CORRELATION THEORY FOR RAYLEIGH SCATTERING OF LIGHT. II

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A number of further consequences which follow from the general formulas obtained in Ref. 1 for the spectral intensities of scattered light are considered. The integrated intensities of the fine structure components of the isotropic part of the scattered light are obtained for a medium with small viscous losses and small heat losses using specific dispersion relations. The dispersion corrections for the Landau-Placzek relation are considered in greater detail for a low-viscosity liquid. The results of the general theory which concern anisotropic scattering are discussed and simple dispersion relations are indicated which yield a satisfactory approximation to the observed intensity distribution in the wing of the Rayleigh line for a low-viscosity liquid. Using these relations the integrated intensities are obtained and an approximation formula is derived for the depolarization coefficient in Rayleigh scattering.

In an earlier paper[1] (referred to hereafter as I) we have developed a spectral theory for Rayleigh scattering, based on correlation theory as applied to the thermal fluctuations in an isotropic medium.[2] General formulas were obtained for observation at right angles to the primary wave and four combinations of the polarization of the primary and scattered light [Eqs. (3.1) of I]. These general formulas apply for any dispersion relation between the parameters which is consistent with the dissipation requirements of the medium. As a first result we have obtained the integrated intensity of the isotropic part of the scattered light and, using certain assumptions, have found the dispersion correction to the Einstein formula for liquids.

In the present part of the work we consider further consequences which follow from the general formulas; first of all, we consider the problem of the integrated intensities of the components of the isotropic part of the scattered light — the central line of the Mandel'shtam-Brillouin doublet and the background. Although the background due to dispersion in the compression modulus $\bar{K}$ and the shear modulus $\bar{\mu}$ is present in isotropic scattering, in low-viscosity liquids it turns out to be extremely weak. In practice, the isotropic part of the scattered light tends to be characterized by the usual triplet in this case. The intensity of the triplet components is calculated in the approximation that there is dispersion only in the elastic moduli; the simplest dispersion relations are assumed — with a single relaxation time for each modulus (τ' for $\bar{K}$ and τ for μ). The neglect of the dispersion background is manifest only in the correction to the intensity I_{MB} of the Mandel'shtam-Brillouin doublet. This correction, due to the dispersion in $\bar{K}$ leads specifically to the intensity ratio for the non-displaced line I_{cent} and the doublet which was obtained earlier by I. L. Fabelinskii[3,4] in an analysis of the doublet based on diffraction by Debye waves.

Essentially new results are obtained in an analysis of the anisotropic scattering (Secs. 3 – 5). The spectral intensities are given by formulas which contain the results of the wing analysis for liquids developed by Leontovich[5] as a particular case corresponding to the assumption that the mechanical-optical coefficient X is directly proportional to the compression modulus $\bar{\mu}$. A departure from this assumption allows a good approximation for the distribution of intensity in the wing, including the intense center part even when the simplest dispersion relations for X and $\bar{\mu}$ are assumed. In this case it is sufficient to assume two relaxation times for X, τ_1 and $\tau_2 \gg \tau_1$, the values of which may be obtained from the spectral intensity distribution. A calculation of the integrated intensities for the dispersion laws which have been indicated leads to an approximation formula (5.9) for the Kabanovsky depolarization factor Δ. An estimate of Δ in benzol based on this formula turns out to be in good agreement with the experimental value.

1. INTEGRATED INTENSITIES OF THE I_{iso} COMPONENT
(FORMULATION OF THE PROBLEM)

The spectral intensity of the isotropic part of the light which is scattered by fluctuations in the density and temperature of the medium is given by the following general expression [Eq. (4.1) of I; all notation is the same as in I]:

$$J_{iso} = \frac{\theta}{2\pi i \Omega} \left\{ \frac{1}{\Delta} \left[|Y|^2 \left(\frac{d_1 A_3}{K^*} - CC^* \overline{K} q^2 \right) - YZ^* C \overline{K} q^2 + Y^* Z \left(C^* A_1 - C \frac{\overline{K}}{K^*} A_3 \right) + |Z|^2 A_1 \right] - \text{comp. conj.} \right\} \qquad (1.1)$$

In calculating the total integrated intensity in I

$$I_{iso} = \int_{-\infty}^{+\infty} J_{iso} \, d\Omega, \qquad (1.2)$$

we have made use of general theorems[2] which make it possible to obtain a definite result by simplifying the dispersion relations for some of the parameters which appear in (1.1); in particular, Y, Z, C and $\overline{K}$. This approach is not adequate in problems in which loss must be taken into account since separating I_{iso} into component intensities means that we must find the residues of the integral (1.2) at separate poles of the integrand; consequently this procedure requires a complete knowledge of the function.

As in I, we shall make the following assumptions with respect to the parameters Y, Z, C and $\overline{K}$: (a) the coefficients Y and Z are independent of Ω, (b) the thermal parameters C, D, and κ are independent of Ω, (c) the elastic moduli $\overline{K}$ and $\overline{\mu}$ are characterized by the simplest dispersion relation with a single relaxation time for each (τ' and τ respectively)

$$\overline{K} = \frac{K_\infty z + K_0 / \tau'}{z + 1 / \tau'}, \qquad \overline{\mu} = \frac{\mu_\infty z + \mu_0 / \tau}{z + 1 / \tau} \qquad (z = i\Omega). \qquad (1.3)$$

We note that by virtue of the assumptions in (a) and (b) the corresponding parameters are given by the real thermodynamic values

$$Y = -\rho_0 (\partial \varepsilon / \partial \rho)_T, \quad Z = T_0 (\partial \varepsilon / \partial T)_\rho, \quad C = \alpha T_0, \quad D = \rho_0 T_0 c_p. \qquad (1.4)$$

Using these assumptions the determinant Δ which appears in the denominator of Eq. (1.1) has the form

$$\Delta = \left(D_1 + \frac{\varkappa T_0 q^2}{i\Omega} \right) \left(\rho_0 \Omega^2 - \frac{4}{3} \overline{\mu} q^2 \right) - \left(D + \frac{\varkappa T_0 q^2}{i\Omega} \right) \overline{K} q^2 = - \frac{\rho_0 D}{z(z + 1/\tau')(z + 1/\tau)} \left\{ v(z) u(z) \right.$$

$$\left. + \frac{K_\infty q^2}{\rho_0} (z + \chi q^2) \left(z + \frac{1}{\tau} \right) (z + a) \right\} = - \frac{\rho_0 D_{1\infty}}{z(z + 1/\tau')(z + 1/\tau)} (z - z_1)(z - z_2)(z - z_3)(z - z_4)(z - z_5), \qquad (1.5)$$

where

$$u(z) = z^2 (z + 1/\tau) + (M_\infty q^2 / \rho_0)(z + b), \quad v(z) = (z + \chi q^2)(z + 1/\tau') - \frac{\gamma - 1}{\gamma} \frac{K_\infty}{K_0} z(z + a),$$

$$a = K_0 / K_\infty \tau', \quad b = \mu_0 / \mu_\infty \tau, \quad \overline{M} = {}^4/_3 \overline{\mu}, \quad D_1 = D - c^2 \overline{K} = D \left(1 - \frac{\gamma - 1}{\gamma} \frac{\overline{K}}{K_0} \right), \qquad (1.6)$$

$\chi = \kappa / \rho_0 c_p$ is the heat conductivity of the medium, $\gamma = c_p / c_v$. Thus, the numerator of Δ contains a polynomial of the fifth degree in z in which all the roots z_j must, in accordance with the dissipation requirements of the system, lie in the left half-plane of z. All dispersion laws for the parameters must satisfy this criterion.

