ENERGY DEPENDENCE OF THE ANGULAR CORRELATION IN THE $\pi^+ - \mu^+ - e^+$ DECAY

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The energy spectrum of positrons from $\pi^+ - \mu^+ - e^+$ decays in emulsions, and the energy dependence of the angular correlation in these decays are studied. It is shown that the coefficient A in the formula $1 + A \cos \vartheta$ for the angular correlation increases rapidly with increasing positron energy, in agreement with the two-component theory of the neutrino.

THE purpose of this work was the study of the dependence of the correlation between the directions of emission of the μ-meson and positron in $\pi^+ \to \mu^+ \to e^+$ decays on the energy of the positron. To obtain this we measured the energy of the positron and the angle between the direction in which it is emitted and that of the μ-meson for 580 decays in NIKFI-R and Ilford G-5 emulsions. Layers of the NIKFI-R emulsions of diameter 10 cm and thickness 400 μ were placed in a double magnetic screen, shielding the external magnetic field to a value less than 0.05 G, and were irradiated with a pulse of monochromatic π^+-mesons, slowed down by an absorber so that they would stop in the emulsion. Layers of Ilford G-5 emulsions of thickness 600 μ constituted part of a large emulsion camera, irradiated at a height of ~ 28 km. They were presented to us by C. Powell. Of 580 measured $\pi^+ - \mu^+ - e^+$ decays, 448 were found in the NIKFI-R emulsions and 132 in the Ilford G-5 ones. The analysis of the data for the first 120 decays in the Ilford G-5 emulsions were given in our preceding article.[1] Here we will consider the complete data, including 580 decays in all.

In the choice of the cases submitted to measurement out of the large number of $\pi^+ - \mu^+ - e^+$ decays, those were taken which satisfied the following criteria: (1) Both parts of the $\pi^+ - \mu^+ - e^+$ decay occurred in the same layer of emulsion. (2) The vertex of the $\mu^+ - e^+$ decay was at a distance greater than 100 μ from both surfaces of the emulsion. (3) The length of the positron track in the layer was not less than 1 mm.

Measurements of the positron energies were carried out by the multiple scattering method. The analysis showed that in both the NIKFI-R and Ilford G-5 emulsions there was substantial distortion, the influence of which strongly diminished in going from the second to the third and fourth differences. The spectra given below were constructed from the fourth differences. Scattering measurements were carried out for cells of 50, 100 or 200 μ, depending on the track length and on the signal-to-noise ratio. The distribution of positron tracks with length are given below:

Length of tracks in mm	1 — 2	2 — 3	3 — 4	4 — 5	5 — 6	>6
Number of tracks	313	126	78	34	23	6

The statistical error in the determination of energy (dispersion) in the measurements with 100 μ cells lay within the limits ~ (30 — 10%) at the beginning and end of the table, respectively.

For each decay, in addition to the energy of the positron, the angle ϑ in the plane of the emulsion between the directions of emission of the μ^+-meson and positron was measured. It should be noted that, thanks to the criteria employed in the choice, the tracks of the positrons and mesons made small angles with the plane of the emulsion; the half-width of the respective distributions was less than 7° for positron and 10° for meson tracks. Therefore, without introducing a considerable error, we may: (1) measure the projection of the volume angle ϑ on the plane of the emulsion instead of the angle itself, and (2) consider that the correlations for particles lying in the plane of the emulsion ("plane case") are investigated.

The measured positron decay spectra are given in Fig. 1 (positrons emitted forward $0° < \vartheta \leq 90°$) and in Fig. 2 (positrons emitted backward, $90° < \vartheta \leq 180°$), where the energy ϵ, expressed as a fraction of the maximum energy $E_{max} \approx 53$ mev, is plotted along the abscissa. The ordinate is the number of particles in an energy interval $\Delta\epsilon = 0.1$. These two spectra are distinctly different. The spectrum of particles emitted forward rises sharply for low energies and, going through a maximum lying between $\epsilon = 0.4 - 0.6$, smoothly drops with larger energies, whereas the spectrum of particles emitted backward grows noticeably more slowly to begin with, going through a maximum in the region $\epsilon = 0.7 - 1.0$ and sharply

