic parts of the wave function are determined by the eigenfunctions $\varphi_n(r, r_1, r_2)$ of Eq. (3), which for $r \to \infty$ have the required asymptotic form, while the corresponding $\epsilon_n(r)$ determine with Eq. (7) the nuclear parts of the wave functions $\chi_n(r)$. The function $\varphi_1 (r, r_1, r_2)$, which goes for $r \to \infty$ into the wave function

$$[\psi (r_1) \psi (r_2) + \psi (| r + r_1 |) \psi (| r - r_2 |)] / \sqrt{2},$$

is a function of the $^1\Sigma_g^+$ state of the hydrogen molecule, while the function $\varphi_2(r, r_1, r_2)$ which changes for $r \to \infty$ into the wave function

$$[\psi (r_1) \psi (r_2) - \psi (| r + r_1 |) \psi (| r - r_2 |)]/\sqrt{2},$$

is a function of the $^3\Sigma_u^+$ state of the hydrogen molecule. The state of the entire systems is a mixture of the singlet state with wave function $\chi_g(r) \varphi_g(r, r_1, r_2)$ and the triplet state with wave function $\chi_u(r) \varphi_u(r, r_1, r_2)$, with probabilities $1/4$ and $3/4$ respectively.* The cross section for elastic scattering in such a systems consists of two parts: scattering in the singlet state and scattering in the triplet state, with

$$d\sigma = \tfrac{1}{4} d\sigma_g + \tfrac{3}{4} d\sigma_u. \tag{9}$$

2. MOLECULAR TERMS

In order to solve the problem it is necessary to know the energy of the terms $^1\Sigma_g^+$ and $^3\Sigma_u^+$ of molecular hydrogen as a function of the distance r between nuclei. The term $^1\Sigma_g^+$ corresponds to the basic bound state of the hydrogen molecule. Rydberg[2] calculated the energy of the term $^1\Sigma_g^+$ in the interval $0.78 < r < 3.75$ from analysis of the vibration spectra of the basic electronic state. The energy of the term outside this interval, and also the energy of the term $^3\Sigma_u^+$ are given in the work of Hirschfelder and Linnett[3] where they also give the most probable energy of the term, taking into account calculations by previous authors.

For dilution of the atoms, i.e., for $r \to \infty$, the above molecular term changes to a state of unexcited atoms: the term $^3\Sigma_u^+$ to a state with parallel spin, and the term $^1\Sigma_g^+$ to a state with antiparallel spin. The energy of interaction of the unexcited atoms at a large distance $r \gtrsim 9$ is determined by the van der Waals force, whereupon for the hydrogen atom[4]

$$U (r) = - 6.47 / r^6. \tag{10}$$

For increased density of atoms, i.e., for $r \to 0$, the term $^1\Sigma_g^+$, after subtraction of the energy of electrostatic repulsion of the nucleus, goes into the ground state $^1S_0$ of the helium atom with energy equal to $- 1.89$ atomic units. Hence, we obtain for the energy of the term $^1\Sigma_g^+$ for small distances

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*Subscripts g and u will designate quantities corresponding respectively to states $^1\Sigma_g^+$ and $^3\Sigma_u^+$.

$$\varepsilon_g = 1 / r - 1.89. \tag{11}$$

Term $^3\Sigma_u^+$ for $r \rightarrow 0$, after subtracting the energy of electrostatic repulsion of the nucleus, changes to the excited state $^3P_0$ of the helium atom having an energy -1.12 atomic units. Therefore, for small distances

$$\varepsilon_u = \frac{1}{r} - 1.12. \tag{12}$$

A table of energies for the terms $^1\Sigma_g^+$ and $^3\Sigma_u^+$ up to $r = 9$ is given in the Appendix.

3. QUANTUM EFFECTS

Besides the total cross section $\sigma = \int d\sigma$ for elastic collision, which determines the departure of the particles from the parallel beam, in kinetic theory of gases there also occur the cross sections σ^* and σ^{**} which determine the coefficients of diffusion, viscosity, and thermal conductivity. These cross sections are determined by the formulae[5]

$$\sigma^* = \int (1 - \cos \theta) \, d\sigma, \tag{13}$$

$$\sigma^{**} = \int (1 - \cos^2 \theta) \, d\sigma, \tag{14}$$

where θ is the angle of variation of the relative velocity with respect to the direction of collision. In the present work we calculate the cross sections σ^* and σ^{**} for elastic collision of unexcited hydrogen atoms.

