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ON THE THEORY OF THE NEUTRINO WITH ORIENTED SPIN

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LEE and Yang[1] have advanced the hypothesis of the nonconservation of parity under spatial inversion in the weak interactions. Developing this idea, Lee and Yang,[2] Salam,[3] and Landau[4] have suggested that this violation of the parity rule can in particular cases be related to special properties of the neutrino, by requiring that it satisfy an equation with the two-rowed Pauli matrices. According to this theory the spin of the neutrino is always parallel to the direction of its momentum, and the spin of the antineutrino is always antiparallel to its momentum. As has been shown by Landau and by Lee and Yang, this theory is invariant with respect to combined inversion. Combined inversion means interchange of particle and antiparticle with simultaneous spatial inversion.

We wish to show that the new theory of the neutrino can be obtained from the Dirac theory, if in the latter one carries out an explicit resolution of the functions in terms of spin states.[5] Then it is not necessary to separate the interaction energy into a sum of main quantities and their pseudo-values (for example, scalar plus pseudoscalar).

The Dirac equation for a free particle has the form

$$(\hat{E} \mp m_0 c^2)\begin{pmatrix}\psi_{1,3}\\ \psi_{2,4}\end{pmatrix} = c(\sigma'\hat{p})\begin{pmatrix}\psi_{3,1}\\ \psi_{4,2}\end{pmatrix}, \quad (1)$$

where $\hat{E}$ and $\hat{p}$ are the operators for energy and momentum, respectively, and σ' is the two-rowed Pauli matrices. Since the mass of the neutrino is zero ($m_0 = 0$), we get a linear relation between the functions,

$$\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} = \varepsilon\begin{pmatrix}\psi_3\\ \psi_4\end{pmatrix}, \quad (2)$$

where $\epsilon = \pm 1$.

We can choose four values for ϵ: (a) $\epsilon = 1$ (states with $E > 0$ and $E < 0$ describe the neutrino), (b) $\epsilon = -1$ (states with $E > 0$ and $E < 0$ describe the antineutrino), (c) $\epsilon = E/|E|$ (states with $E > 0$ correspond to neutrinos and states with $E < 0$ to antineutrinos), and (d) $\epsilon = -E/|E|$ (states with $E > 0$ correspond to antineutrinos, and those with $E < 0$ to neutrinos).

We consider first of all the case $\epsilon = E/|E|$, in which the neutrino is a particle with positive energy and the antineutrino is a hole in the background of negative levels. Equation (1) takes the form

$$(\hat{E} - \varepsilon\sigma(\sigma'\hat{p}))\begin{pmatrix}\psi_{1,3}\\ \psi_{2,4}\end{pmatrix} = 0. \quad (3)$$

The solution of Eq. (1) is of the form (see Ref. 5)

$$\begin{pmatrix}\psi_{1,3}\\ \psi_{2,4}\end{pmatrix} = L^{-3/2}\sum_{\mathbf{k}}\frac{1}{\sqrt{2}}\begin{pmatrix}\cos\frac{\theta}{2}\\ \sin\frac{\theta}{2}\,e^{i\varphi}\end{pmatrix}e^{i\mathbf{kr}}\left[C(\mathbf{k})\,e^{-ickt} \pm \tilde{C}^{+}(-\mathbf{k})\,e^{ickt}\right], \quad (4)$$

where $E = c\hbar k$ is the energy of the particle and $\mathbf{p} = \hbar\mathbf{k}$ is its momentum.

We get the following expressions for the total energy $\bar{E}$, the total momentum $\mathbf{G}$, and the total spin component $\bar{S}$ in the direction of motion:

$$\bar{E} = c\int d^3x \left[(\psi_3^+\psi_4^+)(\sigma'\hat{p}) \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} + (\psi_1^+\psi_2^+)(\sigma'\hat{p}) \begin{pmatrix}\psi_3\\ \psi_4\end{pmatrix} \right] = \sum_{\mathbf{k}} c\hbar k\,[C^+(\mathbf{k})\,C(\mathbf{k}) - \tilde{C}(-\mathbf{k})\,\tilde{C}^+(-\mathbf{k})],$$

$$\bar{\mathbf{G}} = \sum_{\mathbf{k}} \hbar\mathbf{k}\,[C^+(\mathbf{k})\,C(\mathbf{k}) + \tilde{C}(-\mathbf{k})\,\tilde{C}^+(-\mathbf{k})],\quad \bar{S} = \sum_{\mathbf{k}} [C^+(\mathbf{k})\,C(\mathbf{k}) + \tilde{C}(-\mathbf{k})\,\tilde{C}^+(-\mathbf{k})]. \tag{5}$$

If the neutrino amplitudes obey the Fermi type of commutation relations, then we have

$$C^+(\mathbf{k})\,C(\mathbf{k}) = N(\mathbf{k}),\quad \tilde{C}(\mathbf{k})\,\tilde{C}^+(\mathbf{k}) = 1 - \tilde{N}(\mathbf{k}). \tag{6}$$

It follows from Eq. (5) that neutrinos N and antineutrinos $\tilde{N}$ have positive energies, and that the spin of the neutrino is parallel to its momentum, that of the antineutrino antiparallel to its momentum.

In this theory we have only two states (and not four as in the theory of the electron), since the neutrino has no charge. Therefore states with different charges but having the same value for the spin component are not distinguishable.

The trace with neutrino wave functions can be calculated easily from the formula for electron wave functions with fixed spin. This formula can be written in the form:[5]

$$b'^+\beta' b b^+ \beta b' = \frac{1}{16}\,\mathrm{Sp}\left[\beta'\left(1 + \rho_1\varepsilon' s' \frac{k'}{K'} + \varepsilon'\rho_3 \frac{k_0'}{K'}\right)\left(1 + s'\sigma\mathbf{k}'/k'\right)\beta\left(1 + \rho_1 \varepsilon s \frac{k}{K} + \varepsilon\rho_3 \frac{k_0}{K}\right)\left(1 + s\sigma\mathbf{k}/k\right)\right], \tag{7}$$

where b and b' are the spin amplitudes of the functions and β and β' are Dirac matrices.

If one of the particles is a neutrino, the corresponding mass must be set equal to zero ($k_0 = 0$, $K = k$) and $s = 1$. Then the states with $\epsilon = 1$ correspond to neutrinos and those with $\epsilon = -1$ to antineutrinos.

From Eq. (7) it can be seen that in another variant of the theory we can put for neutrinos $\epsilon = 1$, $s = 1$, and for antineutrinos $\epsilon = 1$, $s = -1$. Therefore we can use for their description the Majorana equation with real wave functions. If we substitute into the interaction energy not the general solution for ψ_{neutrino} (as was done by Majorana himself) but the solution separated according to the spin states, then with this procedure double β-decay (i.e., emission from a nucleus of two electrons without neutrinos) will be forbidden, unlike the situation in the old variant of the Majorana theory.

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