$$\hat{H}_{s0} = -\frac{\lambda\hbar^2}{2iM^2c^2}\,\frac{1}{f^2(\cosh^2 u-\cos^2\theta)}\,\frac{\partial V}{\partial u}\left[-\hat{\sigma}_1\left(\sin\varphi\,\frac{\partial}{\partial\theta}+\cot\theta\cos\varphi\,\frac{\partial}{\partial\varphi}\right)+\hat{\sigma}_2\left(\cos\varphi\,\frac{\partial}{\partial\theta}-\cot\theta\sin\varphi\,\frac{\partial}{\partial\varphi}\right)+\hat{\sigma}_3\coth u\,\frac{\partial}{\partial\varphi}\right], \quad (3)$$

where λ is the force constant, $\hat{\sigma}_i$ the Pauli spin matrices, and V the potential, which equals zero for $u < u_0$ and V_0 for $u > u_0$. From the symmetry of the problem it follows that only the third term of (3) contributes to the energy. To obtain the energy spectrum one has merely to solve the transcendental equation for γ, which follows from the matching conditions of the wave function at $u = u_0$:

$$\left[\gamma\frac{J_{m+1}(\gamma S)}{J_m(\gamma S)}\,\frac{dS}{du}-\frac{m+1/2}{S}\,\frac{dS}{du}+\frac{1}{2}\left(\frac{dS}{du}\right)^{-1}\frac{d^2S}{du^2}-g\chi-\frac{1}{2}\,\frac{1}{\chi}\,\frac{d\chi}{du}\right]_{u=u_0}=-A,$$

where

$$g = f\sqrt{\frac{2M}{\hbar^2}(V_0-E)},$$

$$\chi(u) = \sinh u+\frac{1}{g}\,\frac{p}{\sinh u}-\frac{p^2-m^2+1}{2g^2}\left(\frac{1}{2\sinh u}+\frac{1}{\sinh^3 u}\right)-\frac{p}{2g^3}\left(\frac{p^2-m^2+1}{8\sinh u}-\frac{p^2-m^2+7}{2\sinh^3 u}-\frac{p^2-m^2+5}{\sinh^5 u}\right)+\ldots,$$

$$A = \pm\lambda m V_0\coth u_0/(Mc^2),$$

$p = \ell + 1$ for even $\ell - |m|$ and $p = \ell$ for odd $\ell - |m|$, and the two signs of A correspond to the two possible orientations of the nucleon spin with respect to the z axis.

The case of the oblate spheroid can be treated in an analogous manner.

Numerical computations of the energy levels of nucleons in ellipsoidal wells are being carried out at the present time for different deformations.

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DIFFRACTION SCATTERING OF HIGH-ENERGY PROTONS BY PROTONS

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FOWLER et al.[1] attempted a theoretical analysis of the diffraction p—p scattering, starting, however, with the usual spherical nucleon model with distinct boundaries and a definite transparency. Of greater interest is Belenkii's analysis[2] of the diffraction scattering of high-energy pions by nucleons, based on a general theory involving no specific nucleon model. We considered similarly the collisions of high-energy protons and analyzed the known experimental data, making the following assumptions: (1) the spin dependence of the nuclear forces can be neglected at high energies, (2) the imaginary part of the scattering amplitude is much greater than the real part (since it is known from the experimental data of Ref. 1 that the elastic scattering cross section is on the same order as the inelastic one at high energies and that both are on the order of the geometric cross section).

These assumptions simplify considerably the phase analysis, since (see Ref. 2) it follows from

$$|f(x)| = \frac{\lambda}{2}\sum_{l=0}^{\infty}(2l+1)(1-\beta_l)P_l(x). \quad (1)$$

[where f(x) is the scattering amplitude, λ the wavelength of the incident particle, $x = \cos\vartheta$, ϑ the scattering angle, $P_l(x)$ the Legendre polynomials, $\beta_l = \exp(2i\eta_l)$, and η_l is the scattering phase] that

$$\beta_l = 1 - \lambda^{-1}\int_{-1}^{+1}|f(x)|P_l(x)\,dx.$$

By expressing the measured angular distribution in terms of an analytic function it is possible to determine β_l, since the differential cross section of the elastic scattering is $d\sigma/d\omega = |f(x)|^2$. The present-day experimental accuracy does not permit unique determination of the phase, but makes possible an approximate phase analysis, the results of which are given in the table.

We obtained formulas similar to (5) and (6) of Belen'kii's paper. Additional simple transformations yield

$$d\sigma(\vartheta)/d\omega = |f(\vartheta)|^2 = (\sigma_t/4\pi\lambda)^2\exp\{-R^2\vartheta^2/2\lambda^2\}, \quad (2)$$

where $R^2 = \sigma_t^2/8\pi\sigma_e$, where σ_e is the elastic scattering cross section and σ_t is the total cross section of particle interaction.

In the analysis of the experimental data obtained by Fowler et al. for proton energies of 1.5 and 2.75 Bev, we calculated the $d\sigma(\vartheta)/d\omega$ curves from values of σ_t known from investigations with counters[3] and the values of σ_e measured in Ref. 1.