The solution of Eq. (7) with condition (8) can be considered in a quasi-classical approximation. Disregarding for a while small-angle scattering, which is determined by diffraction effects, it is possible to say that approach, at atomic energies on the order of or less than molecular energies, a condition for the validity of the quasi-classical approach is the smallness of the de Broglie wavelength of the atom in relative motion as compared with the radius of the atom, which in atomic units equals unity, i.e.,

$$\lambda = 1 / \sqrt{2ME} \ll 1, \tag{15}$$

where E is the kinetic energy of relative motion of the atom. In the usual units for atomic hydrogen this means that

$$E \gg 0.01 \, \mathrm{ev}. \tag{16}$$

For atoms with energies of relative motion large compared with the molecular energies the interaction of the atoms is determined principally by the Coulomb repulsion between the nuclei. The condition for the applicability of the quasi-classical approximation in this case has the following form[6]

$$1 / v \gg 1, \tag{17'}$$

where v is the relative velocities of the atoms. This introduces a restriction for large energies.

Applicability of the obtained formulae at large energies is further restricted by the condition that the relative velocities of the atoms should be small compared with the electron velocities inside the atom.

$$v = \sqrt{2E/M} \ll 1. \tag{17''}$$

It turns out that the condition of "adiabaticity" (17") coincides in this case with the condition of applicability of the quasi-classical approximation for large energies (17'). In the usual units (17') and (17") give for hydrogen atoms

$$E \ll 25 \, \mathrm{kev}. \tag{18}$$

As to the quantum effect, associated with the identity of the atoms, going over to the quasi-classical case we obtain for the differential cross section for elastic scattering at angles not too close to zero or 180° the classical expressions, and diffraction effects appear for small-angle scattering both forward and backward. These effects make a finite contribution to the total cross section $\sigma = \int d\sigma$. However, we consider not the total cross section σ, but the cross sections σ^* and σ^{**} for which the contribution of the small forward angles is not significant, owing to of the factors $1 - \cos \theta$ and $1 - \cos^2 \theta$, respectively, which vanish for $\theta = 0$. For the cross section σ^{**} small backward angles are also not important since the factor $1 - \cos^2 \theta$ goes to zero also for $\theta = 180°$. The factor $1 - \cos \theta$ in the cross section σ^* does not go

to zero for $\theta = 180°$ and even has a maximum value, so that the quantum effects should remain as we go over to the quasi-classical case. The point, however, is that the cross section $\sigma*$ determines the coefficient of diffusion, and for identical particles diffusion concept has no meaning. Thus, the quantum phenomena associated with small-angle scattering forward and backward, are insignificant in our case, and for the calculation of the cross sections $\sigma*$ and $\sigma**$ it is possible to make use of formulae of classical mechanics.

4. CALCULATION OF CROSS SECTION

We thus proceed to the calculation of the cross sections $\sigma*$ and $\sigma**$ according to classical mechanics. Cross sections $\sigma*$ and $\sigma**$ according to (9), have the form

$$\sigma^* = {}^{1}/_{4}\, \sigma_{g}^{*} + {}^{3}/_{4}\, \sigma_{u}^{**}, \tag{19}$$

$$\sigma^{**} = {}^{1}/_{4}\, \sigma_{g}^{**} + {}^{3}/_{4}\, \sigma_{u}^{**}, \tag{20}$$

where the subscripts g and u correspond to scattering on potentials $\epsilon_g(r)$ and $\epsilon_u(r)$ respectively. The angle of deflection θ for scattering is connected with impact parameter ρ by the relation[7]

$$\theta = \pi - 2 \int_{r_0}^{\infty} \frac{\rho\, dr}{r^2 \sqrt{1 - U(r)/E - (\rho/r)^2}}, \tag{21}$$

where $U(r)$ is potential energy, i.e., $\epsilon_g(r)$ and $\epsilon_u(r)$; r_0 is root of the denominator in the integrand with the maximum absolute values. Cross sections $\sigma^{*}_{g,u}$ and $\sigma^{**}_{g,u}$ are therefore obtained from the formula

$$\sigma^{*}_{g,u} = 2\pi \int_{0}^{\infty} (1 - \cos\theta(\rho))\, \rho\, d\rho, \tag{22}$$

$$\sigma^{**}_{g,u} = 2\pi \int_{0}^{\infty} (1 - \cos^2\theta(\rho))\, \rho\, d\rho. \tag{23}$$

Formula (21), and also (22) and (23), contain only the energy of relative motion of the atoms — energy of the atoms in a system of coordinates where the center of mass is at rest. The mass does not enter into these equations.