Substituting Eqs. (1.1) and (1.5) in Eq. (1.2) and using the notation of (1.6) it is easy to reduce the expression for the integrated intensity of the isotropic scattering to the form

$$I_{iso} = \frac{\theta}{2\pi i D_{1\infty}} \oint_\Gamma \left\{ \frac{g(z)}{f(z)} - \frac{g(-z)}{f(-z)} \right\} \frac{dz}{z}, \quad f(z) = (z - z_1)(z - z_2)(z - z_3)(z - z_4)(z - z_5), \qquad (1.7)$$

where the path of integration Γ goes along the imaginary z-axis and is closed at infinity, encompassing the left half-plane, and where

$$g(z) = \frac{DY^2}{K_\infty} \cdot \frac{(z + \chi q^2)(z^2 - 1/\tau'^2) u(z)}{z - a} - 2YZ_1 C \frac{z(z^2 - a/\tau') u(z)}{z - a} + Z_1^2 z [u(z)(z + 1/\tau') + (K_\infty q^2 / \rho_0)(z + a)(z + 1/\tau)]. \qquad (1.8)$$

Here, in place of Z we introduce the quantity

$$Z_1 = Z + CY, \tag{1.9}$$

by virtue of which the expression for $g(z)$ can be written in shorter form. The thermodynamic significance of Z_1, in accordance with Eqs. (1.4) and (1.9), is the following:

$$Z_1 = T_0 (\partial \varepsilon / \partial T)_\rho - \alpha T_0 (\partial \varepsilon / \partial \rho)_T = T_0 (\partial \varepsilon / \partial T)_p, \tag{1.10}$$

where p is the pressure.

The path of integration Γ chosen in (1.7) encloses all the poles z_j ($j = 1, \ldots, 5$) and, in addition, the pole $z = -a$ of the function $g(-z)$.* Now we find explicit expressions for the roots z_j in order to establish with which roots the specific individual components of I_{iso} are to be associated.

From eqs. (1.5) and (1.6) it is easy to show that one of the roots z_j (suppose this is z_1) becomes zero along with the heat conductivity χ. Actually — at least for liquids — the root z_1 is extremely small in absolute value as compared with the other roots. This allows us to consider values of z_1, calculated only to second order in χq^2, while the values of the other roots are taken for $\chi = 0$, that is, without taking the effect of heat conductivity into account. Thus

$$z_1 = \chi q^2 \gamma (M_0 + K_0)/(M_0 + \gamma K_0) \tag{1.11}$$

and

$$D_{1\infty}(z - z_2)(z - z_3)(z - z_4)(z - z_5) = D \left\{ \left[z + \frac{1}{\tau'} - \frac{\gamma - 1}{\gamma} \frac{K_\infty}{K_0}(z + a) \right] u(z) + \frac{K_\infty q^2}{\rho_0} \left(z + \frac{1}{\tau} \right)(z + a) \right\}. \tag{1.12}$$

In what follows we shall limit ourselves to the case of a medium with rather small viscous losses or [in accordance with (1.3)] what is the same thing, a medium with rather small dispersion differentials $\Delta K = K_\infty - K_0$ and $\Delta M = M_\infty - M_0$. Under these conditions we can use the values of the roots z_j, calculated from Eq. (1.12) with accuracy up to first order in ΔK and ΔM. These values are as follows:

$$z_2 = i\Omega_{MB} + h, \quad z_3 = z_2^*, \quad z_4 = -\frac{1}{\tau'}(1 - \zeta'), \quad z_5 = -\frac{1}{\tau}(1 - \zeta), \tag{1.13}$$

where

$$\Omega_{MB}^2 = (M_0 + \gamma K_0) \frac{q^2}{\rho_0} = \frac{K_0 q^2}{\rho_0} \left(\gamma + \frac{4}{3} \beta_7 \mu_0 \right), \quad -h = -\frac{\gamma^2 \tau' q^2 \Delta K}{2\rho_0 (1 + i\Omega_{MB}\tau')} - \frac{\tau q^2 \Delta M}{2\rho_0 (1 + i\Omega_{MB}\tau)},$$

$$\zeta' = \frac{\Delta K}{K_0} \left(1 - \gamma \frac{1 + \tau'^2 M_0 q^2 / \rho_0}{1 + \Omega_{MB}^2 \tau'^2} \right), \quad \zeta = \frac{\tau^2 q^2 \Delta M}{\rho_0 (1 + \Omega_{MB}^2 \tau^2)}.$$

It is apparent from Eqs. (1.11) and (1.13) that the residue at $z = z_1$ yields the integrated intensity of the central maximum (the undisplaced line). The residues at $z = z_2$ and $z = z_3$ determine the total intensity of the Mandel'shtam—Brillouin doublet, the maxima of which (with an accuracy up to the first-order dispersion corrections) lie at the frequencies $\Omega \pm \Omega_{MB}$. The residues at the poles $z = z_4 \approx -1/\tau'$ and $z = a \approx -1/\tau'$ yield the integrated intensity of that part of the background which is due to dispersion in the compression modulus $\overline{K}$. For brevity we will call this part of the background the "compression wing" to distinguish it from the "shear wing" or, in accordance with established terminology, simply the wing. This latter is the other part of the background, which is caused by dispersion in the shear modulus $\overline{\mu}$; the integrated intensity of this component is determined by the residue at the pole $z = z_5 \approx -1/\tau$. In low-viscosity liquids the extent of the two background components turns out to be considerably different because of the large difference in the relaxation times τ' and τ ($\tau'/\tau \sim 10^3$).

2. INTEGRATED INTENSITIES OF THE COMPONENTS OF I_{iso} (RESULTS)

Taking the residues at the poles or groups of poles indicated above and using the values of the roots (1.11) and (1.13), from Eq. (1.7) we find the following expressions for the integrated intensities.†

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*It is easily shown that $z = 0$ is not a singular point.

†In computing the residues we keep only those terms of order lower than the dispersion corrections in the expressions for the corresponding roots. Thus, taking the residue at $z = z_1$, everywhere, aside from the factor $z + \chi q^2$ we can substitute $z = 0$ in (1.8). In the remaining residues we can assume at the outset that $\chi = 0$ and $z_1 = 0$, keeping terms only of first order in ΔK and ΔM.

