ON THE FORM OF THE BETA-DECAY INTERACTION

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A study is made of the effect of parity nonconservation on the interpretation of the experimental data relating to the Fierz interference and the $\beta-\nu$ angular correlation. Formulas are obtained for the β-decay of polarized nuclei.

UNTIL recently it has been supposed that the β-decay interaction was a combination of the scalar and tensor types. This conclusion was based on the following facts:

(a) The experimental absence in allowed β-ray spectra of the Fierz interference between the scalar (S) and vector (V) types on one hand, and between the tensor (T) and axial-vector (A) types on the other; this leads to the following relations for the coefficients in the combination of the types: $C_SC_V = C_TC_A = 0$.

(b) The positive sign of the $\beta-\nu$ angular correlation coefficient a in the β-decay of He^6 indicates[1] that $C_T \neq 0$. Then it follows from point (a) that $C_A = 0$.

(c) The negative sign of a in the β-decay of Ne^{19} (Refs. 2 and 3) shows that $C_S \neq 0$, and this leads to $C_V = 0$.

(d) The absence of the Fierz interference between V and T in non-unique first forbidden β-ray spectra, together with point (b), also gives $C_V = 0$. The most essential point in this derivation is the absence of the SV and TA interferences, which makes possible the unique solution of the problem of the form of the β-decay interaction on the basis of the sign of a alone.

The discovery of parity nonconservation in β-decay[4,5] has changed the situation somewhat. In fact, the absence of the Fierz interferences, points (a) and (d), now gives only (for real C_i, C_i')

$$C_SC_V + C_S'C_V' = C_TC_A + C_T'C_A' = C_VC_T + C_V'C_T' = 0. \tag{1}$$

Unlike the former situation, one cannot get from Eq. (1) and points (b) to (d) the conclusion that $C_V = C_A = C_V' = C_A' = 0$.* It is obvious that a knowledge of just the sign of a is insufficient for the solution of the problem of the form of the β-decay interaction. For this it is necessary to obtain values of a with greater precision (at present the errors in the determination of a are at least 35 to 45 percent), since for small values of C_V/C_S, C_V'/C_S' and C_A/C_T, C_A'/C_T' the value of a is very insensitive to these ratios.

From what is said above it can be seen that the polarization effects in β-decay must be calculated for a superposition of all the types of interaction. In the present paper the calculation of the polarization effects is carried out for the allowed β-transitions, $\Delta I = 0, \pm 1$ (no). We have carried out the calculations in the Born approximation. For this reason the results can be applied quantitatively only for the lightest nuclei. But the set of experiments presented below as required for the determination of all the constants C_i, C_i' will also be correct in the general case. For generality we regard the coefficients C_i, C_i' as complex, since their reality is connected with the still open question of the invariance of weak-interaction processes with respect to time reversal. All notations are the standard ones.[4]

To calculate the probability $W(\mathbf{p}, \mathbf{q}, \boldsymbol{\eta}, \boldsymbol{\zeta})$ (per unit time, unit energy range of the β-particle, and unit solid angles) of emission of a β-particle with momentum $\mathbf{p}$ and spin along the unit vector $\boldsymbol{\zeta}$ and a neutrino with momentum $\mathbf{q}$, in the β-decay of oriented nuclei with the axis of quantization along the unit vector $\boldsymbol{\eta}$; we use the method of Tolhoek and de Groot.[7]

After averaging over the nuclear spins the formula for W contains the following sorts of expressions:

$$g = 1, \ -I/(I+1), \ 1/(I+1)$$

*We note that this relation will again hold in the two-component theory of the neutrino.[6]

respectively for $I \to I-1$, $I \to I+1$, and $I \to I$ (I is the spin of the β-radioactive nucleus);

$$h = 1, \quad I(2I-1)/(I+1)(2I+3), \quad -(2I-1)/(I+1)$$

respectively for $I \to I-1$, $I \to I+1$, and $I \to I$; f_1 and f_2 are quantities characterizing the orientation of the nuclei,

$$f_1 = \overline{m}/I, \quad f_2 = (3/I(2I-1))\,[\overline{m^2} - I(I+1)/3],$$

(m is the projection of I along the axis of quantization z, and the averaging is taken over the nuclei of the given type contained in the specimen); and the matrix

$$N_{ik} = \begin{pmatrix} -1/2 & 0 & 0 \\ 0 & -1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Finally, the phase α is defined by the formula

$$e^{2i\alpha} = \left(\int 1\right)^+ \left(\int \sigma_z\right) \Big/ \left(\int 1\right)\left(\int \sigma_z\right)^+;$$

it is equal to zero for β-decay transitions in which the state of the nucleon remains unchanged.

