HYDRODYNAMIC FLUCTUATIONS IN A SUPERFLUID LIQUID

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L. Landau and E. Lifshitz[1] have recently calculated the fluctuations of the hydrodynamical parameters in classical hydrodynamics by introducing "outside" terms in the equations of motion.[2]

We will apply the same method to a calculation of the fluctuations of the hydrodynamical parameters in a superfluid liquid. For generality we will assume that there are foreign particles dissolved in the superfluid. In view of the unusual simplicity of the method, this assumption does not complicate the calculations. We write down the hydrodynamical equations of the superfluid solution with "outside" terms

$$\partial\rho/\partial t + \operatorname{div}\mathbf{j} = 0, \quad \partial j_i/\partial t + \partial\Pi_{ik}/\partial x_k = -\partial\tau_{ik}/\partial x_k;$$

$$\partial\rho c/\partial t + \operatorname{div}\rho c\mathbf{v}_n = -\operatorname{div}\mathbf{g}, \quad \partial\mathbf{v}_s/\partial t + \nabla(\mu - cZ/\rho + v_s^2/2) = -\nabla h;$$

$$T\left\{\frac{\partial S}{\partial t} + \operatorname{div}\left(S\mathbf{v}_n - \frac{Z}{\rho T}\mathbf{g}\right)\right\} = -\left\{h\operatorname{div}(\mathbf{j}-\rho\mathbf{v}_n) + \frac{1}{2}\tau_{ik}\left(\frac{\partial v_{ni}}{\partial x_k} + \frac{\partial v_{nk}}{\partial x_i}\right) + \mathbf{g}T\nabla\frac{Z}{\rho T}\right\} - \operatorname{div}\mathbf{q};$$

$$\tau_{ik} = -\eta\left(\frac{\partial v_{ni}}{\partial x_k} + \frac{\partial v_{nk}}{\partial x_i} - \frac{2}{3}\delta_{ik}\frac{\partial v_{nl}}{\partial x_l}\right) - \delta_{ik}\left[\zeta_1\operatorname{div}(\mathbf{j}-\rho\mathbf{v}_n) + \zeta_2\operatorname{div}\mathbf{v}_n\right] + T_{ik}.$$

$$h = -\zeta_3\operatorname{div}(\mathbf{j}-\rho\mathbf{v}_n) - \zeta_4\operatorname{div}\mathbf{v}_n + H; \quad \mathbf{g} = -\rho D\left(\nabla c + \frac{k_T}{T}\nabla T + \frac{k_p}{p}\nabla p\right) + \mathbf{G},$$

$$\mathbf{q} = -\rho DT\left(\nabla c + \frac{k_T}{T}\nabla T + \frac{k_p}{p}\nabla p\right)\left[k_T\frac{\partial}{\partial c}\left(\frac{Z}{\rho T}\right) - T\frac{\partial}{\partial T}\left(\frac{Z}{\rho T}\right)\right] - \varkappa\nabla T + \mathbf{Q}.$$

We use the same notation here as in Refs. 3 and 4.* In these equations we have introduced an "outside" momentum flow tensor T_{ik}, an "outside" potential H, and "outside" flow vector for the dissolved particles **G**, and an "outside" heat flow vector **Q**.

The rate of change of the total entropy of the liquid is, according to our earlier work,[3,4]

$$\dot{S}_{\text{tot}} = -\frac{1}{T}\int\left\{h\operatorname{div}(\mathbf{j}-\rho\mathbf{v}_n) + \frac{1}{2}\tau_{ik}\left(\frac{\partial v_{ni}}{\partial x_k} + \frac{\partial v_{nk}}{\partial x_i}\right) + \frac{\mathbf{q}\nabla T}{T} + \mathbf{g}T\nabla\frac{Z}{\rho T}\right\}dV.$$

The correspondence between the quantities figuring in the theory of fluctuations[5] (equations of motion $\dot{x}_\alpha = \gamma_{\alpha\beta}x_\beta + y_\alpha$), and the quantities in our equations are given by the following table

$\dot{x}_\alpha$	τ_{ik}	h	$\mathbf{g}$	$\mathbf{q}$
X_α	$-\frac{1}{2T}\left(\frac{\partial v_{ni}}{\partial x_k} + \frac{\partial v_{nk}}{\partial x_i}\right)\Delta V$	$-\frac{1}{T}\operatorname{div}(\mathbf{j}-\rho\mathbf{v}_n)\Delta V$	$-\nabla\frac{Z}{\rho T}\Delta V$	$-\frac{\nabla T}{T^2}\Delta V.$
y_α	T_{ik}	H	$\mathbf{G}$	$\mathbf{Q}$

