10^{-4} cm^2/sec. Taking D to be of the same order as the kinematic viscosity, we obtain near the critical point the rough estimate $\gamma/\omega \sim 10^{-9}$, which is less than the separation of the lines of the inner doublet.

In conclusion, the authors wish to express their gratitude to I. E. Dzialoshinskii for his helpful discussions, and K. N. Zinov'eva for communication of results of her measurements prior to their publication.

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SOLUTION OF THE KINETIC EQUATIONS FOR HIGH-ENERGY NUCLEAR CASCADE PROCESSES

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The altitude dependence of high-energy nuclear-active particles and the spectrum of the μ mesons produced by the decay of π mesons are investigated. The elementary act is described hydrodynamically; the energy distribution function used for the particles produced is that of Landau, corrected to take account of the traveling wave in the hydrodynamical solution.

LANDAU'S hydrodynamic theory of the multiple production of particles[1] gives agreement with experiment as to the multiplicity and angular distribution of the secondary particles.[2] But the energy distribution obtained by Landau gives more fractionation of the energy among the particles produced than is observed experimentally. According to Grigorov's data,[3] in high energy nuclear interactions (at about $10^{10} - 10^{12}$ ev) a larger part of the energy remains with one of the particles produced. In this connection Zatsepin and Guzhavin[4] have made numerical calculations of the altitude dependence of the density spectrum of showers, using a phenomenological introduction of such a particle into the description of the elementary act. The results were found to be in good agreement with experiment. In a paper by Chernavskii and the writer[5] it was shown that the inclusion of a traveling wave in the hydrodynamical equation leads to the necessity of introducing a fast particle into the Landau distribution. At present the fraction α of the energy carried away by the fastest of the secondary particles produced cannot be precisely determined theoretically and must be regarded as a parameter.

The disintegration temperature T_k of the hydrodynamical system also appears as a parameter in the theory. In view of the absence of precise data on the index of the energy spectrum of the primary particles and on the interaction distance of particles at high energies, it is also particularly desirable to obtain explicity relations characterizing the passage of high-energy particles through the atmosphere. By the method of successive generations Rozental' has determined the number of particles in an individual shower as a function of α and the fraction of the energy transferred to the soft component. In the present paper we find the solution of the kinetic equations for high-energy ($E \gtrsim 10^{12}$ ev) nuclear cascade processes in the atmosphere and use it to determine the absorption coefficient of particles interacting strongly with nuclei and the spectrum of the μ mesons produced from the decay of π mesons.

The experimental data indicate the production in stars of heavy mesons, hyperons, and nucleons, in addition to π mesons. The most important contribution for a nuclear cascade process is that of the nucleons, since initial nucleons also take part in the development of the cascades. Therefore we shall assume that π mesons and nucleons are produced in the elementary act. Then the numbers of π mesons and of nucleons produced in the energy range $(E, E + dE)$ as the result of the collision of a particle of energy E' with a nucleus of air are written

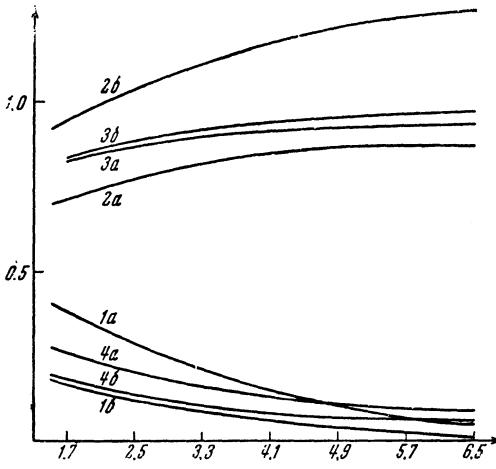
$$dN^{\pi}(E, E') = \pi_1(E')\delta(E - \alpha E')\,dE + dN_{\mathrm{L}}^{\pi}[E, (1-\alpha)E'],\quad dN^{\mathrm{N}}(E, E') = H_1(E')\delta(E - \alpha E')\,dE + dN_{\mathrm{L}}^{\mathrm{N}}[E, (1-\alpha)E']. \quad (1)$$