The expression for $W(\mathbf{p}, \mathbf{q}, \boldsymbol{\eta}, \boldsymbol{\zeta})$ is cumbersome, and we shall not give it here. Other probabilities are obtained by averaging $W(\mathbf{p}, \mathbf{q}, \boldsymbol{\eta}, \boldsymbol{\zeta})$ with respect to different variables (the upper sign refers to β^- and the lower to β^+ transitions; $\hbar = c = m_e = 1$ throughout);

$$W(\mathbf{p}, \boldsymbol{\eta}, \boldsymbol{\zeta}) = (2\pi)^{-4} pEq^2 \Big\{ \left|\int 1\right|^2 \Big[|C_S|^2 + |C_S'|^2 + |C_V|^2 + |C_V'|^2 \pm \frac{2}{E}\,\mathrm{Re}\,(C_S C_V^+ + C_S' C_V'^+) \pm 2\mathrm{Re}\,(C_S C_S'^+ - C_V C_V'^+)\frac{\mathbf{p}\boldsymbol{\zeta}}{E}\Big]$$

$$+ \left|\int \sigma\right|^2 \Big[(|C_T|^2 + |C_T'|^2 + |C_A|^2 + |C_A'|^2)\Big(1 + gf_1\Big(\frac{\boldsymbol{\eta}\boldsymbol{\zeta}}{E} + \frac{(\mathbf{p}\boldsymbol{\eta})(\mathbf{p}\boldsymbol{\zeta})}{E(E+1)}\Big)\Big) \pm 2\mathrm{Re}\,(C_T C_T'^+ - C_A C_A'^+)\frac{\mathbf{p}\boldsymbol{\zeta} + gf_1 \mathbf{p}\boldsymbol{\eta}}{E}$$

$$\pm 2\mathrm{Re}\,(C_T C_A^+ + C_T' C_A'^+)\Big(\frac{1}{E} + gf_1\Big(\boldsymbol{\eta}\boldsymbol{\zeta} - \frac{(\mathbf{p}\boldsymbol{\eta})(\mathbf{p}\boldsymbol{\zeta})}{E(E+1)}\Big)\Big) \pm 2gf_1\,\mathrm{Im}\,(C_T C_A'^+ + C_T' C_A^+)\frac{\mathbf{p}\,[\boldsymbol{\eta}\boldsymbol{\zeta}]}{E}\Big] + 2f_1\sqrt{\frac{I}{I+1}}\left|\int 1\right|\left|\int \sigma\right|$$

$$\times \Big[\mathrm{Re}\,\Big(e^{\mp i\alpha}\Big[(C_S C_A^+ + C_S' C_A'^+ + C_V C_T^+ + C_V' C_T'^+)\Big(\boldsymbol{\eta}\boldsymbol{\zeta} - \frac{(\mathbf{p}\boldsymbol{\eta})(\mathbf{p}\boldsymbol{\zeta})}{E(E+1)}\Big) \pm (C_S C_T^+ + C_S' C_T'^+ + C_V C_A^+ + C_V' C_A'^+)\Big(\frac{\boldsymbol{\eta}\boldsymbol{\zeta}}{E} + \frac{(\mathbf{p}\boldsymbol{\eta})(\mathbf{p}\boldsymbol{\zeta})}{E(E+1)}\Big)$$

$$+ (C_S C_T'^+ + C_S' C_T^+ - C_V C_A'^+ - C_V' C_A^+)\frac{\mathbf{p}\boldsymbol{\eta}}{E}\Big]\Big) + \mathrm{Im}\,(e^{\mp i\alpha}(C_S C_A'^+ + C_S' C_A^+ - C_V C_T'^+ - C_V' C_T^+))\frac{\mathbf{p}\,[\boldsymbol{\eta}\boldsymbol{\zeta}]}{E}\Big]\Big\};$$

