

and the electric current density

$$j = (\sqrt{2\pi}/3\Gamma(3/4)) ne^{1/2}E^{1/2}/\sigma^{1/2}N_r^{1/2}(m\mu)^{1/4}. \quad (19)$$

Finally, for $eEl \gg (\mu/m)^{1/2}kT$ the second term in the expression for A_2^r predominates, and $A_2^r = 2p^3m/15l\mu^2$.

In this case* we obtain for the distribution function the value

$$f_0 \sim \exp\{-p^6/15m\mu^2kT(eEl)^2\}$$

and for the current density

$$j = \frac{2\Gamma(1/3)}{3^{1/2}\sigma^{1/2}V\pi} \frac{ne^{1/2}E^{1/2}}{(N_r\sigma)^{1/2}(m\mu^2kT)^{1/2}}. \quad (20)$$

These formulas contain the unknown parameters λ , σ , the effective mass m , and also the number of electrons per unit volume n . If it were possible to perform measurements in all three ranges of electric field dependence, all of these unknown parameters could be determined. It should be remembered, however, that the magnitudes of the parameters can cause overlapping of the phonon and roton regions and thereby complicate somewhat the interpretation of the experimental results. For example, the condition $(\mu/m)^{1/2}kT \ll eEl$ corresponds to $eEl \gg \Delta$, and the creation of several rotons, as well as phonons, can accompany the scattering of electrons by a roton. In this case the dependence (17) will be observed in place of (20). Experimental investigation of the behavior of particles of small effective mass in He II would be of definite interest.

I would like to express my deep gratitude to L. D. Landau and I. M. Khalatnikov for their helpful discussions of this work.

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STATIONARY CONVECTION IN A PLANE LIQUID LAYER NEAR THE CRITICAL HEAT TRANSFER POINT

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Thermal convection which arises above the limit of stability is investigated for a plane liquid layer. A dependence of the amplitude of motion on a parameter which characterizes the departure from critical conditions has been obtained. Possible symmetry of the flow is discussed.

*An analogous situation can in principle arise for the scattering of electrons by heavy atoms. Here an $\sim E^{2/3}$ dependence for the current is also obtained, but with different numerical coefficients; this is understandable, since A_2 is different for ions. In this case, however, the electrons have sufficient energy for ionization, which must be taken into account.

HEAT transfer in a plane liquid layer with constant temperature gradient was studied experimentally by Benard,¹ who showed that for certain values of a downward-directed temperature gradient pure thermal conduction is replaced by convection with certain characteristics in a stationary mode; the flow is divided among regular vertical hexagonal prisms and thus possesses hexagonal symmetry. The critical gradient at which convection occurs has been calculated in several papers (see Ref. 2), but the utilization of linearized hydrodynamic equations permitted the determination of only the magnitude of the periods. The determination of the symmetry and amplitude of the motion for supercritical gradients requires study of the nonlinear equations. The present paper presents the results of such an investigation for modes close to the critical value.

A plane horizontal liquid layer of thickness h is bounded above and below by two planes between which a constant temperature difference is maintained. The boundary conditions can be taken in three forms: (a) two rigid planes, (b) one rigid plane and one free surface, and (c) two free surfaces. The last case is distinguished by the simplicity of its equations. Benard's experiments were performed under conditions (b). We shall write the basic equations of stationary convection as follows (see Ref. 3):

$$(\mathbf{v}\nabla)\mathbf{v} = -\frac{1}{\rho}\frac{\partial p}{\partial x} - \beta g\theta + \nu\Delta\mathbf{v}, \quad \text{div } \mathbf{v} = 0, \quad (\mathbf{v}\nabla)\theta - Av_z = \chi\Delta\theta. \quad (1)$$

Here p and θ are small deviations, due to convection, from the equilibrium pressure P and temperature T :

$$P = p_0 - \rho gz + p; \quad T = -Az + T_0 + \theta,$$

\mathbf{v} is the velocity of the motion; $\beta = -\rho^{-1}(\partial\rho/\partial T)_p$. The boundary conditions at the rigid surface are:

$$\theta = 0, \quad v_z = 0; \quad \partial v_z / \partial z = 0 \quad (v_x = v_y = 0), \quad (1a)$$

and at the free surface:

$$\theta = 0; \quad v_z = 0; \quad \partial^2 v_z / \partial z^2 = 0 \quad (\sigma_{xz} = \sigma_{yz} = 0). \quad (1b)$$

