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GENERALIZATION OF GAUGE INVARIANCE AND COMBINED INVERSION

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AT the present time it is generally believed that certain particles (π-mesons, K-particles, hyperons, etc.) can exist in different charge states. The appropriate wave functions may be written in the form

$$\psi = \sum_Q c_Q \psi_Q, \tag{1}$$

where ψ_Q is the wave function associated with the state with charge Q and the summation is taken over all possible values of the charge (including zero). A description of particles of this type requires an extension of the notion of gauge invariance. In particles which exist in only one charge state a gauge transformation leads to the multiplication of the wave functions by a trivial phase factor $e^{i\alpha Q}$. In the general case (1) the gauge transformation is obviously given by the formula

$$\psi' = \sum_Q c_Q e^{i\alpha Q} \psi_Q. \tag{2}$$

We may note that the function ψ' depends parametrically on α.

The invariance requirement for infinitely small gauge transformations (2) leads to the law of conservation of charge; this relation is written in the form of the continuity equation for electric current:

$$j_\mu = \sum_Q Q c_Q c_Q^* (\psi_Q^* \Gamma_\mu \psi_Q - \psi_Q \Gamma_\mu \psi_Q^*). \tag{3}$$

Here, we have used the orthogonality of the different charge states ψ_Q and the usual Lagrangian, which gives a first order equation for the wave functions ψ. We may note that the current j_μ is independent of the parameter α, as is to be expected.

Charge conjugation

$$\psi_Q' = \psi_{-Q} \tag{4}$$

in the theory of particles which have only one charge state is defined so as to make the current and charge change sign. In the theory of bosons, with several charge states, the following transformation must be carried out for a change of sign of the current and charge.

$$\psi_Q' = \psi_{-Q}; \quad c_Q' = c_{-Q}, \tag{5}$$

for the case of a single charge state this expression leads to Eq. (4). We may note that charge conjugation (5) is equivalent to the transformation $\alpha' = -\alpha$.

As is well known, the wave functions of particles which exist in only one charge state can be subjected to a gauge transformation in which α is an arbitrary function of the coordinates. In the extension of this transformation to the general case (1) the parameter α is replaced by the quantity α' which is an arbitrary function of coordinates and, in general, the parameter α:

$$\alpha' = f(x, \alpha). \tag{6}$$

In a theory of particles which are free to assume different charge states we must require invariance under generalized gauge transformations (6) and the Lorentz group

$$x' = Lx. \tag{7}$$

In this connection the problem arises of assigning the transformation laws for quantities which are transformed by (6) and (7), which we will call the general group.

Let $f(x, \alpha)$ be an arbitrary numerical function of coordinates x and the parameter α. Then, under transformations of the general group, the quantity $\partial f/\partial\alpha'$ is a linear function of the quantities $\partial f/\partial\alpha$ and $\partial f/\partial x_k$. Hence, we will define a tensor of the general group as a quantity that transforms like products of $\partial f/\partial x_k$ and $\partial f/\partial\alpha$. The quantity $\partial f/\partial\alpha$ appears in the theory since $-i\partial/\partial\alpha$ is essentially the charge operator.

Any tensor of the general group transforms as:

$$T_{\underbrace{0\ldots0}_{s}\, i\ldots k} \sim (\partial f/\partial\alpha)^s (\partial f/\partial x_i) \ldots (\partial f/\partial x_k). \tag{8}$$

In particular, in tensor analysis of the general group there is a totally anti-symmetric "tensor," one component of which we designate by ϵ. It is well known that in the transformations of the general group it is multiplied by Jacobian J of the transformation. Furthermore, from a tensor of the second rank T we can form $\det T$ which under transformations of the general group is multiplied by the square of the Jacobian J^2. Hence, a quantity of the type

$$P = \varepsilon/\sqrt{\det T}$$

is multiplied by the sign of the Jacobian under transformation of the general group and plays the role of a pseudoscalar of the general group.

In the theory of particles which exist in only one charge state we require invariance under the Lorentz group with positive Jacobian and under infinitely small gauge transformations; from this requirement follows conservation of charge. In the theory considered here this requirement is extended in a unique manner: it becomes a requirement for invariance under transformations of the general group with positive Jacobian.

If we require from the theory invariance under transformations of the general group with any sign of the Jacobian, the Lagrangian must be a scalar with respect to the general group. However, experiments on K-meson decay indicate that in decays of this type the theory need not be invariant under space inversion, i.e., under the transformation $x' = -x$, $\alpha' = \alpha$. Whence it follows that in a theory which describes K-mesons we cannot require invariance under the transformations of the general group with negative Jacobian. In this theory the Lagrangian must be the sum of two terms: a gauge scalar and a gauge pseudoscalar $L = S + P$. Thus the Lagrangian will not be invariant under the transformation $\alpha' = -\alpha$, i.e., under charge conjugation the sign of P is changed.

Thus, we may formulate the following general statement. A theory which is not invariant under a change of the orientation of space directions will not be invariant under a change of the sign of the charge. However, such a theory is invariant with respect to the Landau combined inversion, i.e., under the simultaneous transformations $x' = -x$ and $\alpha' = -\alpha$.

We now see that the combined inversion hypothesis, suggested by Landau, is equivalent to the requirement that the theory be invariant with respect to the group of generalized gauge transformations with positive Jacobian.

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