RADIO-FREQUENCY METHOD FOR MEASURING THE MASS OF RELATIVISTIC ELECTRONS

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Submitted to JETP editor January 31, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 267-269 (July, 1957)

THE Einstein-Lorentz formula

$$m = m_0 (1 - \beta^2)^{-1/2}, \tag{1}$$

which describes the velocity dependence of the mass of a moving body has been tested experimentally many times.[1-6] It is usually maintained that Eq. (1) has been verified, with a high degree of accuracy, both by these direct experiments and by certain indirect conclusions based on analyses of accelerator operation. Although there is no reason, at the present time, to doubt relativity theory and its consequences, it cannot be said that the validity of the formula given above has been established absolutely.

Without exception, all efforts at the direct verification of Eq. (1) have met with serious difficulties. An analysis of the most accurate measurements indicates that Eq. (1) has not been rigorously verified; indeed, it is impossible, on the basis of these data, to make a choice between Eq. (1) and, say, the Abraham formula.[7] A similar critical analysis of these experiments has recently been carried out by Farago and Janossy[8] in work as yet unpublished to the best of our knowledge. These authors conclude that measurements of electron mass carried out up to this time can not be used to verify Eq. (1) since even the most accurate experiment has inherent errors which are comparable to the difference in mass which results from the use of Eq. (1) and the Abraham formula. Without dwelling on the remarks of these authors concerning possible deviations from Eq. (1) and the effect of these deviations in certain theoretical problems, we wish to emphasize the desirability of further attempts to verify Eq. (1) by present-day methods. In this connection it is interesting to discuss the possibilities offered by electron-physics techniques. The high sensitivity and accuracy of radio-frequency techniques indicate that it should be possible to measure the mass of electrons over a wide region of velocities with an accuracy exceeding the accuracy of the well-known methods. This type of experiment can be carried out using a scheme very similar to that employed in radio-frequency mass-spectrometers based on the application of the cyclotron resonance or the diamagnetic resonance of charged particles.

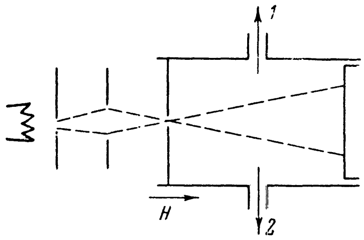

FIG. 1. Diagram of an experiment for measuring relativistic masses in the range $0 \leq \beta < 0.7$. (1) to UHF generator, (2) to detection system.

Suppose that a diverging electron beam (Fig. 1) with a given energy enters a cylindrical resonator in which transverse electric oscillations are excited (for example TE_{01} or TE_{11} mode). Because of the fixed magnetic field H, parallel to the axis of the resonator, the electrons in the resonator move in an helical trajectory, the radius and pitch of which are respectively:

$$\rho = mv_\perp c/eH; \;\; l = v_\parallel T, \tag{2}$$

where $v_\perp$ and $v_\parallel$ are respectively the velocity components perpendicular and parallel to the H field while T is the period of the high-frequency field in the resonator. When the field frequency coincides with the cyclotron frequency resonance absorption of the high-frequency power occurs; this effect can be observed by ordinary radio-frequency spectroscopy methods. Using the values of ν and H at which strong absorption is observed, it is possible to calculate the mass with an accuracy which is limited only by the accuracy with which these two quantities can be measured.

In principle the above scheme can be used to measure the mass of electrons with any energy; in practice, however, this scheme is usless for values of β greater than 0.6—0.7. It follows from Eq. (2) that fixed value of H the diameter and length of the resonator must be increased in proportion to $\beta\sqrt{1-\beta^2}$ and β respectively; an increase in these dimensions requires a larger homogeneous field and leads to

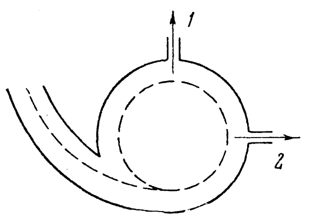

FIG. 2. Diagram of an experiment for measuring relativistic masses in the range $0 < \beta \leq 1.0$; the magnetic field is perpendicular to the plane of the diagram. (1) to UHF generator, (2) to detection system.

greater losses in the walls of the resonator. Hence this arrangement becomes exceedingly cumbersome and less sensitive at high electron velocities ($\beta > 0.7$).

It would appear that there are greater potentialities in a system based on the so-called inverse cyclotron; this scheme is somewhat more difficult but still realizable in practice (Fig. 2).

A half-wave cylindrical (or coaxial) resonator, in which H is paralled to the axis, is placed in a homogeneous field between the pole pieces of an electromagnet. An electron beam, accelerated to any desired value of β (up to $\beta \approx 1$), is introduced into the resonator through a narrow slit in the cylindrical wall. Inside the resonator the electrons move along a spiral, which asymptotically approaches the periphery as the field gradient falls off.