The first terms in Eq. (1) give the probability of formation of one particle with energy $\alpha E'$, and the second terms correspond to the distribution for the other particles: dN_{L} and $dN_{\mathrm{L}}^{\mathrm{N}}$ are the Landau distributions, in which, as also in the first terms, we introduce coefficients corresponding to the ratios of the numbers of particles of different kinds and their energies as calculated by Belen'kii,[6,7] and satisfying the basic relations

$$N = N^{\pi} + N^{\mathrm{N}};\quad \int E^{\pi} dN_{\mathrm{L}}^{\pi} + \int E^{\mathrm{N}} dN_{\mathrm{L}}^{\mathrm{N}} = (1-\alpha)E'.$$

The second relation is related to the parametric statement of the Landau formulas and leads to the appearance in the energy distributions for E^{π} and E^{N} of the factors $\pi_2(E')(1-\alpha)$ and $H_2(E')(1-\alpha)$, respectively, instead of the factors $\pi_1(E')$ and $N_1(E')$ of the distributions of the numbers of particles (cf. Ref. 8).

The diagram shows the energy dependences of π_1, H_1, $\ln\pi_2$, and $\ln H_2$, expressed on a logarithmic scale [$l = \frac{1}{2}(\ln E - 24.2)$], with the same notations as in Ref. 8, here and in what follows.



The curves show the various coefficients as functions of l, as follows: 1 — H_1, 2 — $\ln H_2$, 3 — π_1, 4 — $\ln\pi_2$; curves a are for $kT_k = M_{\pi}c^2$, and curves b for $kT_k = 1/2 M_{\pi}c^2$.

With the distributions (1) one solves the system of kinetic equations for the numbers of nucleons, $\mathscr{P}^{\mathrm{N}}(E, t)$, and of π mesons, $\mathscr{P}(E, t)$, present at the given depth t and with energies between E and $E + dE$:

$$\frac{\partial \mathscr{P}^{\mathrm{N}}(E,t)}{\partial t} = -\mathscr{P}^{\mathrm{N}}(E,t) + \int_E^{\infty} \frac{dN^{\mathrm{N}}(E',E)}{dE}\mathscr{P}^{\mathrm{N}}(E',t)\,dE' + \int \frac{dN^{\mathrm{N}}(E',E)}{dE}\mathscr{P}^{\pi}(E',t)\,dE';$$

$$\frac{\partial \mathscr{P}^{\pi}(E,t)}{\partial t} = -\mathscr{P}^{\pi}(E,t) + \frac{2}{3}\int_E^{\infty} \frac{dN^{\pi}(E'E)}{dE}\mathscr{P}^{\pi}(E',t)\,dE' + \frac{2}{3}\int_E^{\infty} \frac{dN^{\pi}(E',E)}{dE}\mathscr{P}^{\mathrm{N}}(E',t)\,dE'. \quad (2)$$

Here the depth t is measured in units t_0; the factor 2/3 allows for the fact that neutral π mesons play no part in the development of the nuclear-active component. We neglect the decay of the π mesons. As boundary condition for the equations (2) one assumes a power-law spectrum of the primary nucleons:

$$\mathscr{P}^{\mathrm{N}}(E, 0) = B/E^{\gamma+1};\quad \mathscr{P}^{\pi}(E, 0) = 0.$$

Substituting the distributions (1) into Eq. (2) and making the substitution

$$\mathscr{P}^{\mathrm{N}}(E,t) = e^{-t}P^{\mathrm{N}}(E,t),\quad \mathscr{P}^{\pi}(E,t) = e^{-t}P^{\pi}(E,t),$$

we get

$$\frac{\partial P^{\mathrm{N}}(E,t)}{\partial t} = \frac{1}{\alpha}H_1\left(\frac{E}{\alpha}\right)P^{\mathrm{N}}\left(\frac{E}{\alpha}, t\right) + \frac{1}{\alpha}H_1\left(\frac{E}{\alpha}\right)P^{\pi}\left(\frac{E}{\alpha}, t\right) + \int_E^{\infty}\frac{dN_{\mathrm{L}}^{\mathrm{N}}}{dE}P^{\mathrm{N}}(E',t)\,dE' + \int_E^{\infty}\frac{dN_{\mathrm{L}}^{\mathrm{N}}}{dE}P^{\pi}(E',t)\,dE',$$

$$\frac{\partial P^{\pi}(E,t)}{\partial t} = \frac{1}{\alpha}\pi_1\left(\frac{E}{\alpha}\right)P^{\pi}\left(\frac{E}{\alpha}, t\right) + \frac{1}{\alpha}\pi_1\left(\frac{E}{\alpha}\right)P^{\mathrm{N}}\left(\frac{E}{\alpha}, t\right) + \int_E^{\infty}\frac{dN_{\mathrm{L}}^{\pi}}{dE}P^{\pi}(E',t)\,dE' + \int_E^{\infty}\frac{dN_{\mathrm{L}}^{\pi}}{dE}P^{\mathrm{N}}(E',t)\,dE'. \quad (3)$$