For convenience we shall retain the vector notation only for variables in the horizontal plane $\mathbf{x} = (x, y)$, separating the z coordinate; for all quantities we shall employ a Fourier expansion in these variables:

$$\mathbf{v}(x, y, z) = \sum_k \mathbf{v}_k(z) e^{i\mathbf{k}\mathbf{x}} \quad (2)$$

and similarly for v_z , p and θ . By virtue of the boundary conditions $v_{0z} = 0$. The velocity $\mathbf{v}_k(z)$ is resolved into components which are parallel and perpendicular to the wave vector \mathbf{k} :

$$\mathbf{v}_k = \tilde{\mathbf{v}}_k + \mathbf{k}(\mathbf{k}\mathbf{v}_k)/k^2.$$

The second and third equations of (1) become

$$i(\mathbf{k}\mathbf{v}_k) + dv_{kz}/dz = 0, \quad (3)$$

$$\chi \left(\frac{d^2}{dz^2} - k^2 \right) \theta_k + Av_{kz} = \sum_{k'+k''=k} \left\{ i(\mathbf{k}\mathbf{v}_{k'})\theta_{k''} + \frac{d}{dz}(v_{k'z}\theta_{k''}) \right\}. \quad (4)$$

Eliminating the pressure in the first equation,

$$\frac{1}{\rho} p_k = \frac{\nu}{k^2} \left(\frac{d^2}{dz^2} - k^2 \right) \frac{dv_{kz}}{dz} + \frac{i}{k^2} \sum_{k'+k''=k} \left\{ i(\mathbf{k}\mathbf{v}_{k'}) (\mathbf{k}\mathbf{v}_{k''}) + \frac{d}{dz}(v_{k'z}(\mathbf{k}\mathbf{v}_{k''})) \right\},$$

we also obtain

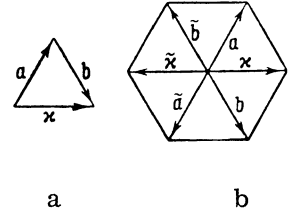
$$-\nu \left(\frac{d^2}{dz^2} - k^2 \right)^2 v_{kz} + k^2 \beta g \theta_k = \sum_{k'+k''=k} \left\{ k^2 \left[i(\mathbf{k}\mathbf{v}_{k'}) v_{k''z} + \frac{d}{dz}(v_{k'z} v_{k''z}) \right] + i \frac{d}{dz} \left[i(\mathbf{k}\mathbf{v}_{k'}) (\mathbf{k}\mathbf{v}_{k''}) + \frac{d}{dz}(v_{k'z}(\mathbf{k}\mathbf{v}_{k''})) \right] \right\}, \quad (5)$$

$$\nu \left(\frac{d^2}{dz^2} - k^2 \right) \tilde{\mathbf{v}}_k = \sum_{k'+k''=k} \left\{ i(\mathbf{k}\mathbf{v}_{k'}) \left[\mathbf{v}_{k''} - \frac{\mathbf{k}(\mathbf{k}\mathbf{v}_{k''})}{k^2} \right] + \frac{d}{dz} \left(v_{k'z} \left[\mathbf{v}_{k''} - \frac{\mathbf{k}(\mathbf{k}\mathbf{v}_{k''})}{k^2} \right] \right) \right\}. \quad (6)$$

Finally, we give the equations for the Fourier components with $k = 0$:

$$\chi \frac{d^2 \theta_0}{dz^2} = \frac{d}{dz} \sum_{k'} (v_{k'z} \theta_{-k'}); \frac{d}{dz} \left\{ \nu \frac{dv_0}{dz} - \sum_{k'} v_{k'z} v_{-k'} \right\} = 0; \quad \frac{1}{\rho} \frac{dp_0}{dz} = \beta g \theta_0 - \frac{d}{dz} \sum_{k'} (v_{k'z} v_{-k'}).$$