Making use of the expression

$$\overline{y_\alpha(t_1)\,y_\beta(t_2)} = k(\gamma_{\alpha\beta} + \gamma_{\beta\alpha})\,\delta(t_1 - t_2),$$

we obtain the fluctuations of the "outside" terms in the hydrodynamical equations of a superfluid liquid

$$\overline{T_{ik}(\mathbf{r}_1t_1)\,T_{lm}(\mathbf{r}_2t_2)} = 2kT\left\{\eta(\delta_{il}\delta_{km} + \delta_{im}\delta_{kl}) + \left(\zeta_2 - \frac{2}{3}\eta\right)\delta_{ik}\delta_{lm}\right\}\cdot\delta(\mathbf{r}_1 - \mathbf{r}_2)\,\delta(t_1 - t_2),$$

$$\overline{T_{ik}(\mathbf{r}_1t_1)\,H(\mathbf{r}_2t_2)} = 2kT\zeta_1\delta_{ik}\,\delta(\mathbf{r}_1 - \mathbf{r}_2)\,\delta(t_1 - t_2), \quad \overline{H(\mathbf{r}_1t_1)\,H(\mathbf{r}_2t_2)} = 2kT\zeta_3\delta(\mathbf{r}_1 - \mathbf{r}_2)\,\delta(t_1 - t_2),$$

$$\overline{Q_i(\mathbf{r}_1t_1)\,Q_k(\mathbf{r}_2t_2)} = 2kT^2\left\{\varkappa + \rho D\left[\frac{\partial}{\partial c}\left(\frac{Z}{\rho T}\right)\cdot k_T - T\frac{\partial}{\partial T}\left(\frac{Z}{\rho T}\right)\right]^2 \Big/ \frac{\partial}{\partial c}\left(\frac{Z}{\rho T}\right)\right\}\,\delta(\mathbf{r}_1 - \mathbf{r}_2)\,\delta(t_1 - t_2),$$

$$\overline{G_i(\mathbf{r}_1t_1)\,G_k(\mathbf{r}_2t_2)} = \left[2k\rho D \Big/ \frac{\partial}{\partial c}\left(\frac{Z}{\rho T}\right)\right]\delta_{ik}\,\delta(\mathbf{r}_1 - \mathbf{r}_2)\,\delta(t_1 - t_2),$$

$$\overline{G_i(\mathbf{r}_1 t_1) Q_k(\mathbf{r}_2 t_2)} = 2kT \left\{\rho D\left[\frac{\partial}{\partial c}\left(\frac{Z}{\rho T}\right) k_T - T\frac{\partial}{\partial T}\left(\frac{Z}{\rho T}\right)\right] \Big/ \frac{\partial}{\partial c}\left(\frac{Z}{\rho T}\right)\right\} \delta_{ik}\delta(\mathbf{r}_1 - \mathbf{r}_2)\ \delta(t_1 - t_2).$$

The fluctuations of the hydrodynamical parameters can be found by expressing them in terms of the "outside" quantities with the aid of the hydrodynamical equations.

In conclusion we note that the equations for fluctuations of the quantities **Q** and **g** will also apply, naturally, to solutions of ordinary (non-superfluid) liquids.

I would like to thank L. D. Landau and E. M. Lifshitz for allowing me to become acquainted with their manuscript before publication.

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*The thermodynamic identity has the form

$$d\epsilon = T\,dS + \mu\,d\rho + Z\,dc + (\mathbf{v}_n - \mathbf{v}_s)\,d(\mathbf{j} - \rho\mathbf{v}_s).$$

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[1] L. D. Landau and E. M. Lifshitz, J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 618 (1957), Soviet Phys. JETP **5**, 512 (1957).

[2] S. M. Rytov, Теория электрических флуктуаций и теплового излучения (Theory of Electrical Fluctuations and Heat Radiation), Academy of Sciences Press, 1953.

[3] I. M. Khalatnikov, J. Exptl. Theoret. Phys. (U.S.S.R.) **23**, 265 (1952).

[4] I. M. Khalatnikov, Usp. Fiz. Nauk **60**, 69 (1946).

[5] L. D. Landau and E. M. Lifshitz, Статистическая физика (Statistical Physics), 3rd Edition, Gostekhizdat, 1951.

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CAUSES OF THERMODYNAMIC IRREVERSIBILITY

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THE statistical interpretation of the second law of thermodynamics was given by Boltzmann more than eighty years ago. In spite of this long period of time, there is no clear explanation for the experimentally observed fact of macroscopic irreversibility.

The laws of elementary mechanics, both classical and quantum, are entirely reversible. This fact causes the general laws of statistical mechanics to be also reversible. Indeed, the Liouville equation is reversible just as is the equation of motion of the density matrix for a mixed state in quantum mechanics. Therefore with any mechanical process developing in time there can always be associated an analogous process moving in the opposite direction. This is accomplished by replacing all the velocities or momenta by their negatives. Thus if the entropy is increasing in time in any process, the reverse process in which the entropy decreases may always exist in principle. One cannot, therefore, assign to the second law of thermodynamics the same universal meaning as to the first law, the conservation of energy. In small-scale events, in fact, one often observes fluctuation phenomena due to the decrease of entropy with time.

It is on the basis of such concepts that Boltzmann arrived at his fluctuation hypothesis. This hypothesis, however, is of a merely speculative nature, being based on no experimental facts other than those it attempts to explain. As is known, it did not achieve recognition.

In order to find genuine reasons for thermodynamic irreversibility, one must first clarify the origin of those statistical anomalous states which approach thermal equilibrium in an irreversible way.

The sources of all the thermal energy with which we deal are nuclear processes. This is above all the energy liberated in radioactive decay; such is, for instance, the source of heat in the interior of the