The merit of both schemes is approximately the same; specifically, both can be used to measure the mass of the electron at practically zero values of β. Assuming that an accuracy of 10^{-4}—10^{-5} is easily attainable in the measurements of ν and H, the electron mass should be determined with an error of less than 0.01 percent. This accuracy makes it possible to detect a relativistic change in mass corresponding to an energy variation of the order of a hundred electron volts. It is apparent that good results can be obtained only by achieving narrow resonance lines. In this sense the second scheme is preferable to the first.

Equations (1) can be verified using the above scheme without measurements of the absolute values of the frequency or accelerating potential; if Eq. (1) holds, the relation $\Delta T = (2\pi/cH)\Delta U$, which expresses the change in frequency as a function of the corresponding potential change, also holds true. To verify the above relation we require only the appropriate stability of the field H and accurate measurements of ΔU. These requirements are easily satisfied.

The intensity of the resonance line, a quantity which is extremely important from the point of view of realizing the best signal to noise ratio, is determined basically by two factors: the charge density of the beam and the ratio of the electron time-of-flight in the resonator to the period of the high-frequency field; if either of these quantities increases the intensity also increases. It is undesirable to increase the current strength of the beam because of possible effects on the width of the resonance line and undesirable secondary effects, namely x-ray radiation and resonator heating. Hence, it is more expedient to increase the number of orbits traversed by an electron in the resonator before it is no longer affected by the effect in question.

There are two effects which might complicate the experiment being discussed: (a) radiation from fast electrons in the magnetic field, which can broaden the resonance line and lead to (in the case in which the electrons move in a volume bounded by conducting walls) the appearance of spurious lines and (b) the effect of the electric field in the resonator on the cyclotron resonance absorption frequency. It has been shown experimentally and theoretically that the intensity of the indicated radiation is inconsequential up to energies of several megavolts and that its effect on the resonance line can be neglected. However, the effect of the electric field on frequency must be taken into account; this correction can be evaluated by measurements at two neighboring frequencies at two values of the magnetic field with the same accelerating potential.

The author is indebted to Ia. P. Terletskii for calling his attention to Ref. 8 and for many valuable discussions.

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Translated by H. Lashinsky
42

GENERALIZATION OF GAUGE INVARIANCE AND COMBINED INVERSION

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Submitted to JETP editor February 15, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 269-271 (July, 1957)

At the present time it is generally believed that certain particles (π-mesons, K-particles, hyperons, etc.) can exist in different charge states. The appropriate wave functions may be written in the form

$$\psi = \sum_Q c_Q \psi_Q, \tag{1}$$

where ψ_Q is the wave function associated with the state with charge Q and the summation is taken over all possible values of the charge (including zero). A description of particles of this type requires an extension of the notion of gauge invariance. In particles which exist in only one charge state a gauge transformation leads to the multiplication of the wave functions by a trivial phase factor $e^{i\alpha Q}$. In the general case (1) the gauge transformation is obviously given by the formula

$$\psi' = \sum_Q c_Q e^{i\alpha Q} \psi_Q. \tag{2}$$

We may note that the function ψ' depends parametrically on α.

The invariance requirement for infinitely small gauge transformations (2) leads to the law of conservation of charge; this relation is written in the form of the continuity equation for electric current:

$$j_\mu = \sum_Q Q c_Q c_Q^* (\psi_Q^* \Gamma_\mu \psi_Q - \psi_Q \Gamma_\mu \psi_Q^*). \tag{3}$$

Here, we have used the orthogonality of the different charge states ψ_Q and the usual Lagrangian, which gives a first order equation for the wave functions ψ. We may note that the current j_μ is independent of the parameter α, as is to be expected.

Charge conjugation

$$\psi_Q' = \psi_{-Q} \tag{4}$$

in the theory of particles which have only one charge state is defined so as to make the current and charge change sign. In the theory of bosons, with several charge states, the following transformation must be carried out for a change of sign of the current and charge.

$$\psi_Q' = \psi_{-Q}; \quad c_Q' = c_{-Q}, \tag{5}$$

for the case of a single charge state this expression leads to Eq. (4). We may note that charge conjugation (5) is equivalent to the transformation $\alpha' = -\alpha$.

As is well known, the wave functions of particles which exist in only one charge state can be subjected to a gauge transformation in which α is an arbitrary function of the coordinates. In the extension of this transformation to the general case (1) the parameter α is replaced by the quantity α' which is an arbitrary function of coordinates and, in general, the parameter α:

$$\alpha' = f(x, \alpha). \tag{6}$$

In a theory of particles which are free to assume different charge states we must require invariance under generalized gauge transformations (6) and the Lorentz group