E	σ_g^*	σ_u^*	σ_g^{**}	σ_u^{**}	σ^*	σ^{**}
0.00184	138	81	87	68	95	73
0.0115	110	48	66	44	63	49
0.0460	76	26	46	26	39	31
0.184	13.7	11.1	21.6	14.6	11.7	16.3

The values of the cross sections are determined from (21) — (23) by numerical integration* for four values of energy (E = 0.00184, 0.0115, 0.0460, 0.184 atomic units) small compared with the molecular energy (the energy of dissociation of molecular hydrogen from the ground state is 0.174 atomic units = 4.46 ev). These four values of atomic hydrogen correspond to relative velocities v = 1/500, 1/200, 1/100 and 1/50 atomic units. For each value of energy we calculate the cross sections $\sigma^{*}_{g,u}$ and $\sigma^{**}_{g,u}$, each for both types of U(r): $\epsilon_g(r)$ and $\epsilon_u(r)$. We obtain the final values of $\sigma*$ and $\sigma**$ from these cross sections by using (19) and (20). The results of the numerical calculation are given in the table (in atomic units), and are marked in the figure by dots.

For atoms with energies of relative motion large compared with the molecular energy (for velocities, as before, small compared with atomic) it is possible to proceed in the following manner. As the energy of relative motion increases, the atoms upon collision gradually come closer to each other. Therefore, the atomic interaction at the ever diminishing distances result in angles that are continuously more significant for the cross sections. At small distances the energy of interaction is given by (11) and (12), and at large distances it tends to zero. At medium distances its order of magnitude is equal to the molecular energy, which in the given case is small compared with the energy of relative motion. Therefore, for large energies it is possible to approximate the energy of interaction by

$$U_{g,u}(r) = 1_j^{\cdot}/r - U_0, \quad r < 1/U_0 = r_0, \quad U_{g,u}(r) = 0, \quad r > 1/U_0 = r_0, \tag{24}$$

upon which we get from (11) and (12) $U_0 = 1.89$ and 1.12 for the terms ${}^{1}\Sigma_g^+$ and ${}^{3}\Sigma_u^+$, respectively, and the differences $\epsilon_g(r) - U_g(r)$ or $\epsilon_u(r) - U_u(r)$ are considered as corrections. Thus, r_0 proves to be a

*The numerical integration was done by D. F. Davidenko.

certain effective radius of interaction.

The problem of scattering in the field U(r) defined by Eq. (24) is solved rigorously. Carrying out the integration in (21) we obtain

$$\sin\frac{\theta}{2} = [(1 - \xi^2)/(1 + a\xi^2)]^{1/2}, \quad \rho < r_0, \quad \theta = 0, \quad \rho > r_0, \tag{25}$$

where

$$\xi = \rho/r_0, \quad a = (4E/U_0)(1 + E/U_0).$$

Integrating (22) and (23) we get

$$\sigma_{g,u}^* = \frac{2\pi r_0^2}{a}\left[\frac{1+a}{a}\ln(1+a) - 1\right], \tag{26}$$

$$\sigma_{g,u}^{**} = 8\pi r_0^2\frac{1+a}{a^2}\left[\frac{2+a}{2a}\ln(1+a) - 1\right]. \tag{27}$$

The values of r_0 and a are different for the terms $^1\Sigma_g^+$ and $^3\Sigma_u^+$. It is now necessary to calculate the correction that results from replacing the interaction energy according to (24). Let

$$V_{g,u}(r) = \varepsilon_{g,u}(r) - U_{g,u}(r). \tag{28}$$

It is possible to calculate the correction to cross sections (26) and (27), necessitated by superimposing a linear potential V(r) on the main field U(r), but this leads to very cumbersome expressions. The equations become simpler if this correction is expanded in powers of 1/E and the first non-vanishing term calculated. It turns out that the first term is $1/E^2$ and, therefore, V(r) enters in this term in a non-linear manner. Actually, let us designate the quantity $\theta(\rho)$, given by (25) and obtained by substituting expression (24) for U(r) into (21), by $\theta_0(\rho)$, and designate correspondingly, the cross section in (26) by σ_0^*, i.e.,

Energy dependence of the elastic collision cross section of hydrogen atoms.

$$\sigma_0^* = 2\pi\int_0^\infty (1 - \cos\theta_0(\rho))\,\rho d\rho. \tag{22'}$$

By $\theta(\rho)$ we now designate the quantity obtained from (21) by replacing U(r) with the exact energy of interaction $\varepsilon(r)$ = U(r) + V(r) (tabulated in the Appendix).

