ON THE SCATTERING OF LIGHT IN $He^3 - He^4$ MIXTURES

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The scattering of light in $He^3 - He^4$ mixtures below the λ-point is considered. It is shown that the spectrum of the scattered light will contain five lines. Formulas are presented for the intensities of these lines, and the question of their width is discussed briefly.

THE scattering of light in $He^3 - He^4$ mixtures has certain peculiarities as compared with the scattering in pure He^4. In He^4, as Ginzburg[1] has shown, two doublets should be observed, corresponding to scattering by first and second sound; the undisplaced line is absent. Unfortunately, the small amount of separation in the inner doublet, and, in particular, its low intensity, make it difficult to observe this effect experimentally. In $He^3 - He^4$ mixtures there would appear to be greater opportunities for observing peculiarities in the scattering of light resulting from superfluid motion. The recent results of Walters and Fairbank,[2] moreover, show that at temperatures below 0.8° a given $He^3 - He^4$ mixture separates into phases of differing concentration, in connection with which there evidently occurs in the diagram of state a critical point situated in the region in which the two superfluid mixtures exist in equilibrium. In the vicinity of this point the phenomenon of critical opalescence should be observed in the scattering of light. The spectrum of the scattered light should contain five lines, of which four compose the two doublets corresponding to scattering by the fluctuations of first and second sound, while the fifth line represents scattering by stationary (if we neglect thermal conductivity and diffusion) fluctuations in concentration and entropy of a special type.

In computing the fluctuations we shall start with the following expression for the probability for thermodynamic fluctuations in the temperature T, the pressure p, and the mass concentration c

$$W \sim \exp\left\{-\frac{1}{2kT}\left[\rho\frac{\partial\sigma}{\partial T}\Delta T^2 + \frac{2}{\rho}\frac{\partial\rho}{\partial T}\Delta T\Delta p + \frac{1}{\rho}\frac{\partial\rho}{\partial c}\Delta\rho^2 + \rho\frac{\partial(Z/\rho)}{\partial c}\Delta c^2\right]\right\}. \quad (1)$$

Here and below we employ the notation used in the work of Khalatnikov:[3] ρ is the density; σ is the entropy per unit mass; $Z = \rho(\mu_3 - \mu_4)$, where μ_3 and μ_4 are the chemical potentials per unit mass, respectively, for He^3 and He^4. In what follows we shall everywhere neglect terms containing $\partial\rho/\partial T$, considering them to be small. With this approximation all of the fluctuations prove to be independent.

The extinction coefficient for total scattering turns out to be

$$h = \frac{\omega^4 kT}{6\pi c^4}\left[\rho\frac{\partial p}{\partial\rho}\left(\frac{\partial\varepsilon}{\partial p}\right)^2 + \left(\frac{\partial\varepsilon}{\partial T}\right)^2\frac{T}{\rho c_p} + \left(\frac{\partial\varepsilon}{\partial c}\right)^2\left(\rho\frac{\partial(Z/\rho)}{\partial c}\right)^{-1}\right], \quad (2)$$

where ϵ is the dielectric constant and c_p is the specific heat at constant pressure. In this expression the third term is the principal one; at the critical point $\partial(Z/\rho)/\partial c$ falls to zero and the intensity of the scattered light rises sharply. Experimental data are completely lacking for nearly all of the quantities entering into the theory, for mixtures in the concentration range (~ 50%) in which we are interested. Rough estimates show that we may regard all terms containing $\partial\epsilon/\partial T$ as small.*

The lines of the doublets corresponding to scattering by the fluctuations of first and second sound are displaced relative to the primary frequency ω by an amount

$$\Delta\omega = \pm 2\omega(u/c)\sin(\theta/2),$$

where θ is the angle between the directions of the incident and scattered waves.