We seek the solution of the equations (3) in the form

$$P^{\mathrm{N}}(E,t)=\frac{B}{E^{\gamma+1}}\exp\left\{\omega\left(\frac{E}{\alpha}\right)\alpha^{\gamma}t\right\}[1+\chi_1^{\mathrm{N}}(E,t)+\cdots],\quad P^{\pi}(E,t)=\frac{B}{E^{\gamma+1}}\exp\left\{\omega\left(\frac{E}{\alpha}\right)\alpha^{\gamma}t\right\}[\chi_1^{\pi}(E,t)+\cdots],$$
$$\omega\left(\frac{E}{\alpha}\right)\equiv H_1\left(\frac{E}{\alpha}\right)+\frac{2}{3}\pi_1\left(\frac{E}{\alpha}\right). \tag{4}$$

The choice of the zeroth approximation

$$P_0^{\mathrm{N}}(E,t)=\frac{B}{E^{\gamma+1}}\exp\left[\omega\left(\frac{E}{\alpha}\right)\alpha^{\gamma}t\right]$$

is due to the assumption that the fraction α is large and that the main terms in Eq. (1) are those containing the δ-function. In the determination of the equations for $\chi_1^{\mathrm{N}}(E,t)$ and $\chi_1^{\pi}(E,t)$ the integrals obtained by substituting $P_0^{\mathrm{N}}(E,t)$ into the right members of the equations (3) are calculated by the method of steepest descents:[8]

$$\int_E^{\infty}\frac{dN_{\mathrm{L}}^{\mathrm{N}}(E',E)}{dE}P_0^{\mathrm{N}}(E',t)\,dE'=P_0^{\mathrm{N}}(E_1^{\alpha\mathrm{N}},t)\,\Delta^{\alpha\mathrm{N}}(E); \tag{5a}$$

$$\int_E^{\infty}\frac{dN_{\mathrm{L}}^{\pi}(E',E)}{dE}P_0^{\mathrm{N}}(E',t)\,dE'=P_0^{\mathrm{N}}(E_1^{\alpha\pi},t)\,\Delta^{\alpha\pi}(E), \tag{5b}$$

where[8]

$$\Delta^{\alpha\mathrm{N}}(E)=H_1(E_1^{\alpha\mathrm{N}})S(\gamma)[H_2(E_1^{\alpha\mathrm{N}})]^{\gamma+\delta(\gamma)}(1-\alpha)^{\gamma+\delta(\gamma)}E^{-\delta(\gamma)};\quad \Delta^{\alpha\pi}(E)=\pi_1(E_1^{\alpha\pi})S(\gamma)[\pi_2(E_1^{\alpha\pi})]^{\gamma+\delta(\gamma)}(1-\alpha)^{\gamma+\delta(\gamma)}E^{-\delta(\gamma)}.$$

The saddle points $E_1^{\alpha\mathrm{N}}$ and $E_1^{\alpha\pi}$ are found from the equations

$$\ln E_1^{\alpha\mathrm{N}}-24.2=2d(\gamma)[\ln E-\ln(C_2(1-\alpha)H_2(E_1^{\alpha\mathrm{N}}))],\quad \ln E_1^{\alpha\pi}-24.2=2d(\gamma)[\ln E-\ln(C_2(1-\alpha)\pi_2(E_1^{\alpha\pi}))]. \tag{6}$$