$$\sigma^* = 2\pi\int_0^\infty (1 - \cos\theta(\rho))\,\rho d\rho. \tag{22''}$$

The expansion in powers of 1/E of $\theta_0(\rho)$ and $\theta(\rho)$, contained in (22') and (22''), has the form

$$\theta_0(\rho) = -\frac{\rho}{E}\int_\rho^\infty \frac{U'(r)\,dr}{\sqrt{r^2 - \rho^2}} + O\left(\frac{1}{E^2}\right), \tag{29}$$

$$\theta(\rho) = -\frac{\rho}{E}\int_\rho^\infty \frac{U'(r) + V'(r)}{\sqrt{r^2 - \rho^2}}\,dr + O\left(\frac{1}{E^2}\right). \tag{30}$$
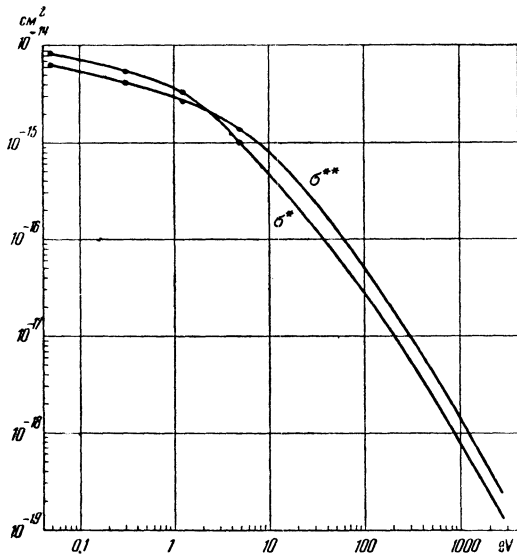
Substituting (29) and (30) in (22') and (22'') we find

$$\sigma^* - \sigma_0^* = 2\pi\int_0^\infty \rho d\rho\,\frac{\rho^2}{E^2}\int_\rho^\infty\int_\rho^\infty \frac{U'(r_1)V'(r_2) + U'(r_2)V'(r_1) + V'(r_1)V'(r_2)}{\sqrt{(r_1^2 - \rho^2)(r_2^2 - \rho^2)}}\,dr_1 dr_2 + O\left(\frac{1}{E^3}\right). \tag{31}$$

Analogous calculations give

$$\sigma^{**} - \sigma_0^{**} = 2(\sigma^* - \sigma_0^*). \tag{32}$$

It is possible to carry out the integration with respect to ρ in formula (31) by substituting expression (24) for U(r). We introduce in the double integral the variables r_1 and $k = r_2/r_1$, which makes for easier calculation, and obtain

$$\sigma^* - \sigma_0^* = \frac{2\pi}{E^2} \int_0^\infty V(r)\,dr + \frac{\pi}{2E^2} \int_0^1 dk \left(\frac{1+k^2}{2} \ln \frac{1+k}{1-k} - k \right) \int_0^\infty dr_1 r_1^3 V'(r_1)\, V'(kr_1). \tag{33}$$

The correction to σ^{**} is obtained from (32). The value of the additional potential $V(r)$ is determined from (24) and (28) and from the table in the Appendix.

Numerical calculation of the first integral in (33) gives 0.19 and 0.53; the double integral in (33) equals 0.23 and 0.22 for the terms $^1\Sigma_g^+$ and $^3\Sigma_u^+$, respectively. Substituting these values in (33) and using (32) we obtain the corrections $\sigma^* - \sigma_0^*$ and $\sigma^{**} - \sigma_0^{**}$.

The cross sections (26) and (27), with the corrections from (33) and (32) added, are determined by

$$\sigma_g^* = \frac{2\pi r_{0g}^2}{a_g}\left[\frac{1+a_g}{a_g}\ln(1+a_g) - 1\right] - \frac{0.83}{E^2}, \qquad \sigma_u^* = \frac{2\pi r_{0u}^2}{a_u}\left[\frac{1+a_u}{a_u}\ln(1+a_u) - 1\right] + \frac{3.68}{E^2}; \tag{34}$$

$$\sigma_g^{**} = 8\pi r_{0g}^2 \frac{1+a_g}{a_g^2}\left[\frac{2+a_g}{2a_g}\ln(1+a_g) - 1\right] - \frac{1.66}{E^2}, \qquad \sigma_u^{**} = 8\pi r_{0u}^2 \frac{1+a_u}{a_u^2}\left[\frac{2+a_u}{2a_u}\ln(1+a_u) - 1\right] + \frac{7.36}{E^2}; \tag{35}$$

with $r_{0g} = 0.529$; $r_{0u} = 0.894$; $a_g = 2.116\, E(1 + 0.529\, E)$; and $a_u = 3.776\, E(1 + 0.894\, E)$.

