

The hyperons Λ ($r = 1$) and Ξ ($r = 2$) belong to this group. The nucleons ($r = 0$) and the Σ -particle ($r = 1$) belong to the second group, for which $t = (r + 1)/2$ and mass equals $1837 + 497.5r$. Instability occurs soon in this group for $r = 2$ or 3 , leading to fast π -meson decay:

The production thresholds in Bev (in the laboratory frame) for hyperons of the first group with various S and for Λ (belonging to the second group) are given in the table. The hyperon of the first group with $S = -3$ and isotopic spin 1, denoted in the table by B , should exist in three charge states: B^0 , B^- , and the doubly charged B^{--} .

If the expressions for the masses given above are at least approximately correct, then the hyperons of the first group, including B , cannot disintegrate by emission of \bar{K} -mesons and, therefore, are metastable cascade particles. Production of such particles is not very probable and would require great energies, but their detection does not present special difficulties.

A simple additivity of the \bar{K} -meson-nucleon interactions requires that the \bar{K} -mesons in hyperons are in the s -state in the nucleon field.

The above considerations, therefore, are correct only if the geometrical spin of all hyperons equals $1/2$ and if they all have even parity in a system in which nucleons and \bar{K} -mesons are even. It should be noted that such parity excludes the treatment of K -particles according to Fermi-Young theory as $(\Lambda + N)$.^{4,5}

In the language of field theory, the proposed scheme corresponds to an interaction term in the Hamiltonian, containing the square of the absolute value of the nucleonic wave function and the square of the absolute value of the \bar{K} -meson wave function. It differs in this from the scheme proposed by Wentzel⁶ which introduces the interaction as linear in the K -meson wave function and, therefore, has to consider the hyperon Λ , together with the nucleons and the K -meson as an elementary particle.

*This agreement is equivalent to validity of Eq. (1), since

$$M = M_N + 2b - a, \quad a = M_\Sigma - M_\Lambda, \quad b = \frac{3}{4} M_\Sigma + \frac{1}{4} M_\Lambda - M_N,$$

from which Eq. (1) follows elementarily.

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MAGNETO-VORTEX RINGS

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IN Ref. 1 we considered magnetohydrodynamical equilibrium configurations. It was noted that the distribution of the field H and the current density j in those configurations is identical with the distribution of the velocity v and vorticity Ω , corresponding to the stationary flow of an incompressible fluid.

Chandrasekhar² paid attention to the existence and showed the stability of the simplest solution of the magnetohydrodynamical equations of an incompressible, perfectly conducting fluid, for the case where the

velocity of flow is connected to the magnetic field by the relation

$$\mathbf{v} = \mathbf{H} / \sqrt{4\pi\rho}; \quad \rho + \rho v^2 / 2 = \text{const.}$$

This solution is a generalization of the solution presenting a magnetohydrodynamical ring.³ It is satisfied by an solenoidal vector \mathbf{v} .

It is natural to generalize all these solutions even further. The magnetohydrodynamical equations of an ideal incompressible fluid of infinite conductivity can be written in the form

$$\partial\mathbf{v}/\partial t = -\nabla(p/\rho + v^2/2) + [\mathbf{j} \times \mathbf{H}]/c - [\boldsymbol{\Omega} \times \mathbf{v}], \quad \partial\mathbf{H}/\partial t = \text{curl}[\mathbf{v} \times \mathbf{H}], \quad \text{curl} \mathbf{H} = (4\pi/c) \mathbf{j}, \quad \text{curl} \mathbf{v} = \boldsymbol{\Omega}. \quad (1)$$

Let us assume that

$$\mathbf{v} = \alpha \mathbf{H} / \sqrt{4\pi\rho}, \quad (2)$$

where α is an arbitrary constant. From the last two equations (1) it follows that

$$\boldsymbol{\Omega} = (\alpha/c) \sqrt{4\pi\rho} \mathbf{j}. \quad (3)$$

From the second equation of (1) we get $\partial\mathbf{H}/\partial t = 0$ and hence by virtue of Eq. (2) $\partial\mathbf{v}/\partial t = 0$. The first equation of (1) takes the form

$$-\nabla(p + \rho v^2/2) + c^{-1}(1 - \alpha^2)[\mathbf{j} \times \mathbf{H}] = 0, \quad (4)$$

whence we obtain the equilibrium condition for magnetohydrodynamical configurations $\text{curl}[\mathbf{j} \times \mathbf{H}] = 0$, or the condition for a stationary flow of an incompressible fluid, $\text{curl}[\mathbf{v} \times \boldsymbol{\Omega}] = 0$. Thus any equilibrium configuration or any stationary flow of an incompressible fluid corresponds to the stationary flow of an incompressible, perfectly conducting fluid in a magnetic field \mathbf{H} , in the direction of the current of the fluid, \mathbf{v} .