Before proceeding to study the equations which have been derived we shall determine the vectors \mathbf{k} of the Fourier series (2). As was shown by the investigation of stability in Ref. 2, when the critical temperature gradient is attained there arises convective motion of the form $f(z) e^{i\mathbf{k}\mathbf{x}}$ with a certain value $k = \kappa_0$ which depends on the experimental conditions (a, b or c). In (2), and similarly in the expansions for v_z and θ , we shall call $v_{\mathbf{k}z}$ with $k = \kappa_0$ the principal term. It is easy to determine all such vectors of the flow. Indeed, as can be seen from (3)–(6), corrections to the principal terms with $k = \kappa_0$ will consist of terms with wave vectors which are again equal in magnitude to κ_0 . Therefore, if (2) contains a term with the wave vector $\kappa (|\kappa| = \kappa_0)$ there must also be terms with the vectors \mathbf{a} and \mathbf{b} ($a = b = \kappa_0$), thus forming the equilateral triangle of Fig. a. It is clear that there are altogether six such vectors $\kappa, \mathbf{a}, \mathbf{b}, \tilde{\kappa}, \tilde{\mathbf{a}}, \tilde{\mathbf{b}}$, which are represented together in Fig. b. The Fourier components which are thus designated, for all quantities, comprise the set of principal terms of the solution; the remaining terms correspond to wave vectors representing various sums of the six principal vectors and will be small for temperature gradients near the critical value.



We shall write the principal terms in the form

$$v_{iz} = X_i f(z); \quad \theta_i = X_i \varphi(z),$$

where²

$$\{(d^2/dz^2 - \kappa_0^2) + (A\beta g \kappa_0^2 / \chi \nu)\} f(z) = 0; \quad \varphi(z) = (\nu / \kappa_0^2 \beta g) (d^2/dz^2 - \kappa_0^2) f(z). \tag{7}$$

It will be sufficient hereinafter to confine ourselves to terms of the third order in X , i.e., to take into account all triple combinations of the principal vectors. This means that all quantities $v_{\mathbf{k}z}$ and $\theta_{\mathbf{k}}$ can be obtained up to terms of the second order in X ; in other words, in addition to the corrections $\delta v_{\mathbf{k}z}$ and $\delta \theta_{\mathbf{k}}$ (and similarly for \mathbf{a}, \mathbf{b} etc.) to the expressions in (7), it is necessary to obtain the Fourier components for the velocities and temperature corresponding to wave vectors each of which is a sum of two principal vectors; for example $\mathbf{f} = \mathbf{\kappa} + \mathbf{b}, 2\mathbf{\kappa}, \mathbf{o}$ etc. We note that by virtue of the equation of continuity (3) expressions of the form $(\mathbf{q}\mathbf{v}_{\mathbf{l}})$ in (4) and (5) can be rewritten as

$$(\mathbf{q}\mathbf{v}_{\mathbf{l}}) = (\mathbf{q}\tilde{\mathbf{v}}_{\mathbf{l}}) + i(\mathbf{q}\mathbf{l}) l^{-2} dv_{l_z} / dz.$$

From (6) it follows, to terms of the second order, that

$$\nu \left(\frac{d^2}{dz^2} - l^2 \right) (\mathbf{q}\tilde{\mathbf{v}}_{\mathbf{l}}) = i \sum_{k'+k''=l} \left\{ \left[\frac{(\mathbf{q}\mathbf{k}'')}{k''^2} - \frac{(\mathbf{q}\mathbf{l})(\mathbf{l}\mathbf{k}'')}{l^2 k''^2} \right] \left[\frac{d}{dz} (v_{k'z} \frac{dv_{k''z}}{dz}) - \frac{(\mathbf{l}\mathbf{k}')}{k'^2} \frac{dv_{k'z}}{dz} \frac{dv_{k''z}}{dz} \right] \right\}.$$

Substituting the expressions in (7) we see that for all required \mathbf{l} and \mathbf{q} the right-hand side vanishes. Second-order terms in $(\mathbf{q}\tilde{\mathbf{v}}_{\mathbf{l}})$ vanish so that in (4) and (5) we can write

$$(\mathbf{q}\mathbf{v}_{\mathbf{l}}) \rightarrow i(\mathbf{q}\mathbf{l}) l^{-2} dv_{l_z} / dz.$$