The velocities of first and second sound are[3]

*It is also obvious that the quantities $\rho(\partial\epsilon/\partial\rho)$ and $c(\partial\epsilon/\partial c)$ are of the same order.

$$u_1^2 = \left(\frac{\partial p}{\partial \rho}\right)_{c,\,T}\left[1 + \frac{\rho_s}{\rho_n}\left(\frac{c}{\rho}\frac{\partial \rho}{\partial c}\right)^2\right], \qquad u_2^2 = \left(\frac{\partial p}{\partial \rho}\right)_{c,T}\frac{\rho_s}{\rho_n u_1^2}\left[\bar{\sigma}^2\left(\frac{\partial T}{\partial \sigma}\right)_{cp} + c^2\frac{\partial (Z/\rho)}{\partial c}\right]$$

($\bar{\sigma} = \sigma - c\partial\sigma/\partial c$). As is well known, $u_1^2 \gg u_2^2$; the separation of the inner doublet is therefore extremely small ($\Delta\omega/\omega \sim 10^{-7}$). In the sound wave, the relations between the deviations of the various quantities from their equilibrium values have the following form:

$$\delta c = -\left[\frac{\partial \rho}{\partial p}u^2 - 1\right]\delta p \Big/ \frac{\partial \rho}{\partial c}u^2,$$

where u is the corresponding sound velocity. Computing the value of the fluctuations $\overline{(\delta\epsilon)^2}$ from Eq. (1), using these relations, we obtain for the intensities of the first and second sound doublets:

$$h_{\mathrm{I}} = \frac{\omega^4}{6\pi c^4}\frac{\rho k T}{(\partial p/\partial\rho)_{T,\,c}}\left[\frac{\partial \varepsilon}{\partial \rho} - \left(\frac{c}{\rho}\frac{\partial \varepsilon}{\partial c}\right)\left(\frac{\rho_s}{\rho_n}\right)\frac{(\partial p/\partial \rho)_{T,\,c}}{u_1^2}\left(\frac{c}{\rho}\frac{\partial \rho}{\partial c}\right)\right]^2, \quad h_{\mathrm{II}} = \frac{\omega^4}{6\pi c^4}\frac{\rho k T}{(\partial p/\partial\rho)_{T,\,c}}\left[\left(\frac{\rho_s}{\rho_n}\right)\left(\frac{c}{\rho}\frac{\partial \varepsilon}{\partial c}\right)^2\frac{(\partial p/\partial\rho)^2_{T,\,c}}{u_1^2 u_2^2}\right].$$

It follows from this that the intensity of the second doublet is $\sim (u_1/u_2)^2$ ($\sim 10^2$) times that of the first sound doublet, in contrast with the situation for pure helium. The intensity of the first doublet will evidently differ only slightly from its value for the case of pure He^4 ($\sim 10^{-8}$). The fifth line in the spectrum of the scattered light arises as a consequence of the possibility for existence in the mixture of fluctuations of a third type, described by the conditions:[3]

$$p = \mathrm{const}, \quad \mu_4 = \mathrm{const}.$$

By virtue of these conditions the variations in the thermodynamic quantities are interrelated in the following manner:

$$\delta p = 0, \quad \delta T = -\frac{c}{\bar{\sigma}}\frac{\partial (Z/\rho)}{\partial c}\delta c.$$

Computing with the aid of these relations the intensity of the undisplaced line, we obtain

$$h_0 = \frac{\omega^4}{6\pi c^4}\frac{\rho k T}{(\partial p/\partial \rho)_{T,c}}\left[\left(\frac{\rho_s}{\rho_n}\right)\left(\frac{c}{\rho}\frac{\partial \varepsilon}{\partial c}\right)^2\frac{\bar{\sigma}^2(\partial T/\partial\sigma)(\partial p/\partial\rho)^2_{T,\,c}}{c^2(\partial(Z/\rho)/\partial c)u_1^2 u_2^2}\right]. \tag{3}$$

Away from the critical point for the mixture the intensity of the central line has approximately the same order of magnitude as the intensity of the inner doublet. For the order-of-magnitude estimation it is convenient to use the relation between the derivatives[3]

$$\left(\frac{\rho_s}{\rho_n}\right)\left[\bar{\sigma}^2\frac{\partial T}{\partial \sigma} + c^2\frac{\partial (Z/\rho)}{\partial c}\right] = u_1^2 u_2^2 (\partial \rho/\partial p)_{T,c}.$$

At the critical point, where $\partial(Z/\rho)/\partial c = 0$, the coefficient h_0 calculated from (3) goes to infinity. In order to treat the scattering in the immediate vicinity of the critical point, therefore, it is necessary to allow for the correlation of fluctuations in the concentration, adding into the exponential of Eq. (1) a term of the form $(b/2)(\nabla c)^2$, where b is some constant. We shall omit presentation of the formulas for the intensity of the scattered light and its angular distribution, which may be obtained from this expression in the usual manner.

