where

$$\varphi_1(k_0;\ p) = (\sqrt{k_0^2 + p^2} + k_0)^{1/2}; \quad \varphi_2(k_0;\ p) = (\sqrt{k_0^2 + p^2} - k_0)^{1/2}.$$

The asymptotic expression analogous to (5) now becomes

$$f(r,\ t) \cong -\frac{e^{-\sigma t}}{\pi^2 r}\frac{\partial}{\partial r}\frac{\vartheta}{r^{3/2}}\int_0^\infty \exp\left(-2\frac{\vartheta}{\sqrt{r}}x\right)\sin x^2 dx$$
$$= -\frac{e^{-\sigma t}}{2\pi r}\frac{\partial}{\partial r}\left\{\frac{1}{\vartheta}\sqrt{\frac{r}{2\pi}}\frac{\partial}{\partial r}\left(\cos\frac{\vartheta^2}{r} - \sin\frac{\vartheta^2}{r}\right) + \frac{\partial}{\partial r}\left[C^2\left(\frac{\vartheta}{\sqrt{r}}\right) + S^2\left(\frac{\vartheta}{\sqrt{r}}\right)\right]\right\}, \quad (6)$$

where

$$\vartheta = \frac{t}{3\tau\sqrt{2k_0}}, \quad S(x) = \frac{2}{\sqrt{2\pi}}\int_0^x \sin t^2 dt, \quad C(x) = \frac{2}{\sqrt{2\pi}}\int_0^x \cos t^2 dt$$

is the Fresnel integral. It follows from (6) that for large optical path lengths

$$f(r,\ t) \approx (4\pi)^{-3/2}(t/\tau\sqrt{k_0 r})\,e^{-\sigma t}/r^3. \quad (7)$$

Comparing (5) and (7) we see that the function for diffusion with redistribution of the photon frequencies decreases much slower than the other. A similar result can be obtained also for the Doppler shape of a spectral line. This is related to the slow decrease of the kernel of the integro-differential equation (1), as has been pointed out by Biberman.[1]

We note further that Ambartsumian's transformation makes it possible to obtain an analytic expression for the Green's function in the problem of diffusion of radiation if one accounts for the motion of the atoms.

In conclusion I express my gratitude to L. M. Biberman for his direction in performing the present work.

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## *EFFECT OF PROTON SIZE ON THE POSITION OF ELECTRONIC LEVELS IN HYDROGEN AND DEUTERIUM*

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RECENT experiments[1] on the scattering of electrons on protons show that the rms radius of the electric charge distribution in the proton is $(0.77 \pm 0.10) \times 10^{-13}$ cm, whereas the rms radius of the neutron is possibly smaller.[2,3] This leads to a reduction of the binding energy of the electron in atoms, i.e., to a correction to the Lamb shift. In the calculation of a similar effect, one may confine oneself to the investigation of the nonrelativistic problem, taking into account additionally the distortions of the electronic wave functions, since the corrections due to these are not large in the cases — of interest to us — of hydrogen and deuterium[4,5] for which experimental data[6] exist.

Let $e\rho(r_1)$ be the electric charge density in the proton, let $\psi_\rho(r)$ be the normalized wave function of the proton in the deuteron (where r is the distance of the proton center from the center of mass of the deuteron and $r_1$ is the distance from the center of the proton to some point within it), and let $\psi(R)$ be the wave function of an atomic s-electron (where in the case of deuterium, R is the distance from the electron to the center of mass of the deuteron, and in hydrogen, R is the distance to the proton center). The reduction, due to the finite size of the proton, of the binding energy of the electron in the hydrogen atom is given (in $cm^{-1}$) by

$$\Delta E_{\mathrm{H}} = \frac{1}{2\pi\hbar c}\int |\psi(R)|^2 d\tau_R \left\{\frac{e^2}{R} - e^2\int \frac{\rho(r_1)\,d\tau_1}{|\mathbf{R}-\mathbf{r}_1|}\right\}. \quad (1)$$

But

$$-e^2\int \frac{\rho(r_1)\,d\tau_1}{|\mathbf{R}-\mathbf{r}_1|} = -\frac{e^2}{R} + \frac{4\pi e^2}{R}\int_R^\infty \rho(r_1)\,r_1(r_1-R)\,dr_1.$$

Moreover, if $\rho$ falls off much more rapidly than $|\psi|^2$, then in (1), one may take $|\psi(0)|^2$ out from under the integral sign. Thus we obtain after integration by parts

$$\Delta E_{\mathrm{H}} = \frac{\alpha}{3}|\psi(0)|^2\langle r^2\rangle, \quad \langle r^2\rangle = 4\pi\int_0^\infty \rho(r_1)\,r_1^4\,dr_1. \quad (2)$$

This result is verified by a method employing more accurate functions in the calculations of E. V. Teodorovich.

In the case of deuterium, the volume effect is given by

$$\Delta E_{\mathrm{D}} = \frac{1}{2\pi\hbar c}\int |\psi(R)|^2 d\tau_R \left\{\frac{e^2}{R} - e^2\int\int \frac{|\psi(r)|^2\rho(r_1)\,d\tau\,d\tau_1}{|\mathbf{R}-\mathbf{r}-\mathbf{r}_1|}\right\}. \quad (3)$$

Upon integration we obtain

$$\Delta E_{\mathrm{D}} = \frac{\alpha}{3}|\psi(0)|^2\{\langle R^2\rangle + \langle r^2\rangle + \langle r_n^2\rangle\}, \quad (4)$$

where the additional last term takes into account the effect of the neutron volume, which has a value less than zero.