The correction to the cross sections for large energy is of the order of $1/\ln E$. The figure shows the cross sections σ^* and σ^{**} obtained from (19) and (20). In the region of intermediate energies the cross sections are obtained by graphical joining.

I wish to thank A. B. Migdal for suggesting the problem and for attention to the work.

APPENDIX

Energy of interaction for terms $^1\Sigma_g^+$ and $^3\Sigma_u^+$ of the hydrogen molecule

r	$\varepsilon_g(r)$	$\varepsilon_u(r)$	r	$\varepsilon_g(r)$	$\varepsilon_u(r)$	r	$\varepsilon_g(r)$	$\varepsilon_u(r)$
0.1	8.135	8.910	2.6	—0.0855	0.0505	5.1	—0.00270	0.00122
0.2	3.195	3.965	2.7	—0.0775	0.0440	5.2	—0.00230	0.00103
0.3	1.600	2.360	2.8	—0.0695	0.0380	5.3	—0.00195	0.000870
0.4	0.855	1.590	2.9	—0.0625	0.0330	5.4	—0.00165	0.000740
0.5	0.450	1.150	3.0	—0.0565	0.0285	5.5	—0.00142	0.000620
0.6	0.207	0.850	3.1	—0.0510	0.0255	5.6	—0.00120	0.000520
0.7	0.0590	0.685	3.2	—0.0460	0.0220	5.7	—0.00102	0.000435
0.8	—0.0355	0.555	3.3	—0.0415	0.0192	5.8	—0.000860	0.000360
0.9	—0.0885	0.460	3.4	—0.0365	0.0168	5.9	—0.000750	0.000305
1.0	—0.1240	0.390	3.5	—0.0315	0.0140	6.0	—0.000640	0.000250
1.1	—0.1475	0.332	3.6	—0.0275	0.0123	6.2	—0.000480	0.000165
1.2	—0.1610	0.288	3.7	—0.0240	0.01075	6.4	—0.000360	0.000107
1.3	—0.1700	0.247	3.8	—0.0210	0.00920	6.6	—0.000265	0.0000680
1.4	—0.1740	0.216	3.9	—0.0185	0.00810	6.8	—0.000195	0.0000400
1.5	—0.1735	0.190	4.0	—0.0160	0.00700	7.0	—0.000147	0.0000222
1.6	—0.1695	0.168	4.1	—0.0138	0.00620	7.2	—0.000110	0.0000110
1.7	—0.1630	0.150	4.2	—0.0120	0.00540	7.4	—0.0000830	0.0000027
1.8	—0.1565	0.134	4.3	—0.0100	0.00460	7.6	—0.0000640	—0.0000027
1.9	—0.1490	0.120	4.4	—0.00800	0.00385	7.8	—0.0000510	—0.0000060
2.0	—0.1410	0.1084	4.5	—0.00660	0.00330	8.0	—0.0000400	—0.0000094
2.1	—0.1325	0.0965	4.6	—0.00570	0.00280	8.2	—0.0000310	—0.0000114
2.2	—0.1230	0.0855	4.7	—0.00490	0.00240	8.4	—0.0000250	—0.0000119
2.3	—0.1130	0.0755	4.8	—0.00420	0.00205	8.6	—0.0000200	—0.0000118
2.4	—0.1035	0.0665	4.9	—0.00365	0.00170	8.8	—0.0000165	—0.0000113
2.5	—0.0940	0.0580	5.0	—0.00320	0.00145	9.0	—0.0000140	—0.0000100

[1] N. F. Mott and H. S. W. Massey, Theory of Atomic Collisions, I.I.L. (Russ. Transl.) (1951), Chapt. VIII, p. 7

[2] R. Rydberg, Z. Physik 73, 376 (1931).

[3] J. O. Hirschfelder and J. W. Linnett, J. Chem. Phys. 18, 130 (1950).

[4] R. Eisenschitz and F. London, Z. Physik 60, 491 (1930).

[5] S. Chapman and T. G. Cowling, The Mathematical Theory of Uniform Gases, Ch. 9.

[6] L. D. Landau and E. M. Lifshitz, Квантовая механика (Quantum Mechanics) GITTL (1948), p. 49.

[7] L. D. Landau L. Piatigorskii, Механика (Mechanics) GITTL (1940), p. 21.