The undisplaced line:

$$I_{cent} = \frac{\Theta\,(\gamma-1)\,q^2}{\Omega^2_{MB}\rho_0\,(1+4\beta_T\mu_0/3)} - \left(Y + \frac{1+4\beta_T\mu_0/3}{\alpha T_0}Z\right)^2 ; \tag{2.1}$$

the Mandel'shtam—Brillouin doublet:

$$I_{MB} = \frac{\Theta q^2}{\rho_0}\left\{\frac{Y_1^2}{\Omega^2_{MB}} - \frac{\Delta K}{K_0}\cdot\frac{\gamma\tau'^2 Y_1}{1+\Omega^2_{MB}\tau'^2}\left[\frac{q^2\gamma\tau'^2 Y_1}{\beta_T\rho_0(1+\Omega^2_{MB}\tau'^2)} + 2\frac{\gamma-1}{\alpha T_0}Z\right] - \frac{\tau^4 q^2\Delta M}{\rho_0(1+\Omega^2_{MB}\tau^2)^2}Y_1^2\right\} ; \tag{2.2}$$

the compression wing:

$$I_{comp.wing} = \Theta\beta_T\frac{\Delta K}{K_0}\left(\frac{1+4\mu_0 q^2\tau'^2/3\rho_0}{1+\Omega^2_{MB}\tau'^2}\right)^2 Y'^2 ; \tag{2.3}$$

the shear wing:

$$I_{wing} = \Theta Y_1^2\tau^4 q^2\Delta M/\rho_0(1+\Omega^2_{MB}\tau^2)^2 . \tag{2.4}$$

In these formulas we use the thermodynamic expressions (1.4) for the parameters C and d (in accordance with the assumption that they do not exhibit dispersion) and the fact that $K_0 = 1/\beta_T$, where β_T is the isothermal compressibility. In addition, we take account of the well-known thermodynamic relations:

$$\alpha^2 T_0 = \rho_0 c_p(\beta_T - \beta_S), \quad \gamma = c_p/c_v = \beta_T/\beta_S, \tag{2.5}$$

where β_S is the adiabatic compressibility. Finally, we use the symbol Y_1 to denote the quantity $Y_1 = Y - ZCK_0/D_1$, the thermodynamic significance of which is easily established by means of (1.4) and (2.5):

$$Y_1 = -\rho_0\left(\frac{\partial\varepsilon}{\partial\rho}\right)_T - \frac{\gamma-1}{\alpha}\left(\frac{\partial\varepsilon}{\partial T}\right)_\rho = -\rho_0\left(\frac{\partial\varepsilon}{\partial\rho}\right)_S . \tag{2.6}$$

Before proceeding to a discussion of Eqs. (2.1) — (2.4) we point out that the sum of these expressions coincides with the intensity I_{iso} obtained earlier by more general methods [Eq. (4.3) in I] if, in the latter, we make the same additional assumptions as to which terms can be neglected in calculating the density of the component (specifically, that D is dispersionless, that the heat conductivity can be neglected and that only first-order terms in the dispersion corrections are needed). Eq. (4.3) of I then assumes the form

$$I_{iso} = \Theta\left\{\frac{Y^2}{K_0+M_0} + \frac{Z^2}{D_{10}} + \frac{\Delta K}{K_0^2}Y_1\left[2\frac{1+(4\mu_0 q^2/3\rho_0)\tau'^2}{1+\Omega^2_{MB}\tau'^2}Y - \left(Y + \frac{CK_0}{D_{10}}Z\right)\right]\right\},$$

which coincides with the sum of the intensities given by (2.1) — (2.4).

It is apparent from Eqs. (2.2) and (2.4) that the intensity of the shear wing compensates for the correction due to the dispersion in the shear modulus which is contained in the intensity expression for the doublet. Hence, I_{iso} is independent of ΔM as should be the case for the explicit dispersion relation given for $\bar\mu$, in accordance with the more general Eq. (4.3) given in I.

In what follows we shall limit ourselves to the case of a liquid ($\mu_0 = 0$). If we introduce the parameter

$$s = 1/\Omega^2_{MB}\tau'^2, \tag{2.7}$$

already used in I [Eq. (5.2)], Eqs. (2.1) — (2.3) assume the form

$$I_{cent} = \Theta\beta_T\left(\frac{Z_1}{\alpha T_0}\right)^2\cdot\frac{\gamma-1}{\gamma} = \Theta\frac{Z_1^2}{\rho_1 T_0 c_p}, \tag{2.8}$$

$$I_{MB} = \Theta\beta_T Y_1^2\left\{\frac{1}{\gamma} - \frac{\Delta K}{K_0(1+s)}\left[\frac{1}{1+s} + 2\frac{(\gamma-1)Z}{\alpha T_0 Y_1}\right]\right\} - I_\kappa; \tag{2.9}$$

$$I_{comp.wing} = \Theta\beta_T Y_1^2\frac{\Delta K}{K_0}\left(\frac{s}{1+s}\right)^2 . \tag{2.10}$$

In the <u>isotropic</u> part of the scattering, which is due to fluctuations in density and temperature only, the intensity of the shear wing (2.4) is extremely small. It is sufficient to compare I_{wing} with the first dispersion correction in the doublet intensity (2.9), i.e., the quantity

$$\Delta I_{MB} = \Theta\beta_T Y_1^2\Delta K/K_0(1+s)^2 .$$

Taking the ratio I_{wing}/I_{MB} and keeping in mind the fact that $\gamma K_0 q^2/\rho_0 = \Omega^2_{MB}$, we have

$$\frac{I_{wing}}{\Delta I_{MB}} = \frac{\Omega^2_{MB}\tau^2 (q^2\tau^2\Delta M/\rho_0)(1+s)^2}{(\gamma\Delta K/K_0)(1+\Omega^2_{MB}\tau^2)^2} .$$

But for liquids the values of the quantities $\Omega^2_{MB}\tau^2$ and $q^2\tau^2\Delta M/\rho_0 = 4\mu_\infty q^2 \tau^2/3\rho_0 = 4q^2\eta_0\tau/3\rho_0$ are of the order of 10^{-4} so that

$$I_{wing}/\Delta I_{MB} \sim (K_0/\gamma\Delta K)\ 10^{-8},$$

i.e., these are vanishingly small quantities even in the case in which $\Delta K/K_0$ amounts to a fraction of a per cent. Hence, we may neglect both the intensity of the shear wing and the corresponding correction in the doublet intensity (2.9).

The intensity of the compression doublet (2.10) is also small as compared with ΔI_{MB}

$$I_{comp.wing}/\Delta I_{MB} = s^2.$$

For the highest value of s which is at our disposal (s = 0.52 in carbon bisulfide) $I_{comp.wing}$ amounts to approximately one-quarter of ΔI_{MB}. Consequently, in comparing the intensities of the doublet and the undisplaced line we need not add $I_{comp.wing}$ to the latter.

