where a ≈ 0.2–0.3 and $\xi \approx 60/45$.

The quantity α can be determined from the results of two independent series of experiments. The first is the direct investigation of the absorption of high-energy protons or neutrons as a function of depth at great heigths; the second is the change in the ratio of the number of stars produced by neutrons and protons as a function of height.

In the figure we compare the calculated flux of charged high-energy particles with $\alpha = 0.5$ and $\alpha = 0.8$ and the experimental data[3] taking account of the geometry in approximate fashion. However, the experimental accuracy is not sufficient for a reliable determination of α. More definite results can be obtained from photoemulsion experiments. The Table lists the results of experiments[4–6] and our calculations for the ratio of the number of stars produced by neutrons and multiply-charged particles to the number of stars produced by protons. It is apparent from the table that α is 0.7–0.8 rather than the usually assumed value of 0.5. More exact measurements will be required, however, before definite conclusions can be reached.

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REMARKS ON THE MULTIPLICATIVE RENORMALIZATION GROUP IN THE QUANTUM THEORY OF FIELDS

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In papers by Bogoliubov and one of us[1–3] an examination had been made of the renormalization group in quantum electrodynamics and meson dynamics. In these papers the group relations were obtained within the framework of perturbation theory. Having in view the practical usefullness of the renormalization method, it is of interest to obtain the group relations without the use of a series expansion — directly from the Schwinger equation for the Green's function. This approach is all the more important because it makes possible a true picture of the origin of the renormalization group. We shall see that the latter is not necessarily connected with the existence of a divergence and can occur in "finite" theories, for example, in the theory of the electron-photon field in a solid body.

We consider a quantum field in which the interaction Lagrangian (in the interaction representation) is of the form ($\hbar = 1$)

$$L(x) = \{g\bar{\psi}(x)\Gamma_0\psi(x) + J(x)\}A(x). \tag{1}$$

Here g is the coupling constant, $\bar{\Psi}$, Ψ and A are Fermi and Bose operators, Γ is the "elementary" vertex part, $x = \{\mathbf{x}, x_0\}$, and J is the "external current." The tensor dimensionality of Γ_0, J and A are not specified; we may note only that A may contain coordinate derivatives of the elementary Bose-field operators. The contractions of the free fields, the total Green's function, and the vertex part are deter-

*Deceased

mined by the relations*

$$G_0(x, y) = i\langle T\{\psi(x)\bar{\psi}(y)\}\rangle_0, \quad D_0(x, y) = i\langle T\{A(x)A(y)\}\rangle_0 \tag{2}$$

$$G(x, y) = i\langle T\{\psi(x)\bar{\psi}(y)S\}\rangle_0/\langle S\rangle_0, \tag{3}$$

$$D(x, y) = \delta a(x)/\delta J(y); \quad \Gamma(x, y, z) = \delta G^{-1}(x, y)/\delta[ga(z)], \quad a(x) = -i\delta\ln\langle S\rangle_0/\delta J(x). \tag{4}$$

In this case the Green's function equations become (they are easily derived, for example, by the method developed by Polivanov[4]):

$$G = G_0 + ig^2G_0\Gamma_0G\Gamma DG + gG_0G\Gamma_0a, \tag{5}$$

$$D = D_0 - ig^2D_0\Gamma_0G\Gamma GD. \tag{6}$$

It is immediately obvious that Eqs. (5) and (6) are invariant under the following multiplicative transformation of the quantities which appear in them:

$$G \to z_2G, \quad D \to z_3D, \quad \Gamma \to z_1^{-1}\Gamma, \quad g \to z_2^{-1}z_3^{-1/2}z_1g. \tag{7}$$

Here z_1, z_2 and z_3 are continuous finite parameters; it should be kept in mind that we are transforming not only the total functions G, D and Γ but also the elementary functions G_0, D_0 and Γ_0. The relations given in Eq. (7) determine in general form the renormalization group in the quantum theory of fields with an interaction Lagrangian of the form in (1).

It should be kept in mind, however, that if the theory includes additional conditions (such as gradient invariance in quantum electrodynamics), some of the constants which appear in Eq. (7) may be interrelated. Thus, for example, in a gradient transformation of the electromagnetic field potentials the fermion Green's functions transforms according to[5]

$$G(x, y) \to G(x, y)\exp(ig^2\{F(x-y) - F(0)\}), \quad F(x) = (2\pi)^{-4}\int e^{ikx}d_l^{(0)}(k)\frac{dk}{(k^2)^2}, \tag{8}$$

where the function $d_l^{(0)}(k)$ determines the longitudinal matching; it is apparent that F transforms in the same way as D. The invariance condition for this relation with respect to a transformation of a type given in (7) yields

$$z_1 = z_2. \tag{9}$$

The relation given in (8) is nothing else than Ward's identity. In this case the relations given in (7) become the well-known renormalization group of quantum electrodynamics.

Thus, the renormalization group actually expresses only the "self-similarity" property of the Schwinger equations. In conclusion we may note that similar analysis can be carried out in theories with an interaction Lagrangian different from that given in (1).

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*As is customary, the Green's functions and vertex part considered here do not contain divergences. They are either finite at the outset (in the case of a solid) or the divergences are removed by an appropriate subtraction procedure.