Regarding $\chi_1^{\mathrm{N}}(E,t)$ and $\chi_1^{\pi}(E,t)$ as slowly varying functions of the energy and neglecting the weak energy dependence of the coefficients and inhomogeneous terms in the equations for χ_1^{N} and χ_1^{π} (it can be verified that the error thus admitted is $\lesssim 10\%$ for $E \gtrsim 10^{12}$ ev), we find the solutions:

$$\chi_1^{\pi}(E,t)=\frac{2\pi_1(E/\alpha)}{3\omega(E/\alpha)}\left[\exp\left\{-\alpha^{\gamma}\omega\left(\frac{E}{\alpha}\right)t\right\}-1\right]+\frac{2\pi_1(E/\alpha)}{3\omega(E/\alpha)}\left\{\frac{\Delta^{\alpha\pi}(E)\left[\exp\left\{-\alpha^{\gamma}\Delta\omega\left(\frac{E_1^{\alpha\pi}}{\alpha}\right)t\right\}-1\right]}{\alpha^{\gamma}[\omega(E_1^{\alpha\pi}/\alpha)-\omega(E/\alpha)]}\right.$$
$$\left.+\frac{\Delta^{\alpha\mathrm{N}}(E)\left[\exp\left\{-\alpha^{\gamma}\Delta\omega\left(\frac{E_1^{\alpha\mathrm{N}}}{\alpha}\right)t\right\}-1\right]}{\alpha^{\gamma}[\omega(E_1^{\alpha\mathrm{N}}/\alpha)-\omega(E/\alpha)]}\right\}+\frac{2\pi_1(E/\alpha)\Delta^{\alpha\mathrm{N}}(E)}{3\omega(E/\alpha)\alpha^{\gamma}\omega(E_1^{\alpha\mathrm{N}}/\alpha)}\left[\exp\left\{-\alpha^{\gamma}\omega\left(\frac{E}{\alpha}\right)t\right\}-\exp\left\{-\alpha^{\gamma}\Delta\omega\left(\frac{E_1^{\alpha\mathrm{N}}}{\alpha}\right)t\right\}\right]$$
$$-\frac{H_1(E/\alpha)\Delta^{\alpha\pi}(E)}{\omega(E/\alpha)\alpha^{\gamma}\omega(E_1^{\alpha\pi}/\alpha)}\left[\exp\left\{-\alpha^{\gamma}\omega\left(\frac{E}{\alpha}\right)t\right\}-\exp\left\{-\alpha^{\gamma}\Delta\omega\left(\frac{E_1^{\alpha\pi}}{\alpha}\right)t\right\}\right];\quad \Delta\omega\left(\frac{E_1^{\alpha\mathrm{N}}}{\alpha}\right)=\omega\left(\frac{E}{\alpha}\right)-\omega\left(\frac{E_1^{\alpha\mathrm{N}}}{\alpha}\right), \tag{7}$$
$$\Delta\omega\left(\frac{E_1^{\alpha\pi}}{\alpha}\right)=\omega\left(\frac{E}{\alpha}\right)-\omega\left(\frac{E_1^{\alpha\pi}}{\alpha}\right),$$

χ_1^{N} is similar in form.

In just the same way one determines $\chi_2^{\mathrm{N}}(E,t)$ and $\chi_2^{\pi}(E,t)$, and so on (with the successive saddle points being also found from Eq. (6), where one substitutes for E the values of the preceding saddle points). The series (4) converge rapidly. We shall not write out the solutions in general form, however, since it is not convenient to use them in practice. The total number of nuclear-active particles $\mathscr{P}=\mathscr{P}^{\mathrm{N}}+\mathscr{P}^{\pi}$ can be represented in the form of a double sum with respect to t (the expansion of $\chi_1(E,t)=\chi_1^{\pi}(E,t)+\chi_1^{\mathrm{N}}(E,t)$ begins with the first power of t, that of $\chi_2(E,t)+\chi_2^{\pi}(E,t)+\chi_2^{\mathrm{N}}(E,t)$ with the second, and so on). To accuracy about 10% for the energies in which we are interested, we can confine ourselves to terms in the expansion up to the second order inclusive. Therefore we get finally