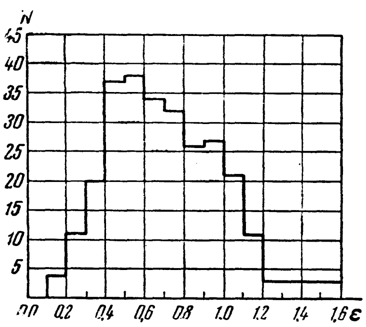

FIG. 1. Spectrum of positrons emitted forwards ($0° \leq \vartheta \leq 90°$).

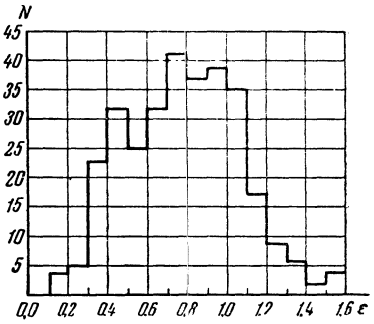

FIG. 2. Spectrum of positrons emitted backwards ($90° < \vartheta \leq 180°$).

drops with larger energies. Beginning with $\epsilon = 0.6$, there is a substantial excess of protons emitted backward, the relative magnitude of which grows with increasing energies. Thus, for example, the ratio of the number of particles emitted backwards to the number of particles emitted forwards is equal, respectively, to 219:163 for $\epsilon > 0.6$, 146:97 for $\epsilon > 0.8$, and 70:44 for $\epsilon > 1.0$.

For a quantitative analysis of the data obtained, we calculated mean values $\overline{\cos\vartheta}$ for each energy interval of the spectra Figs. 1 and 2. For a correlation of the type $1 + A\cos\vartheta$ measured in the plane as in our case the coefficient A is equal to twice the value of $\overline{\cos\vartheta}$. In the table the values are given of $\overline{\cos\vartheta}$ obtained for particles whose energy exceeds a given value.

Statistical errors given in the table were calculated as $0.79/\sqrt{N}$, where N is the number of particles of energy exceeding that given in the table. The data in the table show a monotonic growth in values of $\overline{\cos\vartheta}$ from ~ 0.05 to a value approximately four times larger.

Energy	$\overline{\cos\vartheta}_{>\varepsilon} = A/2$	Energy	$\overline{\cos\vartheta}_{>\varepsilon} = A/2$
0	$-(0.053 \pm 0.033)$	0.6	$-(0.118 \pm 0.040)$
0.1	$-(0.053 \pm 0.033)$	0.7	$-(0.148 \pm 0.045)$
0.2	$-(0.052 \pm 0.033)$	0.8	$-(0.156 \pm 0.051)$
0.3	$-(0.064 \pm 0.034)$	0.9	$-(0.192 \pm 0.059)$
0.4	$-(0.060 \pm 0.035)$	1.0	$-(0.200 \pm 0.074)$
0.5	$-(0.083 \pm 0.037)$	1.1	$-(0.206 \pm 0.104)$

For comparison of the data obtained with the two-component theory of the neutrino, we turn to the formula describing the asymmetry in $\pi - \mu - e$-decay

$$dN = (a + b\alpha\lambda\cos\vartheta)\, d\varepsilon\, d\cos\vartheta, \tag{1}$$

where

$$a = 2\varepsilon^2(3 - 2\varepsilon); \quad b = 2\varepsilon^2(2\varepsilon - 1),$$

λ is a parameter of the theory and α is a coefficient equal to the relative proportion of μ^+-mesons which keep their original direction of spin until the moment of decay. This formula gives, for the "plane case," the following theoretical value of $\overline{\cos\vartheta}$ for particles of energy exceeding ϵ

$$\overline{\cos\vartheta}_{>\varepsilon} = (\alpha\lambda/2)\left(\int_\varepsilon^1 b\, d\varepsilon \Big/ \int_\varepsilon^1 a\, d\varepsilon\right). \tag{2}$$