E_p lab. sys., Bev	$R\times10^{-14}$ cm	σ_t, mb	σ_e, mb	σ_i, mb	$d\sigma(\vartheta)/d\omega$, mb/ster	$1-\beta_l$
1.5	6.9±0.6	47.2±0.9*	20±2**	27**	25.6 exp (−4.3 ϑ^2)	1.7 exp (−0.11 l^2)
2.24	6.6±0.8	44.1±3.7	17.9±5.1	26.2±6.4	33.5 exp (−5.9 ϑ^2)	1.6 exp (−0.09 l^2)
2.75	6.7±0.8	41±1*	15±2**	26**	35.5 exp (−7.5 ϑ^2)	1.5 exp (−0.07 l^2)
4.40	6.9±0.3	33.9±1.8	9.7±1.5	24.2±2.7	38.8 exp (−13 ϑ^2)	1.1 exp (−0.04 l^2)
6.15	7.2±0.3	31.3±1.5	7.5±1.5	23.8±1.8	46.1 exp (−19 ϑ^2)	1.0 exp (−0.03 l^2)
8	7.45	30.0	6.4	23.5	54.6 exp (−27 ϑ^2)	0.9 exp (−0.02 l^2)
10	7.75	28.5	5.4	23.1	62.2 exp (−36 ϑ^2)	0.8 exp (−0.01 l^2)

*Taken from Ref. 3 **Taken from Ref. 1

The calculated curves are in satisfactory agreement with experimental results and are close to the curves calculated by the authors of that reference using an optical nucleon model.

The differential cross sections obtained by Cork et al.[4] for E_p = 2.24, 4.40, and 6.15 Bev make it possible to obtain several values of σ_t^2 and R^2 for each energy, using formula (2). Curves of $d\sigma(\vartheta)/d\omega$ calculated using average values $\overline{\sigma_t^2}$ and $\overline{R^2}$ are in satisfactory agreement with the experimental data; the value σ_t = 44 millibarns for E_p = 2.24 Bev is also in satisfactory agreement with known results on the total cross section of the p−p interaction. Linear extrapolation of the cross section σ_i of the inelastic processes and of the parameter R into the energy region up to 10 Bev was used to calculate the angular distribution of elastic p−p scattering for 8 and 10 Bev (see table.).

It is seen from the table that R increases almost linearly with the energy, starting with 2.24 Bev, but can be assumed constant within experimental accuracy. If R is really constant, it is possible to determine $d\sigma(\vartheta)/d\omega$ by measuring σ_t and vice versa.

The results of the analysis show that the assumptions made by Belen'kii are also true for p−p scattering at $E_p \geq 1.5$ Bev, since the resultant formulas give satisfactory agreement with the experimental data. The values of σ_t, σ_e, and σ_i, calculated for 2.24, 4.40, and 6.15 Bev on the basis of this analysis, can obviously be used as preliminary data.

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PARAMAGNETIC RESONANCE OF NEW ORGANIC RADICALS

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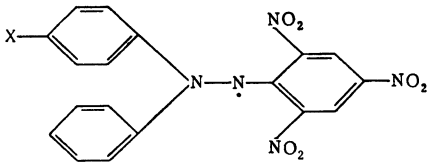
THE narrow line-width of the resonant absorption curve of the organic radical α,α-diphenyl β-picryl hydrazyl has been explained by exchange interaction of the odd electron in the $-N-N$ group.[1] Direct confirmation of the existence of the exchange interaction was obtained from studies of hyperfine structure of this absorption line.[2]

We undertook a set of experiments with an object to investigate the influence of substitutes X in para-position of the α-phenyl ring on exchange interactions in organic radicals. Para-fluoro-, para-chloro-, para-bromo-, and para-methoxy-derivatives of hydrazyl were investigated. Three of these (the para-fluoro-, para-chloro-, and para-methoxy-derivatives) were obtained first.



The line shape of the paramagnetic resonant absorption,[3] characterized by the ratio of fourth to the second moment of the resonant curve, M_4/M_2, served to determine the magnitude of exchange interactions. This ratio was obtained by numerical integration of the experimental absorption curve. The moments were computed separately for the left and right half of the curve and average values were taken.

Substitute X	g-factor	M_4/M_2	ΔH	A
H	2.0042 ± 0.0004	1.43±0.02	1.0 ± 0.15	300
Cl	2.001 ± 0.001	1.42±0.02	1.2 ± 0.15	290
Br	2.002 ± 0.002	1.40±0.02	2.2 ± 0.15	170
OCH_3	2.000 ± 0.002	1.30±0.02	2.6 ± 0.2	120
F	2.000 ± 0.004	small	4.1 ± 0.5	20

Measurement of the paramagnetic resonance absorption was performed using Zavoiskii's grid-current method.[4] To obtain absorption curves without admixture of dispersion, the generator circuit was coupled very weakly to the measuring coil. The generator frequency was varied in a fairly narrow region about a resonant frequency of 621 Mcs and was measured by a type 3003 heterodyne detector with an accuracy of 0.005%. A signal modulated at 224 cps was fed through a broad-band amplifier with a bandwidth 30 cps to 10 Mcs into an oscilloscope or into a narrow-band amplifier of bandwidth 5 cps and a synchronous detector with meter. Absorption curves were photographed from the screen of the oscilloscope. The narrow-band system was used for observing the width of the absorption curve between the points of steepest slope. The measured quantities were the maximum values A of the absorption curves, which yield values proportional to χ''. All the samples investigated contained equal molar amounts of radicals. The field was calibrated against the known resonance of α,α-diphenyl-picryl-hydrazyl, whose g-factor was taken to be 2.0042 ± .0004.[1]

The results are given in the table. It is seen that all the quantities, except the g-factor, which differs only slightly from 2, change gradually from H to F. The exchange interaction thus diminishes in the transition from H to F.