$$\mathscr{P}(E,t)=\frac{B}{E^{\gamma+1}}\exp\left\{-\left[1-\alpha^{\gamma}\omega\left(\frac{E}{\alpha}\right)\right]t\right\}[1+\bar{\chi}_1+\bar{\chi}_2]=\frac{B}{E^{\gamma+1}}\exp\left\{-\left[1-\alpha^{\gamma}\omega\left(\frac{E}{\alpha}\right)\right]t\right\}\left\{1+\left[\Delta^{\alpha\mathrm{N}}(E)+\Delta^{\alpha\pi}(E)\right]t\right.$$
$$\left.-\alpha^{\gamma}\left[\Delta^{\alpha\mathrm{N}}(E)\Delta\omega\left(\frac{E_1^{\alpha\mathrm{N}}}{\alpha}\right)+\Delta^{\alpha\pi}(E)\Delta\omega\left(\frac{E_1^{\alpha\pi}}{\alpha}\right)\right]\frac{t^2}{2}+[\Delta^{\alpha\mathrm{N}}(E)(\Delta^{\alpha\mathrm{N}}(E_1^{\alpha\mathrm{N}})+\Delta^{\alpha\pi}(E_1^{\alpha\mathrm{N}}))+\Delta^{\alpha\pi}(E)(\Delta^{\alpha\mathrm{N}}(E_1^{\alpha\pi})+\Delta^{\alpha\pi}(E_1^{\alpha\pi}))]\frac{t^2}{2}\right\}. \tag{8}$$

The distribution that has been found for the nuclear-active particles in the atmosphere makes it possible to obtain the characteristics of the nuclear cascade process in explicit form and to analyze their dependence on the theoretical parameters α and T_k and also on the index γ of the primary spectrum and the interaction path length t_0.

The form of the energy spectrum of the particles produced in the nuclear interactions has the strongest effect on the altitude distribution of the particles in the atmosphere. By the use of the expression we have determined for $\mathcal{P}(E, t)$ the absorption coefficient of the nuclear-active particles,

$$m = -(1/\mathcal{P}(E,t))\,\partial\mathcal{P}(E,t)/\partial t$$

can be written in the following way:

$$m = \frac{1-D}{t_0}; \quad D = \alpha^{\gamma}\omega\left(\frac{E}{\alpha}\right) + \frac{1}{t}\ln\left[1+\bar{\chi}_1(E,t)+\bar{\chi}_2(E,t)\right]. \tag{9}$$

The first term in D is due to the fastest particle produced in the elementary act. For $\alpha \sim 0.6-0.7$ it amounts to 90 to 95 percent of D, i.e., the absorption of the cascade is almost entirely determined by the value of the fraction of the total energy carried by this particle. The dependence of the main term of D on α is in agreement with that found by Zatsepin.[9] The factor $\omega(E/\alpha)$ makes a statistical allowance for the fact that this particle may be either a nucleon or a π meson.

TABLE I. Absorption Path Length of Nuclear-Active Particles, $1/m$ (in g/cm^2)

t \ E (ev)	10^{12}	10^{13}	α \ kT_k	$M_\pi c^2$	$0.5\,M_\pi c^2$
$\alpha=0.7$;		$kT_k=M_\pi c^2$	$E=10^{12}$ ev;		$t^*=10$
		$\gamma=1.5$;	$t_0=65$ g/cm^2		
10	119.3	100.5	0.6	104.4	104.8
15.9	119	99.9	0.7	119.3	119.5
		$\gamma=1.7$;	$t_0=65$ g/cm^2		
10	110.1	93.8	0.6	96.8	97
15.9	110.1	93.7	0.7	110.1	110.5
$\alpha=0.5$;		$kT_k=M_\pi c^2$	$E=10^{12}$ ev;		$t=8.7$
		$\gamma=1.7$;	$t_0=75$ g/cm^2		
8.7	100.8	93	0.5	100.8	101.5
13.8	100.5	92.8	0.6	111.9	112.5

* The values of t, expressed in units of t_0, correspond to the altitudes at Pamir and at Moscow.

With increase of the energy the role played by the nucleons decreases and $\omega(E/\alpha)$ becomes smaller, approaching the value $2/3$. Values of m computed by Eq. (9) are given in Table I.* According to the data of Kaplon and others,[10] for heights in the range less than 700 g/cm^2 one has $m \sim 1/120$ g/cm^2; for about this same energy value Ryzhkova and Sarycheva[11] found $m = 1/(112 \pm 6)$ g/cm^2 as an average value from Pamir ($t \sim 650$ g/cm^2) to Moscow ($t \sim 1020$ g/cm^2). From Table I it follows that with reasonable assumptions as to γ[12] and t_0[13] values of α between 0.6 and 0.7 give satisfactory agreement with experiment; variation of T_k has practically no effect on the results.