Taking the ratio of the intensities (2.8) and (2.9) with the accuracy given above (to first order in ΔK) we find

$$\frac{I_{center}}{I_{MB}} = \left(\frac{Z_1}{\alpha T_0 Y_1}\right)^2 (\gamma-1)\left\{1 + \frac{\gamma\Delta K}{K_0(1+s)}\left[\frac{1}{1+s} + 2\frac{(\gamma-1)Z}{\alpha T_0 Y_1}\right]\right\}. \tag{2.11}$$

or, neglecting the quantities s and Z in the dispersion correction and making use of (1.10), (2.5), and (2.6)

$$\frac{I_{center}}{I_{MB}} = \left[\frac{1}{\alpha}\left(\frac{\partial\varepsilon}{\partial T}\right)_p \Big/ \rho_0\left(\frac{\partial\varepsilon}{\partial\rho}\right)_S\right]^2 \frac{\alpha^2 T_0}{\rho_0 c_p\beta_S}\left\{1 + \gamma\frac{\Delta K}{K_0}\right\}. \tag{2.12}$$

If we disregard the dispersion correction $\gamma\Delta K/K$ and set $Z = T_0(\partial\varepsilon/\partial T)_\rho = 0$ in accordance with (1.10) and (2.6), the first factor in (2.12) becomes unity, i.e., we obtain the Landau—Placzek formula[6]

$$I_{center}/I_{MB} = \alpha^2 T_0/\rho_0 c_p\beta_S = \gamma - 1.$$

As has already been noted by Fabelinskii,[3,7] without taking dispersion into account, that is, even in the framework of the thermodynamic theory, the first factor in Eq. (2.12) is considerably different from unity if we do not neglect $(\partial\varepsilon/\partial T)_\rho$. We can neglect this quantity without introducing a noticeable error in the value of integrated intensity of the isotropic scattering; however, it turns out that we cannot neglect it in the formula for the ratio of the intensities of the fine-structure components.

It can be shown that the dispersion correction can be taken into account in the ratio I_{cent}/I_{MB} using a formula of the usual "thermodynamic" form if for the adiabatic compressibility we take the quantity

$$\bar\beta_S = \beta_S(1 - \gamma\Delta K/K_0),$$

i.e., the reciprocal of the "adiabatic" compression modulus [cf. Eq. (5.5) of I].

$$\bar K_{ad} = \bar K D/D_1 = D\bar K/(D - C^2\bar K)$$

for $\Omega \to \infty$ or, more precisely, for $\Omega\tau' \gg 1$. The more complete expression for the dispersion correction contained in (2.11) corresponds to the case in which the compression modulus has not completely relaxed at the frequency of the doublet so that it is not very small as compared with unity.

The formula for I_{cent}/I_{MB} which contains the value of the compressibility at the doublet frequency was obtained earlier by Fabelinskii[3,4] from an analysis of the doublet on the basis of the diffraction of light by the Debye waves. He also allows for dispersion in the coefficient $(\partial\varepsilon/\partial\rho)_S$ so that we must also introduce the value of this parameter which corresponds to the doublet frequency in the ratio I_{cent}/I_{MB}. This is certainly proper (cf. below) if we depart from the purely terminological moment: if there is a frequency dependence in $Y_1(i\Omega)$ the use of the thermodynamic notation $(\partial\varepsilon/\partial\rho)_S = -Y_1(0)/\rho_0$ has significance only at sufficiently low frequencies. In concluding this section we consider the problem of how to introduce dispersion in Y and Z into the results of the calculation of the integrated intensities of the fine-structure components.

The derivation of Eqs. (2.1) — (2.4) indicates that the dispersion in Y and Z may appear in two ways. Firstly, in the component intensity we introduce the values $Y(z)$ and $Z(z)$ at the appropriate poles — at $z = z_1$ for the undisplaced line (i.e., the quasi-static values), at $z = z_2$ and $z = z_3$ for the doublet (i.e., the values of Y and Z at the frequency Ω_{MB}), and at $z = z_4 \approx -1/\tau'$ and $z = z_5 \approx -1/\tau$ respectively for the compression wing and the shear wing. Secondly, additional background components appear, given by the residues at the poles $z = \tilde{z}_W$ of the functions $Y(z)$ and $Z(z)$. If, however, the dispersion in Y and Z is small, wherever the intensity $J_{iso}(z)$ is still noticeable, these additional terms will be small and, in practice, we need consider only the first of the indicated effects. In other words, the additional terms do not play an important role in the case in which the characteristic frequencies of the coefficients Y and Z lie beyond the limits of the observed spectrum, i.e., $|\tilde{z}_W| \gg |z_j|$. This is the case even for the root $z_5 \approx -1/\tau$ of the dispersion equation ($|z_5| \gg |z_j|$, $j = 1, \ldots 4$) as a result of which it turns out that the shear wing is absent in practice and the observed region in the spectrum of J_{iso} is limited to the doublet components.

3. ANISOTROPIC SCATTERING. REMARKS ON THE ANALYSIS GIVEN BY M. A. LEONTOVICH[5]

Turning now to scattering by anisotropic fluctuations, which are responsible for the wing of the Rayleigh line (i.e., the shear wing), we may note, first of all, that the results of the well-known analysis by Leontovich[5] can be obtained from Eq. (3.1) of I as a particular case for the following assumptions as to the parameters. In Eq. (2.1) of I, which relates fluctuations of the dielectric constant to fluctuations in deformation and temperature

$$\varepsilon_{\alpha\beta} = X u'_{\alpha\beta} + Y u \delta_{\alpha\beta} + Z \vartheta \delta_{\alpha\beta} \qquad (u'_{\alpha\beta} = u_{\alpha\beta} - {}^1\!/_3 u \delta_{\alpha\beta}) \tag{3.1}$$

it is assumed that

(a) the coefficient X is proportional to the complex shear modulus

$$X(i\Omega) = B\mu(i\Omega), \quad B = \text{const}; \tag{3.2}$$

(b) the coefficient Y is dispersionless and is given by the real thermodynamic value (1.4);

(c) the quantity $\varepsilon_{\alpha\beta}$ is independent of temperature (for fixed density) i.e. z = 0.

With respect to the mechanical and thermal parameters of the medium it is assumed that

(d) there is no thermal expansion (C = 0);

(e) there is no dispersion in the compression modulus ($\overline{K} = K_0 = 1/\beta_T$, $\zeta = 0$);

(f) the shear modulus obeys the simplest dispersion law with a single relaxation time τ and $\mu_0 = 0$ (liquid)

$$\overline{\mu} = \mu + i\Omega\eta = \mu_\infty i\Omega / (i\Omega + 1/\tau). \tag{3.3}$$

It should be emphasized that some of these assumptions [(c), (d) and (e)] were made only to simplify the basic problem — the explanation of the wing in the case of liquids — thus dispensing beforehand with descriptions of the undisplaced line and the doublet.