$$W(\mathbf{p}, \mathbf{q}, \boldsymbol{\zeta}) = \frac{1}{2}(2\pi)^{-5} pEq^2 \Big\{\left|\int 1\right|^2 \Big[(|C_S|^2 + |C_S'|^2)\Big(1 - \frac{\mathbf{pq}}{qE}\Big) + (|C_V|^2 + |C_V'|^2)\Big(1 + \frac{\mathbf{pq}}{qE}\Big)$$

$$\pm \frac{2}{E}\,\mathrm{Re}\,(C_S C_V^+ + C_S' C_V'^+) - 2\,\mathrm{Im}\,(C_S C_V^+ + C_S' C_V'^+)\frac{\boldsymbol{\zeta}\,[\mathbf{pq}]}{qE} \pm 2\mathrm{Re}\,(C_S C_S'^+ - C_V C_V'^+)\frac{\mathbf{p}\boldsymbol{\zeta}}{E}$$

$$- 2\mathrm{Re}\,(C_S C_V'^+ + C_S' C_V^+)\Big(\frac{\mathbf{q}\boldsymbol{\zeta}}{q} - \frac{(\mathbf{p}\boldsymbol{\zeta})(\mathbf{pq})}{qE(E+1)}\Big) \mp 2\mathrm{Re}\,(C_S C_S'^+ + C_V C_V'^+)\Big(\frac{\mathbf{q}\boldsymbol{\zeta}}{qE} + \frac{(\mathbf{p}\boldsymbol{\zeta})(\mathbf{pq})}{qE(E+1)}\Big)\Big]$$

$$+ \left|\int \sigma\right|^2 \Big[(|C_T|^2 + |C_T'|^2)\Big(1 + \frac{\mathbf{pq}}{3qE}\Big) + (|C_A|^2 + |C_A'|^2)\Big(1 - \frac{\mathbf{pq}}{3qE}\Big) \pm \frac{2}{E}\,\mathrm{Re}\,(C_T C_A^+ + C_T' C_A'^+) + \frac{2}{3}\,\mathrm{Im}\,(C_T C_A^+ + C_T' C_A'^+)\frac{\boldsymbol{\zeta}\,[\mathbf{pq}]}{qE}$$

$$\pm 2\mathrm{Re}\,(C_T C_T'^+ - C_A C_A'^+)\frac{\mathbf{p}\boldsymbol{\zeta}}{E} \pm \frac{2}{3}\,\mathrm{Re}\,(C_T C_T'^+ + C_A C_A'^+)\Big(\frac{\mathbf{q}\boldsymbol{\zeta}}{qE} + \frac{(\mathbf{p}\boldsymbol{\zeta})(\mathbf{pq})}{qE(E+1)}\Big) + \frac{2}{3}\,\mathrm{Re}\,(C_T C_A'^+ + C_T' C_A^+)\Big(\frac{\mathbf{q}\boldsymbol{\zeta}}{q} - \frac{(\mathbf{p}\boldsymbol{\zeta})(\mathbf{pq})}{qE(E+1)}\Big)\Big]\Big\};$$

$$W(\mathbf{p}, \mathbf{q}, \boldsymbol{\eta}) = (2\pi)^{-5} pEq^2 \Big\{\left|\int 1\right|^2 \Big[(|C_S|^2 + |C_S'|^2)\Big(1 - \frac{\mathbf{pq}}{qE}\Big) + (|C_V|^2 + |C_V'|^2)\Big(1 + \frac{\mathbf{pq}}{qE}\Big) \pm \frac{2}{E}\,\mathrm{Re}\,(C_S C_V^+ + C_S' C_V'^+)\Big]$$

$$+ \left|\int \sigma\right|^2 \Big[(|C_T|^2 + |C_T'|^2)\Big(1 + \frac{\mathbf{pq}}{3qE}\Big) + (|C_A|^2 + |C_A'|^2)\Big(1 - \frac{\mathbf{pq}}{3qE}\Big) \pm \frac{2}{E}\,\mathrm{Re}\,(C_T C_A^+ + C_T' C_A'^+)$$

