uum space of the Dewar on a fixed heat conductor. A detailed report on work being carried out will be published in the near future.

[1] V. L. Ginzburg and G. P. Motulevich, Usp. Fiz. Nauk **55**, 469 (1955).
[2] K. G. Ramanathan, Proc. Phys. Soc. **A65**, 532 (1952).
[3] M. A. Biondi, Phys. Rev. **96**, 534 (1954); **102**, 964 (1956).
[4] P. G. Strelkov, J. Exptl. Theoret. Phys. (U.S.S.R) **24**, 248 (1953).

Translated by H. Lashinsky
51

CONVERGENCE OF THE PERTURBATION-THEORY SERIES FOR A NON-RELATIVISTIC NUCLEON

A. G. GALANIN and IU. N. LOKHOV

Submitted to JETP editor April 4, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 285-286 (July, 1957)

In the quantum theory of interacting fields the perturbation-theory series is an asymptotic series.[1] On the other hand, the problem of a non-relativistic nucleon interacting with a neutral scalar-meson field (n.s. theory) has an exact solution.[2] In this case the exact Green's function of the nucleon is an analytic function of the coupling constant with an infinite radius of convergence (in the coordinate representation; in the momentum representation the radius of convergence is finite[3]).

The problem of a non-relativistic nucleon which interacts with a symmetric pseudoscalar meson field (s.p.s. theory) does not as yet have an exact analytical solution. In spite of this fact an analysis of the convergence of the perturbation-theory series can be carried out.

For the interaction

$$(g_0/2\mu)\,\psi^*[(\sigma\nabla)(\tau_i\varphi_i)]\,\psi \tag{1}$$

there is a definite rule for writing the Feynman diagrams. The rule can be completely stated if we write only one matrix element, corresponding to the self-energy diagram in the first approximation

$$M(E) = -i\left(\frac{g_0}{2\mu}\right)^2\frac{1}{8\pi^2}\int\frac{(\sigma\,\mathbf{k})\,\tau_j\,(\sigma\,\mathbf{k})\,\tau_j\,d^3k\,dk_4}{(E-k_4-i\eta)\,(k_4^2-\mathbf{k}^2-\mu^2+i\varepsilon)}, \tag{2}$$

where E is the nucleon energy (we neglect the kinetic energy of the nucleon), and the integration extends to some upper limit. We go around the poles in the complex plane K_4 by infinitesimal increments $\epsilon > 0$ and $\eta > 0$. The symbol η corresponds to the appropriate Green's function in the non-relativistic case.[4] In the upper half plane of k_4 there is only one pole $k_4 = -\sqrt{\mathbf{k}^2+\mu^2}$. Closing the integration path in the k_4 plane in an upward direction, it is easy to calculate the integral over k_4 in (2). After integrating over the angles we have

$$M(E) = \frac{1}{6}\left(\frac{g_0}{2\mu}\right)^2\sigma_i\tau_j\sigma_i\tau_j\int_0^{\Lambda}k^4dk/\left(E+\sqrt{k^2+\mu^2}\right)\sqrt{k^2+\mu^2}, \tag{3}$$

where Λ is the cut-off momentum. In what follows we neglect the meson mass; going over to dimensionless variables of integration we have

$$M(z) = \frac{1}{6}\left(\frac{g_0\,\Lambda}{2\,\mu}\right)^2\Lambda\sigma_i\tau_j\sigma_i\tau_j\int_0^1\frac{x^3dx}{z+x}, \tag{4}$$

($z = E/\Lambda$). The rules for forming more complicated diagrams can be obtained easily by generalizing this example.

Similarly, in n.s. theory with the interaction $g_1\psi\varphi\psi$, we obtain in place of (4)

$$\frac{1}{2} g_1^2 \Lambda \int_0^1 \frac{x dx}{z + x}. \quad (5)$$

In this case the theory can be renormalized and the cut-off momentum Λ can approach infinity after renormalization. However, since the expressions under the integrals are positive (for a given diagram), by introducing a finite cut-off momentum we reduce the magnitude of the integral and improve the convergence of the power series in g_1^2. We now compare (4) and (5) and the more complicated diagrams. We denote an arbitrary diagram of order n s.p.s. theory by $I_n^{(p)}$ and the same diagram in n.s. theory by $I_n^{(s)}$. Inasmuch as all the variables of integration in (4) and in the more complicated diagrams are less than unity, we have $I_n^{(p)} < I_n^{(s)} A_n$ if $g_1^2 = {}^1/_3 (g_0 \Lambda/2\mu)^2$ and A_n denotes a combination of the matrices σ and τ. Since the matrices σ and τ have the same properties,

$$A_n = (\sigma_{i_1} \sigma_{i_2} \ldots \sigma_{i_{2n}})^2, \quad (6)$$

in which the indices contain n identical pairs for which we carry out a summation from 1 to 3. Hence the expression in the parentheses in (6) consists of 3^n terms. We consider one of these terms. Let the first matrix be σ_x while the next successive matrix after σ_x stands in the k'th position. Having made k—2 substitutions we change sign k—2 times and, further, since $\sigma_x^2 = 1$, we have reduced the number of matrices by 2. Carrying out this operation n times, we calculate one of the terms in $\sigma_{i_1} \ldots \sigma_{i_{2n}}$. The quantity A_n increases only if we drop the factor $(-1)^{k-2}$ which arises from the fact that the σ matrices do not commute. Since the number of terms is 3^n, $A_n < 3^{2n}$ and

$$I_n^{(p)} < I_n^{(s)}, \quad (7)$$

if now $g_1^2 = 3(g_0 \Lambda/2\mu)^2$.

Thus, the power series in g_1^2 in n.s. theory is the majorant for s.p.s. theory.

In n.s. theory the Green's function has, in the p-representation, a finite radius of convergence (cf. Ref. 3) if the cut-off momentum $\Lambda \to \infty$. If Λ remains finite, however, (the inequality in Eq. (7) is obtained specifically in this case), the radius of convergence in n.s. theory also becomes infinite for a Green's function written in the p-representation. Hence, from Eq.(7) it follows that the perturbation-theory series in s.p.s. theory has an infinite radius of convergence.

The convergence of the perturbation series in non-relativistic theories is due to the fact that the integral which corresponds to a diagram of n'th order, has a denominator of the following form:

$$(E + k_1)(E + k_1 + k_2) \ldots (E + k_1 + \ldots + k_n),$$

where k_i is the energy of the i'th virtual meson ($k_i > 0$).

This denominator leads to a very rapid, factorial reduction of each diagram of n'th order with increasing n. This means that the n'th term of the power series in g_0^2 falls off with increasing n in spite of the fact that the number of diagrams of n'th order increases as n!

Thus, in non-relativistic theories it is assumed that the energy of a nucleon in virtual states increases with an increase in the number n. It is obvious that relativistic theories, in which pair production is possible, will differ in a fundamental way from non-relativistic theories at this point. Hence, in the example of a relativistic theory chosen by Thirring each matrix element of n'th order does not fall off factorially with increasing n and the perturbation theory series becomes asymptotic.

The authors wish to express their gratitude to B. L. Ioffe for some illuminating discussions.

[1] W. Thirring, Helv. Phys. Acta **26**, 33 (1953).
[2] S. F. Edwards and R. E. Peierls, Proc. Roy Soc. (London) **224**, 24 (1954).
[3] Проблемы современной физики, (Problems of Contemporary Physics) IIL **3** (1955) p. 119.
[4] R. F. Feynman, Phys. Rev. **76**, 749 (1946).

Translated by H. Lashinsky
52