Using the assumptions enumerated above and the notation

$$B = A/\mu_\infty, \quad \Omega_T^2 = \mu_\infty q^2/\rho_0, \quad \Omega_L^2 = K_0 q^2/\rho_0, \quad \Omega_S^2 = \Omega_L^2 + \frac{4}{3}\Omega_T^2, \quad \Delta_L = \Omega^2 - \Omega_L^2 + i\Omega\tau(\Omega^2 - \Omega_S^2), \quad \Delta_T = i\Omega + \tau(\Omega^2 - \Omega_T^2),$$

after several transformations Eq. (3.1) of I leads to the same expression which was obtained in the cited work of Leontovich, namely,

$$J_x^\rightarrow = \frac{\Theta A^2 \tau}{16\pi\mu_\infty} \left\{ \frac{3(\Omega^2 - \Omega_L^2)^2}{|\Delta_L|^2} + \frac{1}{1 + \Omega^2\tau^2} \right\}, \quad J_z^\rightarrow = J_x^\uparrow = \frac{\Theta A^2 \tau}{8\pi\mu_\infty} \left\{ \frac{\Omega^2}{|\Delta_T|^2} + \frac{1}{1 + \Omega^2\tau^2} \right\},$$

$$J_z^\uparrow = \frac{\Theta\tau}{2\pi\mu_\infty} \left\{ \frac{A^2}{2} \left[\frac{(\Omega^2 - \Omega_L^2)^2}{3|\Delta_L|^2} + \frac{1}{1 + \Omega^2\tau_2} \right] - \frac{4AY\Omega_T^2(\Omega^2 - \Omega_L^2)}{3|\Delta_L|^2} + \frac{8Y^2\Omega_T^4}{3|\Delta_L|^2} \right\}.$$

We note that because of assumptions (d) and (e) the frequency dependence of the isotropic part of $J_z^\uparrow$, i.e. of the quantity

$$J_{iso} = (\Theta\tau/2\pi\mu_\infty) \cdot 8Y^2\Omega_T^4/3|\Delta_L|^2,$$

cannot give the intensity distribution in the region of the doublet and the undisplaced line; nevertheless,

the integral of J_{iso} over Ω yields the Einstein formula, as can be shown by direct computation. In accordance with the general expression for the integrated intensity I_{iso} [cf. Eq. (4.3) I] this should be the case since a liquid is being considered ($\mu_0 = 0$) and (c) and (e) are assumed.

In that region of the wing which lies beyond the limits of the doublet, i.e., for $\Omega^2 > \Omega_S \approx \Omega_{MB}$, Eq. (3.4) yields*

$$J_x^{\rightarrow} = J_z^{\rightarrow} = J_x^{\updownarrow} = \frac{3}{4} J_z^{\updownarrow} = (\Theta A^2 \tau / 4\pi\mu_\infty) / (1 + \Omega^2 \tau^2),$$

so that the degree of depolarization in the wing is

$$J_x / J_z = (J_x^{\rightarrow} + J_x^{\updownarrow}) / (J_z^{\rightarrow} + J_z^{\updownarrow}) = 6/7,$$

and the reciprocal intensities are proportional to $1 + \Omega^2 \tau^2$. The slope of the line in the curve which shows the dependence of $1/J$ on Ω^2 obviously allows us to determine the relaxation time τ. This determination has been carried out by Fabelinskii[3,8] whose measurements have shown that in a number of liquids there is actually a wide region ($40 - 60$ cm^{-1}) in which the dependence of $1/J$ on Ω^2 is linear. The deviations from linearity occur in the region of Ω smaller than $15 - 25$ cm^{-1} and larger than 70 cm^{-1}. In this connection special attention is merited by the assumption in (3.2) which is important in the Leontovich analysis.

Generally speaking, it is difficult to find any basis for the fact that the frequency behavior of the mechanical-optical parameter $X(i\Omega)$ is completely determined by the dispersion of the purely mechanical quantity — the shear modulus $\bar{\mu}$. In Ref. 5, Leontovich starts with the assumption that

$$\overset{aniso}{\varepsilon_{\alpha\beta}} = A\zeta_{\alpha\beta} \quad (A = \text{const}),$$

where $\zeta_{\alpha\beta}$ is the so-called anisotropy tensor which is related to the pure shear deformations $u'_{\alpha\beta}$ by the relaxation equation

$$\dot{\zeta}_{\alpha\beta} + \zeta_{\alpha\beta}/\tau = \ddot{u}'_{\alpha\beta},$$

or spectrally by

$$\zeta_{\alpha\beta} = i\Omega u'_{\alpha\beta} / (i\Omega + 1/\tau). \tag{3.4}$$

Consequently, $\overset{aniso}{\epsilon_{\alpha\beta}} = X u'_{\alpha\beta}$, where

$$X = A i\Omega / (i\Omega + 1/\tau).$$

On the other hand, Leontovich has assumed that in the equations of motion of the liquid the pure shear stresses are also determined by the tensor $\zeta_{\alpha\beta}$, specifically, $\sigma'_{\alpha\beta} = 2\mu_\infty \zeta_{\alpha\beta}$. According to Eq. (3.4) this relation means that the complex shear modulus $\bar{\mu}$ obeys the dispersion law in (3.3). As a result X is expressed by Eq. (3.2) where $B = A/\mu_\infty$.

The fact that the observed dependence of the reciprocal intensity of the wing on Ω^2 departs from a linear relation may indicate both a more complicated dispersion for $\bar{\mu}$ if the assumption in (3.2) is to be retained and/or the inapplicability of (3.2), i.e., in the final analysis it may not be valid to assume that $\overset{aniso}{\epsilon_{\alpha\beta}}$ and $\sigma'_{\alpha\beta}$ depend on the <u>same internal parameter</u> in the same way. It is reasonable, at <u>sufficiently low frequencies</u> when

$$X(i\Omega) \approx X_0 + (\partial X / i\partial\Omega)_0 i\Omega, \quad \bar{\mu}(i\Omega) \approx \mu_0 + i\Omega\eta_0,$$

to assume for solid bodies that X_0 is proportional to μ_0 while for liquids ($X_0 = 0$, $\mu_0 = 0$) $(\partial X / i\partial\Omega)_0$ is proportional to η_0. <u>Formally</u> both cases are connected by the relation in (3.2).