Note that if the electron is not treated as a point particle, then in (2) and (4) one should replace $\langle r^2\rangle$ with the sum $\langle r^2\rangle + \langle r_e^2\rangle$. Regardless of this, however, in the present approximation, the difference $\Delta E_D - \Delta E_H$ does not depend upon the structure of the proton and electron, but is determined only by the properties of the deuteron and by the structure of the neutron:

$$\Delta E_{\mathrm{D}} - \Delta E_{\mathrm{H}} = \frac{\alpha}{3}|\psi(0)|^2\{\langle R^2\rangle + \langle r_n^2\rangle\}\,\mathrm{cm}^{-1}.$$

Regarding the computation of the magnitude of $\langle R^2\rangle$, the following remarks apply. In the first place $\langle R^2\rangle$, which is characteristic of the distribution of charge relative to the center of mass of the deuteron, is four times as small as the quantity $\langle R_1^2\rangle$, which characterizes the distribution of charge relative to the neutron. Secondly, although the wave function of the proton in the deuteron, which determines $\langle R_1^2\rangle$, depends also on the properties of the nuclear forces, which have not yet been completely explained, the various assumptions about the form of the nuclear potential give almost exactly the same results if the parameters of the potentials are chosen to fit the data on the scattering of low-energy neutrons. In particular, in the case of the square well, we obtain

$$\langle R^2\rangle = \frac{1}{8\gamma^2}\left\{1 + \frac{\gamma(\beta R_0 - {}^1/_2\sin 2\beta R_0)}{\beta\sin^2\beta R_0}\right\}^{-1}\left\{1 + 2\gamma R_0 + 2\gamma^2 R_0^2 + \frac{\gamma^3}{\beta^3\sin^2\beta R_0}\left(\frac{2}{3}\beta^3 R_0^3 - \beta^2 R_0^2\sin 2\beta R_0 - \beta R_0\cos^2\beta R_0 + \frac{1}{2}\sin 2\beta R_0\right)\right\}, \quad (5)$$

where $\gamma = (2\pi/\hbar)\sqrt{m\mathscr{E}}$; $\beta = (2\pi/\hbar)\sqrt{m(V_0 - \mathscr{E})}$, m is the proton mass, $\mathscr{E} = 2.226$ Mev is the binding energy[7] of the deuteron, and $V_0$ and $R_0$ are the well depth and width, respectively, for which values[2,8] of 35.2 Mev and $2.04 \times 10^{-13}$ cm were taken.

For these data the numerical values of the volume effect are: $\Delta E_H = 0.117 \pm 0.03$ Mc/sec for hydrogen, in agreement with our previous estimate.[4] For deuterium we obtain a correction of 0.74 + 0.117 = 0.86 Mc/sec plus $\delta_n = (\alpha/3)|\psi(0)|^2\langle r_n^2\rangle$ Mc/sec, which also improves the agreement with experiment.

The isotopic volume effect, $\Delta E_D - \Delta E_H$, is equal to 0.74 Mc/sec + $\delta_n$, i.e., together with the mass effect it amounts to 1.33 Mc/sec + $\delta_n$, which is in good agreement with the measurements[6] of Lamb (1.23 ± 0.20 Mc/sec). Before drawing final conclusions about the magnitude of the Lamb shift, however, it would be desirable to ascertain to what extent possible corrections, e.g., higher-order quantum-electrodynamic terms, might modify these results.

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## *APPLICATION OF CHARGE INVARIANCE TO POLARIZATION PHENOMENA*

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As is well known, the hypothesis of charge invariance leads to relations between experimentally observable quantities. Up to the present time, however, the only relations that have been derived connect the cross-sections of different processes. In connection with experiments on change of polarization it is also of interest to examine the relations involving the polarization which follow from charge invariance. We shall consider an extremely simple method for finding such relations.

Suppose that the isotopic spin is conserved in the interactions that cause the process $a + A \rightarrow b + B$. Let us denote the respective isotopic spins of the particles by $j_a$, $j_A$, $j_b$, $j_B$, and the values of a particular component by $m_a$, $m_A$, $m_b$, $m_B$. The amplitude describing the transition $m_a, m_A \rightarrow m_b, m_B$ can be written in the form

$$R_{m_a m_A;\, m_b m_B} = \sum_j (j_a j_A m_a m_A \mid j_a j_A j m)\, R_j\, (j_b j_B m_b m_B \mid j_b j_B j m). \tag{1}$$

by the use of the Clebsch-Gordan coefficients. Here $R_j$ is the amplitude in a state with a definite total isotopic spin; it depends on the angles, the spins, and the energy.

Let $T_k$ be operators acting on the spin variables of particles b and B and forming a complete set of matrices (for example, 1 and the Pauli matrices $\sigma_x$, $\sigma_y$, $\sigma_z$ in the case of a spinless particle and a particle with spin ½). The experimentally observable quantities are average values $\langle T_k\rangle$. From Eq. (1) we have:

$$(\langle T_k\rangle \sigma)_{m_a m_A;\, m_b m_B} = \sum_{j, j_1} (j_a j_A m_a m_A \mid j_a j_A j m)(j_a j_A m_a m_A \mid j_a j_A j_1 m)\, R_j^+ T_k R_{j_1}\, (j_b j_B m_b m_B \mid j_b j_B j m)(j_b j_B m_b m_B \mid j_b j_B j_1 m), \tag{2}$$