ON THE THEORY OF A POSITIVE COLUMN IN A LONGITUDINAL MAGNETIC FIELD

M. V. KONIUKOV

Tula Pedagogical Institute

Submitted to JETP editor February 6, 1957; resubmitted March 29, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) 33, 408-411 (August, 1957)

This article considers the contraction of the positive column in a gas discharge located in an external magnetic field directed along the discharge axis.

IT has been shown experimentally[1,2] that in the positive column of a gas discharge in an external magnetic field there occurs a redistribution of the current density through the cross section. The current density increases at the axis, and decreases at the walls. A generalization[3] of Schottky's ambipolar diffusion theory to a column in a longitudinal magnetic field failed to explain the observed phenomena. Fabrikant[4] feels that this is due to insufficiencies in the boundary condition used by Tonks. Similar conclusions can be reached on the basis of the recent work of Engel and Bicerton.[5] Since the behavior of a plasma in a magnetic field is of great interest, we feel that a theoretical treatment of the current-density redistribution due to a longitudinal magnetic field is of value.

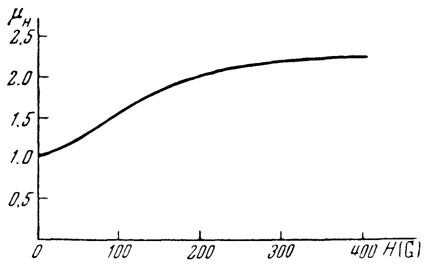

FIG. 1. The root of the transcendental equation (6) as a function of the magnetic field. The graph is for argon, with p = 0.0037 mm Hg, I = 300 ma, and R = 2.2 cm.

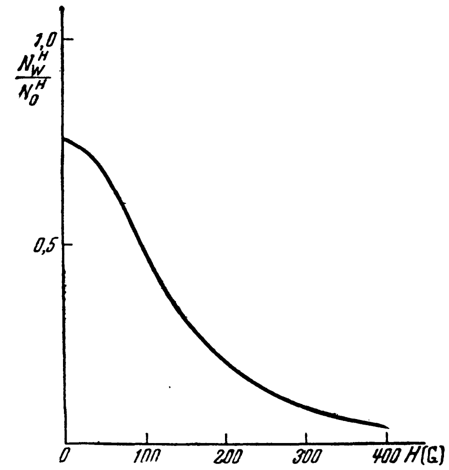

FIG. 2. Wall-to-axis concentration ratio as a function of the magnetic field in argon with the same parameters as those of Fig. 1.

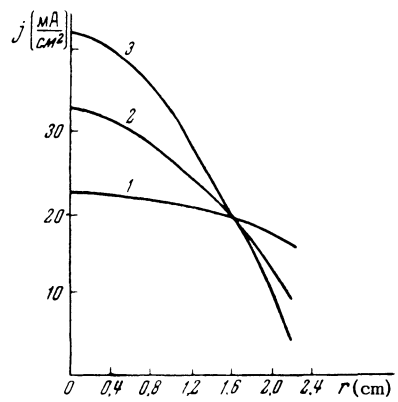

FIG. 3. Current density distribution over the cross section (with the magnetic field as a parameter). The calculations are performed for argon with the same conditions as in Fig. 1. Curve 1 — H = 0; 2 — H = 167 gauss; 3 — H = 365 gauss.

1. As in the previous work of Terletskii and the author[6] we start our consideration of the positive-column plasma with the hydrodynamic equations for ideal electron and ion gases uniformly distributed through the cross section in a neutral gas. The magnetic field is accounted for* by adding to the right side of the equations of motion the forces exerted by the magnetic field on the moving charged particles. We consider the radially symmetric case with the field directed along the Z axis. Then on the assumption of a stationary state for ambipolar diffusion,† we obtain

*For sufficiently low currents one may neglect the magnetic field due to the current, compared with the external field.

†The equations referring to flow along the axis are not included. They are not changed by the addition of the magnetic field.

$$\Theta_e \frac{\partial N_e}{\partial r} + \frac{eN_e}{m_e} E_r + \alpha_e N_e v_{er} + \frac{eN_e}{m_e c} H v_{e\varphi} = 0, \quad \alpha_e N_e v_{e\varphi} - \frac{eN_e}{m_e c} H v_{er} = 0, \quad \frac{1}{r}\frac{\partial}{\partial r}(rN_e v_{er}) = Z^H N_e,$$

$$\Theta_p \frac{\partial N_p}{\partial r} - \frac{eN_p}{m_p} E_r + \alpha_p N_p v_{pr} - \frac{eN_p}{m_p c} H v_{p\varphi} = 0, \quad \alpha_p N_p v_{p\varphi} + \frac{eN_p}{m_p c} H v_{pr} = 0, \quad \frac{1}{r}\frac{\partial}{\partial r}(rN_p v_{pr}) = Z^H N_e, \tag{1}$$

where N_e and N_p are the charged-particle concentrations, H is the magnetic field strength, m_e, v_{er}, $v_{e\varphi}$, α_e, and Θ_e are the mass, velocity components, coefficient of friction, and temperature (in ergs) of the electrons (and similarly for the positive ions), and Z^H is the number of ionization events per second per electron. After eliminating the velocities and field from Eq. (1), the assumption of quasi-neutrality $N_e \approx N_p = N$ for the plasma leads to the equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial N^H}{\partial r}\right) + \frac{Z^H}{D_a^H} N^H = 0, \tag{2}$$

for the charged-particle distribution through the cross section, where

$$D_a^H = \frac{D_e b_p + D_p b_e}{b_p\{1 + (\omega_e/\alpha_e)^2\} + b_e\{1 + (\omega_p/\alpha_p)^2\}} \tag{3}$$

is the ambipolar diffusion constant in the longitudinal magnetic field, ω_e and ω_p are the Larmor frequencies for the electrons and ions, and b_e and b_p are the mobilities of the electrons and ions in the absence of the magnetic field.