The optical anisotropy in a solid body, which arises as a result of the static (or sufficiently low-frequency) stresses, is expressed by the following empirical formula[9] which we do not write in the usual way (in terms of the stresses) but in terms of the shear deformations

$$\overset{aniso}{\varepsilon_{\alpha\beta}} = 4\sqrt{\varepsilon_0}\, c\mu_0 u'_{\alpha\beta}.$$

Here c is the so-called relative optical stress coefficient, measured in Brewsters (1 Brewster $= 10^{-13}$ cm^2/dyne). An empirical formula for the anisotropy due to flow in the liquid (the Maxwell effect) is of the form[10]

$$\overset{aniso}{\varepsilon_{\alpha\beta}} = 2\, i\Omega\varepsilon_0\eta_0 M u'_{\alpha\beta}, \tag{3.5}$$

*It is sufficient if Ω exceeds Ω_{MB} by several line-widths of the doublet.

where M is the Maxwell constant. According to (3.2) at low frequencies we should have $B = \sqrt{\epsilon_0 c} = 2\epsilon_0 M$, i.e., the following relation between the photo-elastic constant for solid bodies c and the Maxwell constant for liquids M:

$$c = {}^{1}\!/_{2} \sqrt{\epsilon_0} M. \tag{3.6}$$

It is possible to establish the validity of this relation only in bodies which, in the low frequency region being considered, behave as "intermediate cases" between solids and liquds, i.e., bodies which allow measurement of both anisotropic effects. In typical liquids and typical bodies in which the comparison indicated above cannot be made, (3.6) is essentially meaningless. Without going any further into this problem we wish to emphasize once again that even if there is an actual relation like (3.6) at low frequencies there is no reason to expect that this relationship is maintained at frequencies of the order of 10^{11} and that the relation in (3.2) is satisfied.

4. SPECTRAL INTENSITIES IN THE SHEAR WING

We now turn to a consideration of the shear wing and certain consequences which follow from the general formulas (3.1) in I. We are interested in frequencies Ω which exceed both the doublet frequency Ω_{MB} as well as the characteristic frequency of the compression modulus $1/\tau'$. In this case, from Eq. (3.2) in I we have

$$d \approx D_\infty, \quad d_1 \approx D_{1\infty} = D_\infty - C_\infty^2 K_\infty, \quad \Delta \approx D_{1\infty}\rho_0\Omega^2, \quad A_1 \approx A_3 \approx A_4 \approx \rho_0\Omega^2,$$

as a result of which the general expressions for the intensities (3.1) from I are reduced to the following:

$$J_x^{\rightarrow} = J_z^{\rightarrow} = J_x^{\uparrow} = \frac{3}{4} J_z^{\uparrow} = \frac{\Theta \mid X \mid^2}{8\pi i \Omega}\left(\frac{1}{\mu^*} - \frac{1}{\mu}\right) = \frac{\Theta}{4\pi}\left|\frac{X}{\mu}\right|^2 \eta. \tag{4.1}$$

Thus, the degree of polarization J_x/J_z is ${}^6\!/_7$ in the wing regardless of the frequency behavior of X and $\overline{\mu}$.

According to Eq. (4.1), a graph on which is plotted the reciprocal value of any of the intensities as a function of Ω^2 represents the quantity $|\overline{\mu}/X|^2/\eta$ as a function of Ω^2. If we grant the validity of the relation given in (3.2), i.e., X = B$\overline{\mu}$, this quantity is simply const/η which indicates a linear dependence of $1/\eta$ on Ω^2 if the dispersion relation given in (3.3) for μ is assumed. Since, as was indicated above, the relation given in (3.2) is open to doubt, it is reasonable to interest ourselves in those relations which give simple dispersion laws for X and $\overline{\mu}$ in the event that (3.2) is inapplicable.

Using the dispersion relation for $\overline{\mu}$ given in (3.3) we assume that

$$X(i\Omega) = (X_\infty i\Omega + X_0/\tau_1)/(i\Omega + 1/\tau_1). \tag{4.2}$$

Then, as can easily be shown, each of the intensities (4.1) has a frequency dependence given by the expression

$$\frac{1}{J} \sim \left|\frac{\overline{\mu}}{X}\right|^2 \frac{1}{\tau_1} \sim x\frac{x+1}{x+\beta}, \quad x = \Omega^2\tau_1^2, \quad \beta = \frac{X_0^2}{X_\infty^2}. \tag{4.3}$$
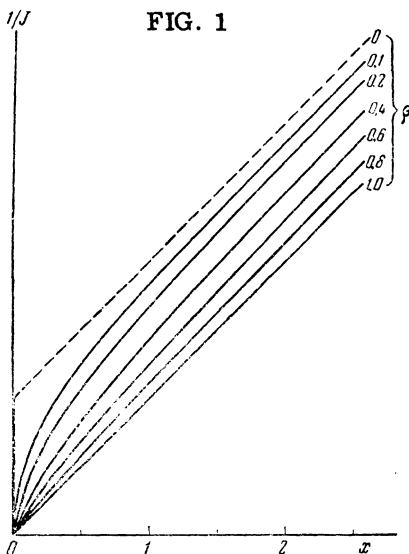

FIG. 1

For different values of $\beta < 1$ the relation in (4.3) is shown by the solid curves in Fig. 1. The dashed line corresponds to the value $\beta = 0$ and is inclined at the same angle as the asymptotes of the other curves for increasing x. This dashed line corresponds, in particular, to the Leontovich theory, since it is assumed that X is proportional to $\overline{\mu}$ and consequently that $\tau_1 = \tau$, $\beta = \mu_0^2/\mu_\infty^2 = 0$. However, the departure of the experimental curves in the low frequency region from the linear behavior is precisely that which is to be expected from (4.3) for $\beta \neq 0$. Assuming the relation in (4.2), we have the possibility of determining from the experimental curves not only τ_1 but also the value of the parameter β. Thus, for example, the curve for carbon bisulfide given in Fig. 2 of Ref. 8, which is incorrect according to Eq. (4.30), gives $\tau_1 \approx 1.5 \times 10^{-13}$ sec from the slope of the asymptote whereas the fourth point in the region in which the curve slopes downward gives a value of $\beta \approx 0.09$ with a spread of not more than 10 per

cent. From the data of Weiler,[11] we obtain in benzol a value of $\tau_1 \approx 2.16 \times 10^{-13}$ sec and $\beta \approx 0.385$.

It is clear, however, that in the liquid case being considered the dispersion law for X given in (4.2) cannot remain effective to any arbitrarily low frequency because when $\Omega \to 0$, this relation gives a value $X = X_0$ different from zero. According to Eq. (3.1) this would denote a photo-elastic anisotropy, which is possible only in a solid body. Assuming that the relation given in (4.2) applies only at sufficiently high values of x we may assume that deviations from this relation are due to the existence of other relaxation times $\tau_2 \gg \tau_1$ in X. The simplest dependence, for which we introduce only one other additional relaxation time τ_2 and which by giving $X \to 0$ as $\Omega \to 0$ maintains (4.2) for $\Omega \gg 1/\tau_2$, is of the form

$$X(i\Omega) = \frac{i\Omega}{i\Omega + 1/\tau_2} \cdot \frac{X_\sigma i\Omega + X_0/\tau_1}{i\Omega + 1/\tau_1}. \tag{4.4}$$

Then, in place of (4.3) the reciprocal intensity is given by the expression

$$\frac{1}{J} \sim \left(\frac{\tau_1^2}{\tau_2^2} + x\right)\frac{x+1}{x+3}, \tag{4.5}$$

which gives the former behavior of the curve (Fig. 1) at $x \gg \tau_1^2/\tau_2^2$ but a value of $1/J$ different from zero as $x \to 0$.