We shall now determine the spectrum of the μ mesons produced by the decay of the π mesons. The main contribution to the intensity of the μ mesons with energies $\gtrsim 10^{12}$ ev is made by the μ mesons from the decay of the π mesons with spectrum given by the first term of the series for $\mathcal{P}^\pi(E, t)$:

$$\mathcal{P}_1^\pi(E,t) = \exp\left\{-\left[1-\alpha^\gamma\omega\left(\frac{E}{\alpha}\right)\right]t\right\}\chi_1^\pi(E,t), \tag{10}$$

(here in expanding the curly brackets in $\chi_1^\pi(E, t)$ in terms of t we can confine ourselves to the first power of t).

The number of μ mesons with energies greater than E at the depth t is calculated in the following way:[14]

$$\mathcal{P}^\mu(>E,t) = \int_0^t e^{-t'}t'^{-1}dt'\int_E^\infty dE' \int_{E'}^{E'(M_\pi/M_\mu)^2} P^\pi(E'',t')\,k_\pi(E'')\,D_\mu^\pi(E'',E')\,dE''. \tag{11}$$

Here $k_\pi(E)$ is a coefficient characterizing the probability of decay of a π meson with energy E. Following Ref. 15, we take

$$k_\pi \equiv E_\pi/E = (t_0/65\ \text{g/cm}^2)\ 1.17\cdot 10^{11}\ \text{ev}/E,$$

*The writer is grateful to Z. S. Maksimova for aid with the numerical calculations.

$D_{\mu}^{\pi}(E'', E')\,dE'$ is the probability for formation of a μ meson with its energy in the range dE' from the decay of a π meson with energy E''. If we assume that the μ mesons are distributed isotropically in the reference system of the π meson, then

$$D_{\mu}^{\pi}\,dE' = dE'/[1-(M_{\pi}/M_{\mu})^2]\,E'$$

(M_{μ} and M_{π} are the masses of the μ meson and the π meson). The limits of integration in the integral with respect to E'' correspond to the π-meson energies necessary for the production of a μ meson of energy E' when it is emitted forward and backward, respectively, with maximum energy in the system of the π meson. In calculating by Eq. (11) we take advantage of the weak dependence on the energy of the quantities π_1, H_1, ω, $\Delta^{\alpha\pi}(E)$, and $\Delta^{\alpha N}(E)$, and also of the closeness of the limits of the integration, bring these factors outside the sign of integration with respect to E''. Using the fact that

$$\int_0^t [e^{-xt'} - e^{-t'}]\frac{dt'}{t'} \approx \ln x$$

for sufficiently large t, we get:

$$\mathscr{F}^{\mu}(>E, t) = \frac{BE_{\pi}[1-(M_{\mu}/M_{\pi})^{2(\gamma+2)}}{(\gamma+2)(\gamma+1)E^{\gamma+1}[1-(M_{\mu}/M_{\pi})^2]}\left\{\frac{2\pi_1(E/\alpha)}{3\omega(E/\alpha)}\right.$$

$$\times\ln\frac{1}{[1-\alpha^{\gamma}\omega(E/\alpha)]} + \frac{2\pi_1(E/\alpha)[\Delta^{\alpha\pi}(E)+\Delta^{\alpha N}(E)]\left[1-\exp\left\{-\left[1-\alpha^{\gamma}\omega\left(\frac{E}{\alpha}\right)\right]t\right\}\right.}{3\omega(E\ \ /\alpha)\ \ -\alpha^{\gamma}\omega(E/\alpha)]}$$

$$\left.-\frac{2\pi_1(E/\alpha)\Delta^{\alpha N}E}{3\omega(E/\alpha)\alpha^{\gamma}\omega(E_1^{\alpha N}/\alpha)}\ln\frac{1}{[1-\alpha^{\gamma}\omega(E_1^{\alpha N}/\alpha)]} + \frac{H_1(E/\alpha)\Delta^{\alpha\pi}(E)}{\omega(E/\alpha)\alpha^{\gamma}\omega(E_1^{\alpha\pi}/\alpha)}\ln\frac{1}{[1-\alpha^{\gamma}\omega(E_1^{\alpha\pi}/\alpha)]}\right\}$$

(the contribution to the total number of μ mesons from the decay of the π mesons from the part of the spectrum given by the remaining terms of the series for $\mathscr{F}^{\pi}(E, t)$ can be neglected).