For brevity we introduce the notation

$$D_{\mathbf{x}} = (d^2/dz^2 - \kappa_0^2).$$

From (4) and (5), including third-order terms in X , we obtain

$$D_{\mathbf{k}}^3 \theta_{\mathbf{k}} + \frac{k^2 A \beta g}{\chi \nu} \theta_{\mathbf{k}} = \frac{1}{\chi} \sum_{k'+k''=k} D_{\mathbf{k}}^2 \left\{ \frac{d}{dz} (v_{k'z} \theta_{k''}) - \frac{(\mathbf{k}\mathbf{k}')}{k'^2} \frac{dv_{k'z}}{dz} \theta_{k''} \right\} + \frac{A}{\chi \nu} \sum_{k'+k''=k} \left\{ k^2 \left[\frac{d}{dz} (v_{k'z} v_{k''z}) - \frac{(\mathbf{k}\mathbf{k}')}{k'^2} \frac{dv_{k'z}}{dz} v_{k''z} \right] + \frac{(\mathbf{k}\mathbf{k}'')}{k''^2} \frac{d}{dz} \left[\frac{(\mathbf{k}\mathbf{k}')}{k'^2} \frac{dv_{k'z}}{dz} \frac{dv_{k''z}}{dz} - \frac{d}{dz} (v_{k'z} \frac{dv_{k''z}}{dz}) \right] \right\}. \tag{8}$$

The magnitude of the dimensionless parameter $\gamma = A\beta g h^4 / \chi \nu$ determines the possibility of the existence of stationary convective motion. The critical value γ_0 at which instability arises² is for two rigid surfaces ≈ 1708 , for one free surface ≈ 1100 and for two free surfaces ≈ 657 . We shall write (8) for the κ component as follows:

$$D_{\kappa}^3 \theta_{\kappa} + \gamma_0 \theta_{\kappa} = -\Delta \gamma \theta_{\kappa} + R_{\kappa} (\theta^3)$$

where $R_\kappa(\theta^3)$ is the right-hand side of (8) and $\Delta\gamma = \gamma - \gamma_0$. The homogeneous equation has the solution $\varphi(z)$, with γ_0 possessing the same meaning. The condition

$$\int f \{R_x(\theta^3) - \Delta\gamma\theta_x\} dz = 0, \quad (9)$$

which is required for the solution of (8), leads to the determination of the amplitude of motion X . This condition has the form

$$\begin{aligned} \frac{\chi_0^2 \Delta\gamma}{h^4} X_x \int f \varphi dz = \frac{1}{\chi} \int (D_x^2 f) \sum_{k'+k''=x} \left\{ \frac{d}{dz} (v_{k'z} \theta_{k''}) - \frac{(\chi k')}{k'^2} \frac{dv_{k'z}}{dz} \theta_{k''} \right\} dz + \frac{A\chi_0^2}{\chi^2} \int f \sum_{k'+k''=x} \left\{ \frac{d}{dz} (v_{k'z} v_{k''z}) - \frac{(\chi k')}{k'^2} \frac{dv_{k'z}}{dz} v_{k''z} \right\} dz \\ + \frac{A}{\chi^2} \int \frac{df}{dz} \sum_{k'+k''=x} \left\{ \frac{(kk'')}{k''^2} \left[\frac{d}{dz} (v_{k'z} \frac{dv_{k''z}}{dz}) - \frac{(kk')}{k'^2} \frac{dv_{k'z}}{dz} \frac{dv_{k''z}}{dz} \right] \right\} dz. \end{aligned}$$

Second-order terms in X drop out, as is shown by substitution of (7) into the right-hand side;* for the purpose of calculating third-order terms it is necessary to obtain the corrections to v_z and θ . Such corrections appear for the following quantities (see Fig. b):

$$\begin{aligned} \delta\theta_a = X_{\tilde{c}} X_x \xi_1(z); \quad \delta\theta_b = X_{\tilde{c}} X_x \xi_1(z); \quad \delta\theta_f = X_x X_b \xi_2(z); \quad \delta\theta_g = X_a X_x \xi_2(z); \quad \delta\theta_{2x} = X_x^2 \xi_3(z); \\ \delta\theta_0 = (|X_a|^2 + |X_b|^2 + |X_x|^2) \xi_4(z). \quad \delta v_{az} = X_{\tilde{c}} X_x \eta_1(z); \quad \delta v_{bz} = X_{\tilde{c}} X_x \eta_1(z); \quad \delta v_{fz} = X_x X_b \eta_2(z); \\ \delta v_{gz} = X_x X_a \eta_2(z); \quad \delta v_{2x} = X_x^2 \eta_3(z). \end{aligned}$$

Using these designations (9) becomes

$$\begin{aligned} \frac{\chi^2 \Delta\gamma}{\beta g h^4} \int (f D^2 f) dz = (|X_a|^2 + |X_b|^2) \int \left\{ \frac{A}{2v} [Df'f(\eta_1 + \eta_2) + 2f'Df\eta_1] - f D^2 f'(\xi_1 + \xi_2 + \xi_4) - \frac{D^2 f}{2} f'(\xi_1 - \xi_2 + 2\xi_4) \right\} dz \\ + |X_x|^2 \int \left\{ \frac{A}{2v} [f''f - f'f'] \eta_3 + \left[\xi_3 (D^2 f f' - D^2 f' f) - \xi_4 \frac{d}{dz} (f D^2 f) \right] \right\} dz \end{aligned}$$

where primes denote differentiation with respect to z . Two other equations are then obtained directly by the substitutions: $\kappa \rightarrow a$, $a \rightarrow b$, $b \rightarrow \kappa$, or $\kappa \rightarrow b$, $a \rightarrow \kappa$, $b \rightarrow a$.