The first factor in (4.5) determines the behavior of $1/J$ in the narrow and most intense central part in which both the width of the central part as well as the maximum value of the intensity depend on τ_2 for $x = 0$. The value of τ_2 can be estimated from any of these parameters; however, in view of the difficulty in measuring the intensity of the wing at zero, more reliable results can be obtained by estimating the width of the central part of the wing.

At low frequencies ($\Omega \ll 1/\tau_2$), in accordance with (4.4) we have $X = i\Omega\tau_2 X_0$ so that the anisotropic part of $\epsilon_{\alpha\beta}$ is found to be equal to $\epsilon_{\alpha\beta}^{aniso} = i\Omega\tau_2 X_0 u'_{\alpha\beta}$. Comparison with Eq. (3.5) indicates that in the relation given in (4.4)

$$X_0 = 2\varepsilon_0 \eta_0 M/\tau_2. \tag{4.6}$$

In evaluating the applicability of Eq. (4.4) at low frequencies special interest attaches to the observation of the diffraction of linear polarized light on ultrasonic waves, propagating at an angle (say, 45°) to the direction of the crossed axes of the analyzer and polarizer. An experiment of this type may give some idea of the frequency dependence of $|X|^2$ up to $\Omega \sim 10^8$.

As has already been noted, the experimental curves which show the dependence of $1/J$ on Ω^2 also exhibit a departure from a linear dependence in the high-frequency region. This result also indicates a departure in the dispersion relations for and/or $\bar\mu$ from the simple forms taken above but we will not dwell on this problem at this point.

5. INTEGRATED INTENSITIES OF THE ANISOTROPIC SCATTERING

Since the dispersion relations given in (3.3) for $\bar\mu$ and in (4.4) for X permit a good approximation to the behavior of the intensity over the entire extent of the wing, with the exclusion of the high-frequency part, where the intensity is small,* it is of interest to calculate the integrated intensities of the anisotropic scattering using (3.3) and (4.4). For this purpose we turn to Eq. (3.1) of I noting, first of all, that in the case of a low-viscosity liquid, to which we are limiting ourselves, the quantity $\Omega_T^2 \tau^2 = q^2\eta_0\tau/\rho_0$ is of the order of 10^{-4} thus making it possible to neglect certain terms and to simplify considerably the expression for the spectral intensities.† In particular, in the expression for $J_{\vec x}$ we can neglect terms which contain the determinant Δ in the denominator while in the expression for $J_{\vec z} = J_{\vec x}^t$ it may be assumed that $A_4 = \rho_0\Omega^2 - \bar\mu q^2 \approx \rho_0\Omega^2$. Then Eq. (3.1) of I assumes the form

$$J_{\vec x} = J_{\vec z} = J_{\vec x}^t = \frac{\Theta|X|^2}{8\pi i\Omega}\cdot\left(\frac{1}{\bar\mu^*} - \frac{1}{\bar\mu}\right), \qquad J_{\vec z}^t = \tfrac{4}{3}J_{\vec x} + J_{cross} + J_{iso},$$

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*Actually, this intensity falls off more rapidly than as given by (4.5), i.e., more rapidly than $1/\Omega^2$.

† The validity of neglecting these terms was verified in calculations of the integrated intensities which were carried out without eliminating small terms.

where J_{cross} denotes "cross" terms of I_z^t which contain products of X and Y or Z and where I_{iso} is given by Eq. (1.1).

To compute the integral of J_{cross} over Ω we neglect terms with Z and assume that Y, C and $\overline{K}$ are dispersionless. Then the integrated intensities are

$$I_x^{\rightarrow} = I_z^{\rightarrow} = I_x^t = \frac{\Theta}{4} \frac{1}{2\pi i} \oint_\Gamma |X|^2 \left(\frac{1}{\mu^*} - \frac{1}{\mu}\right) \frac{dz}{z}, \qquad I_z^t = {}^4/_3 I_x^{\rightarrow} + I_{cross} + I_{iso}, \tag{5.1}$$

$$I_{cross} = \frac{\Theta Y q^2}{3} \frac{1}{2\pi i} \oint_\Gamma \left\{\left(X + X^* \frac{\overline{\mu}}{\mu^*}\right) \frac{d_1}{\Delta} - \text{comp. conj.}\right\} \frac{dz}{z}.$$

Here $z = i\Omega$ while the path of integration Γ goes along the imaginary z axis and is closed at infinity, enclosing the left-hand plane. The expression for I_{iso} has been found in I [Eqs. (5.1) or (5.3)] but for the following estimates we will simply use the Einstein formula

$$I_{iso} = \Theta Y^2 \beta_T. \tag{5.2}$$

Substituting Eqs. (3.3) and (4.4) in the top formula in Eq. (5.1) we find

$$I_x^{\rightarrow} = \frac{\Theta X_\infty^2}{4\mu_\infty} \frac{1}{2\pi i} \oint_\Gamma \left\{\frac{g(z)(z-\alpha)(z-\alpha')}{f(z)(z-a)(z-a')} - \frac{g(-z)(z+\alpha)(z+\alpha')}{f(-z)(z+a)(z+a')}\right\} \frac{dz}{z}, \tag{5.3}$$

where we have introduced the notation

$$\frac{g(z)}{f(z)} = \frac{(z+\alpha)z}{(z+a)(z+a')}, \quad \alpha = \frac{\sqrt{3}}{\tau_1}, \quad \alpha' = \frac{1}{\tau} \quad a = \frac{1}{\tau_1} \quad a' = \frac{1}{\tau_2}. \tag{5.4}$$

As to I_{cross}, the integral of $(Xd_1/\Delta - \text{comp. conj.})$ is obtained by the theorem given in (7.5) of Ref. 2 and is $X_0/K_0 q^2$ while the integral of $((x^*\mu d_1/\mu^*\Delta) - \text{comp. conj.})$, after substitution of (3.3) and (4.4), assumes the same form as the integral in (5.3) except that

$$g(z)/f(z) = X_\infty z d_1/(z+\alpha')\Delta.$$

Thus, all the calculations lead to computation of an integral of the type given in (5.3) where all the roots of $f(z)$ are in the left half-plane while the poles $z = a$ and $z = a'$ are in the right half-plane. An appropriate extension of the theorem given in (7.4) of Ref. 2 (which provides for only the existence of one root on the right) is not difficult; specifically:

$$\frac{1}{2\pi i} \oint_\Gamma \left\{\frac{g(z)(z-\alpha)(z-\alpha')}{f(z)(z-a)(z-a')} - \frac{g(-z)(z+\alpha)(z+\alpha')}{f(-z)(z+a)(z+a')}\right\} \frac{dz}{z}$$

$$= \frac{g(\infty)}{f(\infty)} - 2\frac{g(a)}{f(a)}\left(1 - \frac{\alpha}{a}\right)\frac{a-\alpha'}{a-a'} - 2\frac{g(a')}{f(a')}\left(1 - \frac{\alpha'}{a'}\right)\frac{a'-\alpha}{a'-a} - \frac{g(0)\alpha\alpha'}{f(0)aa'}. \tag{5.5}$$