The boundary condition proposed by Schottky[7] for the walls does not lead to the current distribution found in experiment,* so that we have chosen another condition, mentioned by van de Groot and used by Granovskii[8] in de-ionization theory. If one neglects the creation of particles in a layer at the walls whose thickness is equal to the mean free path,[5] this boundary condition can be written

$$-D_a^H dN^H/dr|_{r=R} = {}^1/_2 N^H c_p - D_p^H \partial N^H/dr|_{r=R}, \tag{4}$$

where c_p is the thermal velocity of the positive ions.

2. A solution of Eq. (2) with a finite concentration on the axis is of the form

$$N^H = N_0^H J_0\left(\sqrt{Z^H/D_a^H}\, r\right), \tag{5}$$

where N_0^H is the charged-particle concentration on the axis in the presence of the magnetic field. If one accounts for the fact that $D_a^H \gg D_p^H$, Eq. (4) leads to the transcendental equation †

$$2D_a^H \mu^H / c_p R = J_0(\mu_H)/J_1(\mu_H), \tag{6}$$

which defines $\mu_H = \sqrt{Z^H/D_a^H}\, R$. Graphical solution makes it possible to determine μ_H as a function of the magnetic field (Fig. 1).

3. In order to compare the calculated results with experiment, let us consider the charged-particle concentration and current density in the tube cross section. On the assumption that the total current is the same with and without a magnetic field, as well as that the fraction of the total energy expended on ionization is field independent, we obtain the following relation between the concentrations on the axis of discharge:

$$N_0^H/N_0 = \sqrt{D_a/D_a^H} \cdot J_1(\mu)/J_1(\mu_H). \tag{7}$$

This equation makes it possible to express the charged particle distribution in the magnetic field in terms of the concentration at the axis in its absence, namely

$$N^H = N_0 \sqrt{\frac{D_a}{D_a^H}} \frac{J_1(\mu)}{J_1(\mu_H)} J_0\left(\sqrt{\frac{Z^H}{D_a^H}}\, r\right). \tag{8}$$

---

*It should be noted that for a stationary current-density distribution, the concentration increases over the whole cross section.

†For small values of the mean free path perpendicular to the tube axis, Eq. (6) can be reduced to a linear equation for μ_H. In general one cannot obtain an explicit expression for μ_H as a function of the discharge parameters and the magnetic field, and it was therefore necessary to perform a graphical solution.

Equation (8) can be used to calculate the wall-to-axis concentration ratio as a function of the magnetic field:

$$N_W^H/N_0^H = J_0(\mu_H), \tag{9}$$

where N_W^H is the charged-particle concentration at the wall. The graph of this function shown in Fig. 2 gives satisfactory qualitative agreement with experimental results for helium.*

Assuming that the current passing through the column is due only to the drift motion of the electrons and that the longitudinal gradient is determined by the equation

$$E_Z^H = (\mu_H/R)\sqrt{D_a^H/\alpha}, \tag{10}$$

we obtain the following expression for the current density distribution through the cross section:

$$j_H = \frac{I}{2\pi R^2}\frac{\mu_H}{J_1(\mu_H)}J_0\left(\frac{\mu_H}{R}r\right), \tag{11}$$

Here I is the total current passing through the cross section of the conductor. Curves of the current density distribution (Fig. 3) show that the magnetic field redistributes the current, increasing its density at the axis and decreasing it at the wall. This is in qualitative agreement with the observations of Reikhrudel' and Spivak.[1] The difference between the theoretical and experimental results can be due to uncertainties, but it would seem that it is partly caused also by the neglect of cascade processes and the magnetic-field dependence of such parameters as the electron temperature. The calculations performed indicate the significant role that processes taking place at the walls of the discharge tube play in the contraction of the column. This role is not the same at different gas pressures, and should be decreased by a pressure rise for a given magnetic field strength. The disappearance of effects due to the magnetic field, which one may expect at high pressures,† may not occur, since the current redistribution is determined not only by processes that take place at the wall. Among the determining factors, is, in particular, the magnetic-field dependence of the discharge parameters.

In conclusion I consider it my pleasant duty to thank Professor Ia. P. Terletskii and Lecturer A. A. Zaitsev for their valuable advice and comments in performing the work, as well as to Professors G. V. Spivak and E. M. Reikhrudel' for discussing results. I express my gratitude also to Professor A. Engel for acquainting me with his work.

[1]E. M. Reikhrudel' and G. V. Spivak, J. Phys. (U.S.S.R.) **4**, 211 (1941).

[2]V. A. Fabrikant and G. N. Rokhlan, J. Exptl. Theoret. Phys. (U.S.S.R.) **9**, 804 (1939).

[3]L. Tonks, Phys. Rev. **56**, 360 (1939).

[4]V. A. Fabrikant, Dokl. Akad. Nauk SSSR **29**, 557 (1940).

[5]R. J. Bicerton and A. von Engel, Proc. Phys. Soc. (London) **69**, 468 (1956).

[6]M. V. Koniukov and Ia. P. Terletskii, J. Exptl. Theoret. Phys. (U.S.S.R.) **27**, 542 (1954).

[7]W. Schottky, Physik. Z. **25**, 342, 637 (1924).

[8]V. L. Granovskii, Dokl. Akad. Nauk SSSR **23**, 224 (1939).

Translated by E. J. Saletan
80

*It is impossible to speak of a quantitative comparison in the present case, since the calculation was performed for argon.

†The author is familiar with no experiments in which the influence of the magnetic field vanishes in this way at sufficiently high pressures.