The solution of these equations is obviously $|X_\kappa| = |X_a| = |X_b|$; the amplitude of convective motion is then proportional to the square root of the parameter $\Delta\gamma$ which characterizes the supercritical heat transfer mode. But our equations contain only the absolute value of X so that the phase relation cannot be determined. It is clear how this has come about: when we write (9) for θ_κ , say, we must select in R all combinations corresponding to any three principal vectors whose vector sum is x . It is easily seen from the figure that any such combination must be of the form $X_\kappa |X_\kappa|^2$ so that X_κ is cancelled and (10) contains only the square of the modulus of X . Fourth-order terms in X in (9) obviously drop out. In the fifth order it is possible to obtain a combination which is not proportional to X_κ : the sum $2a + 2b + \tilde{\kappa}$ in the right-hand side gives a term which is proportional to $X_\kappa \sim X_a^2 X_b^2$. Writing $X = |X| e^{i\delta}$, we obtain because of the reality of the coefficients the following phase relation:

$$\delta_a + \delta_b - \delta_x = 0, \quad \pi/2.$$

In the first case it is possible to make all phases equal to zero by shifting the coordinate origin. The principal term in the solution becomes

$$v_z = B \left(\frac{\chi}{v} \right) f(z) \sqrt{\frac{\Delta\gamma}{\gamma}} \left\{ \cos \alpha_0 y + \cos \frac{\alpha_0}{2} (y + \sqrt{3}x) + \cos \frac{\alpha_0}{2} (y - \sqrt{3}x) \right\} \quad (11a)$$

where the y axis is parallel to κ . This solution possesses hexagonal symmetry; in the (x, y) plane it represents a periodic structure of regular hexagonal prisms of the length $4\pi/\kappa_0$ along each side of the base; the liquid flows upward at the center of each prism and downward along the walls.²

In the second case, with a suitable choice of the coordinate origin, the solution can be written as follows:

$$v_z = B \left(\frac{\chi}{v} \right) f(z) \sqrt{\frac{\Delta\gamma}{\gamma}} \left\{ \sin \frac{\alpha_0}{2} (y + \sqrt{3}x) + \sin \frac{\alpha_0}{2} (y - \sqrt{3}x) - \sin \alpha_0 y \right\}. \quad (11b)$$

This flow permits three-fold rotations around the vertical axis (the z axis) and is symmetric with respect to the x axis, but changes sign with the substitution $y \rightarrow -y$. In the horizontal plane it breaks up

*For symmetrical boundary conditions this is clear from simple considerations, since the solution of $f(z)$ is symmetrical with respect to $z = 0$.

into alternate equilateral triangles of side length $4\pi/\sqrt{3}\kappa_0$, in each of which the liquid flows upward or downward, respectively.

A decision as to which of the two types of flow will occur in reality could be reached theoretically only by investigating fifth-order terms in X in the basic equations of convection, but in actuality it would be difficult to do this. For a liquid layer on a rigid plate experiment¹ favors the first type of flow.

The equations for the ξ functions are

$$\begin{aligned} \left[D^3 + \frac{\kappa_0^2 \gamma}{h^4} \right] \xi_1 &= \frac{2\nu}{\kappa_0^2 \beta g \chi} \left\{ D^2 \left(f D^2 f' + \frac{f'}{2} D^2 f \right) - \frac{A \beta g \kappa_0^2}{\nu^2} \left(f D f' + \frac{f'}{2} D f \right) \right\}; \\ \left[(D - 2\kappa_0^2)^3 + \frac{3\kappa_0^2 \gamma}{h^4} \right] \xi_2 &= \frac{2\nu}{\kappa_0^2 \beta g \chi} \left\{ (D - 2\kappa_0^2)^2 \left(f D^2 f' - \frac{f'}{2} D^2 f \right) - \frac{3A \beta g \kappa_0^2}{2\nu^2} f D f' \right\}; \end{aligned} \quad (12)$$