Our data give the possibility of determining the quantity $\alpha\lambda$. However, a more precise determination of this quantity comes from the consideration of available work with emulsions, where the angular correlation, averaged over energy, was measured[2-6] (see also the private communication of Gurevich). In this case the coefficient A in the correlation formula $1 + A\cos\vartheta$ is equal to $1/3\alpha\lambda$, which is easy to obtain by integrating (1) over the energy. The data which have been available to us are given below:

Authors	Number of Particles	$A = \alpha\lambda/3$
Chadwick, Durrani *et al.*[2]	3021	$-(0.149 \pm 0.033)$
Bodlik *et al.*[2]	1562	$-(0.080 \pm 0.044)$
Biswas *et al.*[3]	2003	$-(0.095 \pm 0.039)$
Castaghnoli *et al.*[4]	1028	$-(0.222 \pm 0.054)$
Friedman and Telegdi[5]	1300	$-(0.120 \pm 0.048)$
Gurevich *et al.* (private communication)	2000	$-(0.120 \pm 0.039)$

The statistical errors indicated here were calculated according to the formula $1.73/\sqrt{N}$, where N is the total number of particles.

The mean weighted value of the coefficient A from these data is $A = -0.127 \pm 0.017$, corresponding to $\alpha\lambda = -0.380 \pm 0.050$. We used this value of $\alpha\lambda$ for comparison of the experimental values of $\overline{\cos\vartheta}$ in the table with the theory.

Curve 1 on Fig. 3 gives the theoretical values of $\cos\vartheta/\alpha\lambda$; they vary from 0.167 for $\epsilon = 0$ to 0.500 for $\epsilon = 1$. A direct comparison of the experimental data with this curve is difficult, because our spectra (see Figs. 1 and 2) are smeared out by the dispersion in energy measurements and bremsstrahlung. We neglected the bremsstrahlung and calculated the smearing out in curve 1, starting from dispersions in the energy measurements of 15 and 30%; almost all of our measurements are confined to the region between these values of the dispersion. The results obtained are given in curves 2 and 3 which differ only slightly from each other. These curves whould be used for comparison with the experimental data. The experimental points on Fig. 3 correspond to values of $\overline{\cos\vartheta}$ from the table, divided by $\alpha\lambda = -0.38$. The encircled point at the beginning of the curve corresponds to the normalized value $\alpha\lambda = -0.380 \pm 0.050$. The experimental data show the sharp rise in the mean value of $\overline{\cos\vartheta}$ with energy predicted by the two-component theory of the neutrino. From Fig. 3 it can be seen that the experimentally observed increase somewhat exceeds the theoretical values. The accuracy of the experiment was, however, insufficient to make this significant.

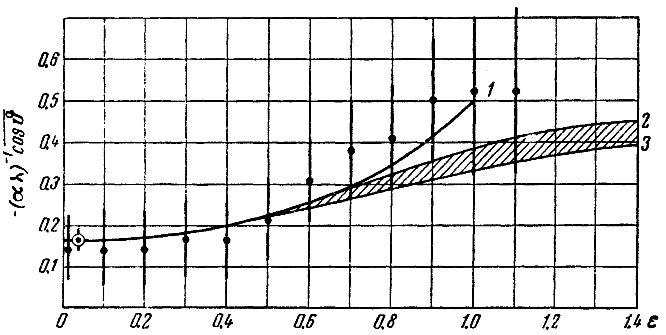

FIG. 3

We consider separately the data in the region of small energies. Because of the dispersion of the measurements, the change in sign of the correlation predicted by the theory is displaced from $\epsilon = 0.5$ in the direction of an ϵ lying in the region 0.3 — 0.4, depending on the magnitude of the dispersion. In the range $\epsilon < 0.4$, 34 positrons emitted forwards, 32 backwards were observed, which agrees with the strong decrease in correlation and change in sign predicted by the theory.

A detailed description of the work and a through statistical analysis will be given in the near future.

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