$$+ 2gf_1\Big(\pm\frac{1}{E}\,\mathrm{Re}\,(C_T C_T'^+ - C_A C_A'^+)\,\mathbf{p}\boldsymbol{\eta} \pm \frac{1}{q}\,\mathrm{Re}\,(C_T C_T'^+ + C_A C_A'^+)\,\mathbf{q}\boldsymbol{\eta} + \frac{1}{qE}\,\mathrm{Re}\,(C_T C_A'^+ + C_T' C_A^+)\,\mathbf{q}\boldsymbol{\eta}\Big) + 2hf_2(|C_T|^2 + |C_T'|^2$$

$$- |C_A|^2 - |C_A'|^2)\frac{N_{ik} q_i p_k}{qE}\Big] + 2f_1\sqrt{\frac{I}{I+1}}\left|\int 1\right|\left|\int \sigma\right|\Big[\mathrm{Re}\,\Big(e^{\mp i\alpha}\Big((C_S C_T'^+ + C_S' C_T^+ - C_V C_A'^+ - C_V' C_A^+)\frac{\mathbf{p}\boldsymbol{\eta}}{E} - (C_S C_T'^+ + C_S' C_T^+$$

$$+ C_V C_A'^+ + C_V' C_A^+)\frac{\mathbf{q}\boldsymbol{\eta}}{q} \mp (C_S C_A'^+ + C_S' C_A^+ + C_V C_T'^+ + C_V' C_T^+)\frac{\mathbf{q}\boldsymbol{\eta}}{qE}\Big)\Big) + \mathrm{Im}\,(e^{\mp i\alpha}(C_S C_T^+ + C_S' C_T'^+ - C_V C_A^+ - C_V' C_A'^+))\frac{\boldsymbol{\eta}\,[\mathbf{pq}]}{qE}\Big]\Big\}.$$

A special feature is the appearance in W(p, η, ζ) of a term in p [ηζ]; this quantity is not invariant with respect to time reversal, but nevertheless it is present in the formulas for $\alpha \neq 0$, becuase, as can be seen without difficulty, α changes sign on reversal of the time.

The first comparison of theory with experiment is conveniently carried out for the β-decay transition ΔI = ± 1 (no). Measurements of the β-ray spectrum and the β−ν correlation fix the values of

$$|C_T|^2 + |C_T'|^2, \quad |C_A|^2 + |C_A'|^2, \quad \mathrm{Re}\,(C_T C_A^+ + C_T' C_A'^+).$$

Furthermore, measurement of W(p, ζ) [or of W(p, η)] makes it possible to determine $\mathrm{Re}\,(C_T C_T' - C_A C_A')$, and measurement of W(p, q, ζ) gives

$$\mathrm{Re}\,(C_T C_T'^+ + C_A C_A'^+), \quad \mathrm{Re}\,(C_T C_A'^+ + C_T' C_A^+), \quad \mathrm{Im}\,(C_T C_A^+ + C_T' C_A'^+).$$

By carrying out measurements of the β-ray spectrum and the β−ν correlation in the case of the transition ΔI = 0 (no) one can find the quantities

$$|C_S|^2 + |C_S'|^2, \quad |C_V|^2 + |C_V'|^2, \quad \mathrm{Re}\,(C_S C_V^+ + C_S' C_V'^+).$$

Measurement of W(p, ζ) and W(p, q, ζ) gives

$$\mathrm{Re}\,(C_S C_S'^+ - C_V C_V'^+), \quad \mathrm{Re}\,(C_S C_S'^+ + C_V C_V'^+), \quad \mathrm{Re}\,(C_S C_V'^+ + C_S' C_V^+) \text{ and } \mathrm{Im}\,(C_S C_V^+ + C_S' C_V'^+).$$

Thus we get 14 relations for the determination of 15 quantities (8 absolute values and 7 phase differences). The additional relation necessary for the determination of the relative phase of the Fermi and Gamow-Teller interactions can be found by measuring W(p, η) for the β-decay transitions ΔI = 0 (no).

Note added in proof (September 19, 1957). After this paper had been submitted for publication, there appeared a paper [J. D. Jackson, S. B. Treiman, and H. W. Wyld, Jr., Phys. Rev. 106, 517 (1957)] devoted to the same problem. But these authors differ from us in that they take the phase α to be zero.

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