$$\left[(D - 3\kappa_0^2)^3 + \frac{4\kappa_0^2 \gamma}{h^4} \right] \xi_3 = \frac{\nu}{\kappa_0^2 \beta g \chi} \left\{ (D - 3\kappa_0^2)^2 (f D^2 f' - f' D^2 f) + \frac{2A \beta g \kappa_0^2}{\nu} (f' f'' - f f''') \right\}; \quad d^2 \xi_4 / dz^2 = (2\nu / \kappa_0^2 \beta g \chi) d(f D^2 f) / dz,$$

and the expressions for the η functions are

$$A\eta_1 = -\chi D \xi_1 + 2(f\varphi' + \frac{1}{2}\varphi f'); \quad A\eta_2 = -\chi(D - 2\kappa_0^2)\xi_2 + 2(f\varphi' - \frac{1}{2}\varphi f'); \quad A\eta_3 = -\chi(D - 3\kappa_0^2)\xi_3 + (f\varphi' - f'\varphi). \quad (13)$$

The calculation of these quantities subject to the boundary conditions 1a and 1b would be very laborious for the general case. The problem becomes simpler only in the somewhat hypothetical case of a liquid with two free surfaces. We present the results in so far as they are of a general character.

In this case the critical value is $\gamma_0 = 27\pi^4/4$, $\kappa_0^2 = \pi^2/2h^2$ and $f(z)$, which determines the dependence of the vertical velocity component on height, can be taken in the form

$$f(z) = \cos \frac{\pi z}{h} \quad \text{or} \quad \varphi(z) = \frac{9\nu\pi^2}{2\beta g h^2} \cos \frac{\pi z}{h}.$$

Equations (12) and (13) can be solved simply and give for ξ and η

$$\begin{aligned} \left. \begin{aligned} \xi_1 &= \frac{9\nu\pi [9 + \chi/\nu]}{52\chi\beta g h} \\ \xi_2 &= \frac{9\nu\pi [121 + 27\chi/\nu]}{2500\chi\beta g h} \\ \xi_3 &= \frac{9\nu\pi}{4\chi\beta g h} \end{aligned} \right\} \times \sin \frac{2\pi z}{h}, \quad \left. \begin{aligned} \eta_1 &= \frac{h [1 + 3\chi/\nu]}{26\pi\chi} \\ \eta_2 &= \frac{9h [3 + 11\chi/\nu]}{1250\pi\chi} \end{aligned} \right\} \times \sin \frac{2\pi z}{h}, \\ \xi_4 &= 0. \end{aligned}$$

By substituting these expressions in (10) we can determine X . For example, the temperature distribution for the first type of flow is given by

$$\theta(xyz) = \frac{4Ah}{3\pi} \sqrt{\frac{\Delta\bar{\gamma}}{\gamma}} \frac{\cos(\pi z/h)}{\sqrt{1.35 + 0.08\chi/\nu + 0.14\chi^2/\nu^2}} \left\{ \cos \frac{\pi y}{2h} + \cos \frac{\pi}{4h} (y + \sqrt{3}x) + \cos \frac{\pi}{4h} (y - \sqrt{3}x) \right\} \quad (14)$$

The development of convection is conveniently characterized by the ratio of the maximum temperature change in the (x, y) plane, $T_{\max} - T_{\min}$, to the vertical temperature difference $T_1 - T_2 = Ah$:

$$\frac{T_{\max} - T_{\min}}{T_1 - T_2} = \frac{4}{\pi} \sqrt{\frac{\Delta\bar{\gamma}}{\gamma}} \left(1.35 + 0.08 \frac{\chi}{\nu} + 0.14 \frac{\chi^2}{\nu^2} \right)^{-1/2}.$$

We note in conclusion that the form of the dependence of the coefficient in (14) on the Prandtl number is the same for any boundary conditions. This results from the fact that the solution of the linear equation $f(z) \equiv f(z/h)$ depends only on the critical value γ_0 , and also on how (12), (13) and (10) contain the different dimensional quantities. Thus the strength of stationary thermal convection in a plane liquid layer for modes near the critical point is proportional to the square root of the parameter which characterizes supercriticality, whereas dependence on the Prandtl number appears in a denominator as the square root of a polynomial of not higher than the second degree. The flow belongs to one of the two types of symmetry represented by (11a) and (11b).

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