We shall consider briefly the question of the width of the lines. The width of the lines in the first and second doublets is determined in the usual manner, using the attenuation coefficients for first and second sound, respectively.

The width of the undisplaced line is determined from the rate of resorption of the corresponding fluctuations, and is expressed in terms of the diffusion and thermal conductivity coefficients. Calculations yield for the width the value

$$\gamma = \frac{2\omega^2}{c^2}(1 - \cos\theta)\frac{\dfrac{\partial (Z/\rho)}{\partial c}\left\{\varkappa + \dfrac{\rho D T}{\partial (Z\,\rho)/\partial c}\left[c\dfrac{\partial \sigma/c}{\partial c} + \dfrac{k_T \partial (Z/\rho)}{T\ \ \partial c}\right]^2\right\}}{\rho\left[T\left(c\dfrac{\partial \sigma/c}{\partial c}\right)^2 + c_p\dfrac{\partial (Z/\rho)}{\partial c}\right]},$$

where κ, k_T, and D are the thermal conductivity, thermodiffusion, and diffusion coefficients for the mixture. At the critical point γ is determined by the diffusion coefficient. From unpublished measurements by Zinov'eva the viscosity coefficient for He^3 is of the same magnitude as for He^4, and is on the order of

10^{-4} cm^2/sec. Taking D to be of the same order as the kinematic viscosity, we obtain near the critical point the rough estimate $\gamma/\omega \sim 10^{-9}$, which is less than the separation of the lines of the inner doublet.

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SOLUTION OF THE KINETIC EQUATIONS FOR HIGH-ENERGY NUCLEAR CASCADE PROCESSES

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The altitude dependence of high-energy nuclear-active particles and the spectrum of the μ mesons produced by the decay of π mesons are investigated. The elementary act is described hydrodynamically; the energy distribution function used for the particles produced is that of Landau, corrected to take account of the traveling wave in the hydrodynamical solution.

LANDAU'S hydrodynamic theory of the multiple production of particles[1] gives agreement with experiment as to the multiplicity and angular distribution of the secondary particles.[2] But the energy distribution obtained by Landau gives more fractionation of the energy among the particles produced than is observed experimentally. According to Grigorov's data,[3] in high energy nuclear interactions (at about $10^{10} - 10^{12}$ ev) a larger part of the energy remains with one of the particles produced. In this connection Zatsepin and Guzhavin[4] have made numerical calculations of the altitude dependence of the density spectrum of showers, using a phenomenological introduction of such a particle into the description of the elementary act. The results were found to be in good agreement with experiment. In a paper by Chernavskii and the writer[5] it was shown that the inclusion of a traveling wave in the hydrodynamical equation leads to the necessity of introducing a fast particle into the Landau distribution. At present the fraction α of the energy carried away by the fastest of the secondary particles produced cannot be precisely determined theoretically and must be regarded as a parameter.

The disintegration temperature T_k of the hydrodynamical system also appears as a parameter in the theory. In view of the absence of precise data on the index of the energy spectrum of the primary particles and on the interaction distance of particles at high energies, it is also particularly desirable to obtain explicity relations characterizing the passage of high-energy particles through the atmosphere. By the method of successive generations Rozental' has determined the number of particles in an individual shower as a function of α and the fraction of the energy transferred to the soft component. In the present paper we find the solution of the kinetic equations for high-energy ($E \gtrsim 10^{12}$ ev) nuclear cascade processes in the atmosphere and use it to determine the absorption coefficient of particles interacting strongly with nuclei and the spectrum of the μ mesons produced from the decay of π mesons.