For instance, there corresponds to a ring current in a magnetic field and a circular vortex an analogous configuration (which can be called a magneto-vortex ring) with an identical distribution of the magnetic field and the fluid velocity. Let R and a be the large and the small radius of this torus, and let $R \gg a$. If the current density j_φ and, correspondingly, the vorticity Ω_φ are constant along a cross section, then according to Ref. 1 the radii of the torus and the current through it $J = j_\varphi/\pi a^2$ are connected with the external field $H_z = H_0$, perpendicular to the plane of the ring, through the relation

$$H_0 = -(J/cR) [\ln(8R/a) - 1/4]. \quad (5)$$

If we use Eqs. (2) and (3) to express the field and the current in formula (5) in terms of the velocity and vorticity, we find the velocity of the fluid current relative to the ring \mathbf{v}_0 , or in the frame of reference in which the fluid at infinity is at rest, the velocity of the torus $\mathbf{v}_k = -\mathbf{v}_0$,

$$v_k = (\kappa/4\pi R) [\ln(8R/a) - 1/4], \quad (6)$$

where $\kappa = \pi a^2 \Omega$ is the vorticity intensity. The absolute value of the velocity of the magneto-vortex ring for a given external field H_0 can be arbitrary. In the case $\alpha = 0$ the magneto-vortex ring becomes a magnetohydrodynamical equilibrium configuration at rest, while for $\alpha = \infty$ it turns into the normal vortex ring.

The distribution of the pressure in the ring in the case when $R \gg a$ is not essentially different from the distribution in a cylindrical magneto-vortex filament of radius a . This last distribution is easily found from Eq. (4) and has the form

$$\rho = \rho_0 - \frac{H_a^2}{8\pi} \left(1 - \frac{a^2}{2}\right) \frac{r^2}{a^2}; \quad (r < a); \quad \rho = \rho_0 - \frac{H_a^2}{8\pi} \left(1 - \frac{a^2}{2}\right) + \frac{H_a^2}{8\pi} \frac{a^2}{2} \left(1 - \frac{a^2}{r^2}\right); \quad (r > a).$$

where ρ_0 is the pressure on the axis of the cylinder and $H_a = 2J/ca$.

The stability of a magneto-vortex configuration depends on the value of the parameter α . It is, for instance, well known that a cylindrical vortex is stable⁴ while a straight current is not. The vortex corresponds to $\alpha = \infty$ and the current to $\alpha = 0$. Apparently the value $\alpha \sim 1$ separates the regions of stability and instability of a magneto-vortex filament.

Another kind of solution — a magneto-vortex configuration at rest — can be obtained from the equilibrium configurations which are maintained by gravitation⁴ or by external pressure.⁵ In those cases the magnetic lines of force, and hence also the lines of the fluid current are close (they do not go to infinity)

and the center of gravity of the magneto-vortex configuration is at rest.

In conclusion I want to express my gratitude to S. I. Braginskii for a discussion which stimulated the present paper.

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ERRATA TO VOLUME 6

Page	Line	Reads	Should Read
643	16 from bottom	where $\kappa = \pi a^2 \Omega - \dots$	where $\kappa = \pi a^2 \Omega \varphi - \dots$
690	8 from bottom	$\dots \sin [- \dots$	$\dots \sin \vartheta [- \dots$
	5 from bottom	$\dots \sin 2\vartheta \sqrt{\frac{1}{3}} \dots$	$\dots \sin 2\vartheta \left[\sqrt{\frac{1}{3}} \dots \right.$
809	9 from top	$\dots \left(\frac{1}{2 \sinh u} + \dots \right.$	$\dots \left(\frac{1}{\sinh u} + \dots \right.$
973	unnumbered equation	$\dots C_{n\mu-\mu'}^{S'-\mu'} S_{\mu} T_{\mu'-\mu}^{(n)}$	$\dots C_{n\mu-\mu'}^{S'-\mu'} S_{\mu} \langle S' \ T^{(n)} \ S^{-1} \rangle \times T_{\mu'-\mu}^{(n)}$
975	5 from bottom	\dots of a particle by a nucleus \dots	\dots of a particle in state a by a nucleus \dots
992	Eq. (18)	$\dots \tau_1 \tau_2^{-2} / 2\hbar^1 \dots$	$\dots \tau_1 \tau_2^{-1} / 2\hbar^2 \dots$