where s_0 is the value of s given before in (7). With $n \sim 10^{22}$ the temperature effect is insignificant although, in principle, the degree of polarization should increase with increasing temperature.

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PROBABILITY FOR CHARGE EXCHANGE IN NUCLEONS WITH ENERGIES OF $3 \times 10^9 - 10^{10}$ ev IN INTERACTIONS WITH AIR NUCLEI

V. A. VOROBIEV

Institute for Applied Geology, Academy of Sciences, U.S.S.R.

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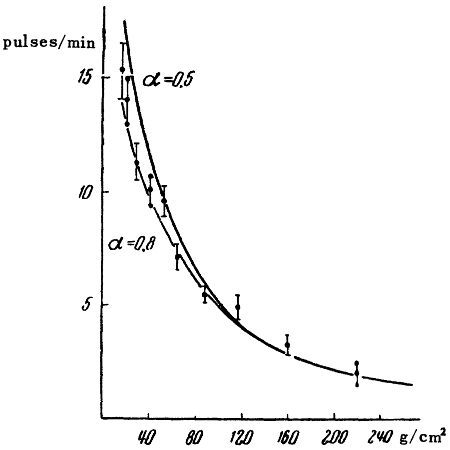
In the last several years a number of papers have been published in which the authors, investigating nuclear-cascade showers in cosmic rays, have reached the conclusion that in the interaction of high-energy nucleons (several Bev and higher) with air nuclei, the main part of the energy (about 70 percent) in the stars which are produced is carried off by one of the emitted nucleons, which in turn also produces stars.[1,2] It is of interest to examine the dependence of the charge of this nucleon on the charge of the incoming nucleon.



We shall assume that protons and neutrons with the energies in question ($3 \times 10^9 - 10^{10}$ ev) interact with air nuclei in the same way. We shall denote the probability of charge exchange for a high-energy nucleon in collision with an air nucleus by the quantity $1-\alpha$, i.e., α is the probability that the main energy of the star produced by the proton (neutron) is carried away by a nuclear-active proton (neutron). We neglect deviations from the direction of the primary nucleon. Then the cascade equations which describe the nucleon history can be solved rather simply if it is assumed that the nucleon energy spectra are independent of depth. The latter assumption is in satisfactory agreement with the experimental data obtained in investigations of high-energy cosmic rays.

Depth	$\alpha=0.5$	$\alpha=0.8$	Experiment
0.9	0.6	0.4	0.35±0.1[4] 0.45±0.2[6]
11	0.99	0.91	0.83±0.1[5] 0.72±0.15[6]

We denote the flux of primary particles incident on the boundaries of the atmosphere by $i_0(E)$. Then the dependence of the global proton flux $P(E, x)$ and neutron flux $N(E, x)$ on the depth x assumes the following form

$$P(E, x) = {}^1/_2\, i_0(E)\,[\mathscr{E}_1(\mu x) + \mathscr{E}_1(\eta x)],\quad N(E, x) = {}^1/_2\, i_0(E)\,[\mathscr{E}_1(\mu x) - \mathscr{E}_1(\eta x)]$$

(x is expressed in units of free path ≈ 60 g/cm^2) where $\eta = 1-(2\alpha-1)(1-\mu)$ and $\mathscr{E}_1$ is the Gol'dovskii integral. The absorption coefficient μ, found from numerous experiments, is approximately $60 \text{ g} \times \text{cm}^{-2}/120 \text{ g} \times \text{cm}^{-2} = 0.5$. The presence of multiply-charged nuclei in primary cosmic radiation (chiefly He) tends to increase the number of protons and neutrons by an amount which is independent of α and equal to

$$\xi a i_0(E)\,[\mathscr{E}_1(\mu x) - \mathscr{E}_1(\xi x)]\,/\,(\xi - \mu),$$

where a $\approx$ 0.2–0.3 and $\xi \approx 60/45$.

The quantity α can be determined from the results of two independent series of experiments. The first is the direct investigation of the absorption of high-energy protons or neutrons as a function of depth at great heigths; the second is the change in the ratio of the number of stars produced by neutrons and protons as a function of height.

In the figure we compare the calculated flux of charged high-energy particles with $\alpha = 0.5$ and $\alpha = 0.8$ and the experimental data[3] taking account of the geometry in approximate fashion. However, the experimental accuracy is not sufficient for a reliable determination of α. More definite results can be obtained from photoemulsion experiments. The Table lists the results of experiments[4-6] and our calculations for the ratio of the number of stars produced by neutrons and multiply-charged particles to the number of stars produced by protons. It is apparent from the table that α is 0.7–0.8 rather than the usually assumed value of 0.5. More exact measurements will be required, however, before definite conclusions can be reached.

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REMARKS ON THE MULTIPLICATIVE RENORMALIZATION GROUP IN THE QUANTUM THEORY OF FIELDS

V. Z. BLANK,* V. L. BONCH-BRUEVICH, and D. V. SHIRKOV

Moscow State University

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IN papers by Bogoliubov and one of us[1-3] an examination had been made of the renormalization group in quantum electrodynamics and meson dynamics. In these papers the group relations were obtained within the framework of perturbation theory. Having in view the practical usefullness of the renormalization method, it is of interest to obtain the group relations without the use of a series expansion — directly from the Schwinger equation for the Green's function. This approach is all the more important because it makes possible a true picture of the origin of the renormalization group. We shall see that the latter is not necessarily connected with the existence of a divergence and can occur in "finite" theories, for example, in the theory of the electron-photon field in a solid body.

We consider a quantum field in which the interaction Lagrangian (in the interaction representation) is of the form ($\hbar = 1$)

$$L(x) = \{g\bar{\psi}(x)\Gamma_0\psi(x) + J(x)\}A(x). \tag{1}$$

Here g is the coupling constant, $\bar{\Psi}$, Ψ and A are Fermi and Bose operators, Γ is the "elementary" vertex part, $x = \{\mathbf{x}, x_0\}$, and J is the "external current." The tensor dimensionality of Γ_0, J and A are not specified; we may note only that A may contain coordinate derivatives of the elementary Bose-field operators. The contractions of the free fields, the total Green's function, and the vertex part are deter-

*Deceased