TABLE II. Intensity of μ Mesons at Sea Level (particles $cm^{-2}\,sec^{-1}\,sterad^{-1}$)

$kT_k = M_{\pi}c^2$, $\gamma = 1.7$

$t_0 = 65\ g/cm^2$			$\alpha = 0{,}7$		
E, ev \ γ	1.5	1.7	E, ev \ kT_k	$M_{\pi}c^2$	$0.5\,M_{\pi}c^2$
10^{12} $3\cdot10^{12}$	$4.8\cdot10^{-7}$ $3{,}8\cdot10^{-8}$	10^{1} [illegible] $2.5\cdot10^{-}$ [illegible]	10^{12} $3\cdot10^{12}$	$4.8\cdot10^{-7}$ $2.5\cdot10^{-8}$	$4.8\cdot10^{-7}$ $2.6\cdot10^{-8}$
$t_0 = 75\ g/cm^2$			$\alpha = 0{,}6$		
E, ev \ α	0.6	0.5	E, ev \ t_0	$65\ g/cm^2$	$75\ g/cm^2$
10^{12} $3\cdot10^{12}$	$4\cdot10^{-7}$ $2{,}1\cdot10^{-8}$	$2{,}9\cdot10^{-7}$ $1{,}5\cdot10^{-8}$	10^{12} $3\cdot10^{12}$	$3{,}5\cdot10^{-7}$ $1.8\cdot10^{-8}$	$4\cdot10^{-7}$ $2{,}1\cdot10^{-8}$

In the formula (12), just as in Eq. (9), the main term is that caused by the fastest particle [it gives ~ 90–95% of $\mathscr{F}^{\mu}(>E, t)$ for $\alpha \sim 0.6-0.7$]. Here the coefficient $2\pi_1(E/\alpha)/3\omega(E/\alpha)$ is a manifestation of the fact that the presence of nucleons reduces $\mathscr{F}^{\mu}$. Values calculated by Eq. (12) for the numbers of μ mesons* at sea level with energies 10^{12} ev and 3×10^{12} ev for various values of α, T_k, γ, and t_0 are shown in Table II; the coefficient B is determined for the various values of γ from the normalization of the spectrum used to the intensity[4] of the primary particles at $E = 10^{12}$ ev. Gorchakov[16] has made numerical calculations of $\mathscr{F}^{\mu}(10^{12}\,ev)$ for several values of α, T_k, γ, and t_0; his results agree with ours.

Comparing the values of $\mathscr{F}^{\mu}$ from Table II with the experimental values obtained on the basis of the work of George,[17]

$$\mathscr{F}^{\mu}(>10^{12}\,ev) \approx 2\cdot10^{-7};\quad \mathscr{F}^{\mu}(>3\cdot10^{12}\,ev) \approx 10^{-8}\frac{\text{particles}}{\text{cm}^2\ \text{sec steradian}}$$

we see that the theoretical values of the intensity of the μ mesons have turned out too high. But no great significance is to be attached to this discrepancy, since the data on μ mesons of such high energies are

*In the derivation of Eqs. (9) and (12) α was assumed not to depend on the energy. In actual fact[5] $\alpha \sim E^{-(2c_0-c)/4c_0} = E^{-0.065}$, i.e., it decreases slowly with increasing energy. In the computations for Tables I and II we have used this dependence directly in Eqs. (9) and (12) [the values indicated in Tables I and II are for $\alpha \equiv \alpha(10^{12}\,ev)$].

extremely rough. The attempt to diminish $\mathscr{P}^{\mu}$ by assuming that the fastest particle is always a nucleon leads to a decided lowering of the absorption coefficient. At the same time the data on the absorption coefficient, which can be accounted for without additional assumptions, appear more reliable than those on the μ mesons. Because of the strong dependence of both characteristics on γ and t_0 and the uncertainties in these quantities we cannot establish strict limits on the variation of α. From Table I it follows that satisfactory agreement with experiment is given by $\alpha \sim 0.6-0.7$, but we cannot exclude also 0.5 and 0.8. From the estimates for the traveling wave[8] one found $\alpha \sim 0.5$. Thus the hydrodynamic theory of multiple production of particles is not in contradiction with the existing experimental data on nuclear cascade processes in the atmosphere.

The present work was inspired by the late S. Z. Belen'kii, whose advice was especially valuable to me. I am extremely grateful to I. L. Rozental' and G. T. Zatsepin for helpful discussions.

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