Using this formula, from (5.3) we have

$$I_x^{\rightarrow} = (\Theta X_\infty^2/4\mu_\infty)\alpha'(\alpha^2 + aa')/aa'(a+a'), \tag{5.6}$$

while for I_{iso}, calculating the values of Δ and $d_1 = D_1 + \kappa T_0 q^2/z$ for $z \to \infty$, $z = a$ and $z = a'$ by means of Eqs. (1.5), (1.6) and (3.3), we find

$$I_{cross} = -\frac{\Theta X_0 Y}{3K_0}\left\{1 - \frac{2\Omega_{MB}^2 X_\infty}{(a-a')X_0}\left[\frac{(a-\alpha')(a-\alpha)}{(a^2+\Omega_{MB}^2)(a+a')+\frac{4}{3}\Omega_T^2 a} - \frac{(a'-\alpha')(a'-\alpha)}{(a'^2+\Omega_{MB}^2)(a'+\alpha')+\frac{4}{3}\Omega_T^2 a'}\right]\right\},$$

where, as before, $\Omega_{MB}^2 = \gamma K_0 q^2/\rho_0$, $\Omega_T^2 = \mu_\infty q^2/\rho_0 = \eta_0 q^2/\rho_0\tau$. If we now take account of the fact that the quantity $a \sim \alpha \sim \alpha'$ is much larger than $\Omega_{MB} \sim \Omega_T$, the first correction term in I_{iso} turns out to be very small (smaller than $\Omega_{MB}^2\tau_1^2 \sim 10^{-4}$) and may be disregarded while the second term can be simplified considerably

$$I_{cross} = -\frac{\Theta X_0 Y}{3}\beta_T\left\{1 - \frac{2X_\infty\Omega_{MB}^2\alpha}{X_0 a(a'^2+\Omega_{MB}^2)}\right\}. \tag{5.7}$$

In Eqs. (5.6) and (5.7), returning to the original notation, using (5.4) and replacing X_∞ by $\sqrt{\beta}X_0$, from Eqs. (5.1) and (5.2) we have the following final result:

$$I_x^{\dagger} = I_x^{\updownarrow} = I_z^{\rightarrow} = \frac{\Theta X_0^2}{4\eta_0} \frac{\tau_2(\tau_2 + \tau_1/\beta)}{\tau_2 + \tau_1}, \qquad I_z^{\updownarrow} = \frac{4}{3} I_x^{\rightarrow} - \frac{\Theta X_0 Y}{3} \beta_T \frac{1 - \Omega_{MB}^2 \tau_2^2}{1 + \Omega_{MB}^2 \tau_2^2} + \Theta Y^2 \beta_T. \qquad (5.8)$$

The increase in the anisotropic intensities with increasing τ_2 is completely understandable since the finite value of τ_2 specifically limits the intensity of the brightest central part of the curve. The intensity I_{cross} can take on different values between $\pm \Theta X_0 Y \beta_T /3$ depending on the magnitude of $\Omega_{MB}\tau_2$, i.e., on the ratio between the width of the central part of the wing and the frequency of the doublet. In low-viscosity liquids $\Omega_{MB}^2 \tau_2^2 \ll 1$ so that the correction to $I_{cross} = -\Theta X_0 Y \beta_T /3$ can, in general, be discarded.

Finally, we have the following expression for the depolarization coefficient in Rayleigh scattering in low-viscosity liquids:

$$\Delta = \frac{I_x^{\rightarrow} + I_x^{\updownarrow}}{I_z^{\rightarrow} + I_z^{\updownarrow}} = \frac{6}{7} \left[1 + \frac{12}{7} \beta_T \eta_0 k \frac{\tau_2 + \tau_1}{\tau_2 + \tau_1/\beta} \left(k\tau_2 + \frac{1}{3} \right) \right]^{-1}, \qquad (5.9)$$

where [cf. Eq. (4.6)]

$$k = |Y| / X_0 \tau_2 = \rho_0 (\partial \varepsilon / \partial \rho)_T / 2\varepsilon_0 \eta_0 M. \qquad (5.10)$$

We estimate the value of Δ for benzol, assuming that* $\beta_T = 9.36 \times 10^{-11}$, $\eta_0 = 6.5 \times 10^{-3}$, $\epsilon_0 = 2.32$, $\rho_0 (\partial \varepsilon / \partial \rho)_T = 1.62$, $M = 9 \times 10^{-11}$, $\tau_1 = 2.16 \times 10^{-13}$, $\beta = 0.385$, $\tau_2 = 3 \times 10^{-12}$. The condition that $\Omega_{MB}^2 \tau_2^2$ be small is thus satisfied (this quantity is approximately 7×10^{-3}). Eq. (5.10) gives $k = 5.97 \times 10^{11}$. From Eq. (5.9) we find $\Delta = 0.39$ while the experimental value is $\Delta = 0.43$. The latter is obtained from Eq. (5.9) if, maintaining all the other parameters fixed we set $\tau_2 = 2.5 \times 10^{-12}$. Considering the inaccuracy of the determination of τ_2† and the presence of a number of (apparently less important) assumptions which were made in deriving Eq. (5.9),‡ the result which has been obtained is completely satisfactory. In the present paper we are concerned with only approximate values. A more complete comparison of the available experimental data for low-viscosity liquids with the formulas which have been obtained as well as the consideration of high-viscosity liquids and solid bodies will be carried out in a separate paper.

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*The values of the quantities written in the upper line are taken from Ref. 3 where the statistical value was used for $\rho_0 (\partial \varepsilon / \partial \rho)_T$, in accordance with the neglect of dispersion assumed above in estimating I_{cross} and I_{iso}. The values for τ_1 and β were obtained in Section 4 from the data of Weiler.[11] The Maxwell constant M is taken from Ref. 12 (for 20°C).

†We assume the value of τ_2, calculated from the results of the measurements made by Fabelinskii[3] from the width of the central part of the plane (at the half-intensity level). In this procedure the frequency interval from 0 to 20 cm^{-1} alone is used. The estimate of τ_2 made by Fabelinskii considering the region from 0 to 8 cm^{-1} gives a close value 3.3×10^{-12}. According to the data of Ref. 11, using the slope of the tangent to the curve $1/J(\Omega^2)$ at the point $\Omega = 0$ we find $\tau_2 = 2 \times 10^{-12}$.

‡The neglect of dispersion in Y and K and the fact that the anisotropic intensities fall off faster than $1/\Omega^2$ in the high-frequency part of the wing.