*DIFFRACTION SCATTERING AND NUCLEON STRUCTURE**

S. Z. BELEN'KII †

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor May 17, 1957

J. Exptl. Theoret. Physics (U.S.S.R.) 5, 1248-1250 (November, 1957)

The scattering of high-energy particles, such as the collision of two nucleons or the collision of a π meson with a nucleon, is analyzed. It is shown that, at high energies, each of these processes is characterized by a single scattering amplitude. Only the imaginary part of this amplitude plays an essential role, and, generally speaking, it can be determined from experiments on diffraction scattering.

1. Elastic meson-nucleon and nucleon-nucleon scattering is in general a very complicated affair. The functions characterizing the scattering amplitudes have both real and imaginary parts. They differ for states of different isotopic and ordinary spin. A phase-shift analysis is very difficult to carry out even at moderate energies, and its relation to the structure of nucleons is quite uncertain. It is possible, however, to make the following statement: When the scattered particles carry large energies, the analysis becomes simplified, only one scattering amplitude plays an essential role, and in general only its imaginary part. In that case only one real function is unknown, and it can be fully and completely determined from meson-nucleon and nucleon-nucleon diffraction scattering experiments. This function leads to definite conclusions about the size and form-factor (structure) of nucleons.

The proof of this statement is based on the following three conditions: (1) the isotopic invariance of nuclear interaction, (2) the high probability of multiple production of π mesons at high energies; (3) the fact that at high energies the total interaction cross section tends to a value σ_0 which differs from zero and is independent of energy (and equal, furthermore, to that for the interaction of π^+ and π^- mesons with nucleons). It should be noted that an attempt has already been made[1] to obtain some information on the nucleon "radius" from experimental data on the elastic and non-elastic scattering of π mesons having energies of 1.4 Bev and greater. This analysis, however, was based on a crude model of the nucleon represented by a sphere of constant density with abrupt boundaries. As was shown earlier,[2] the problem of determining the nucleonic structure can be treated more consistently by starting from the general scattering theory for particles which can also be absorbed by the scattering center. In that case the spatial distribution of the scattering and absorbing field may be obtained from experimental data on the elastic and inelastic scattering of π mesons by nucleons, if these data are sufficiently detailed.

Essential to this proof is the premise that in the imaginary part of the scattering amplitude is considerably smaller than the real part in the corresponding energy region.[3] These questions will be considered here in greater detail.

2. We consider first the scattering of a proton by a proton (pp) and of a proton by a neutron (pn). We denote the wave function of the isotopic state as follows:

$$(pp) = \chi_{1,1}, \quad (nn) = \chi_{1,-1}, \quad (pn) = (\chi_{0,0} - \chi_{1,0}) / \sqrt{2}, \quad (np) = (\chi_{0,0} - \chi_{1,0}) / \sqrt{2}. \tag{1}$$

The first index denotes the total isotopic spin T, and the second index its z-component T_z.

The following three basic processes may occur in elastic scattering:[3]

$$\begin{aligned} &1)\ p + p \to p + p, (pp) \to (pp);\\ &2)\ p + n \to p + n, (pn) \to (pn);\\ &3)\ p + n \to n + p, (pn) \to (np). \end{aligned} \tag{2}$$

*Prepared for publication by V. P. Silin and E. L. Feinberg from notes found among the manuscripts of S. Z. Belen'kii. This work was reported at a seminar of the Theoretical Division of the Physics Institute, Academy of Sciences, September 7, 1956.

†Deceased.

The last process includes an exchange of nucleons (exchange scattering).

The differential cross section for these reactions is obtained from the scattering amplitudes in the state T = 1 and T = 0, denoted[3] by f and g, as follows:

$$\sigma_1 = |f|^2, \quad \sigma_2 = {}^1/_4 |f+g|^2, \quad \sigma_3 = {}^1/_4 |f-g|^2. \tag{3}$$

At small energies, say on the order of 10^8 ev, it is well known that there is a considerable amount of exchange scattering. This means that σ_2 and σ_3 are of the same order of magnitude. In that case σ_1 is also of the same order of magnitude. Therefore, $|f| \gg |g|$.

At large energies, where the main process consists of meson production, i.e. an inelastic interaction, the elastic exchange scattering is relatively small, i.e. $\sigma_3 \ll \sigma_{\text{inel}}$. As for σ_2, the elastic scattering cross section with no charge exchange, it is always of the order of σ_{inel}, since it contains diffraction scattering which is considerable even for a "gray" nucleus. Thus, at large energies, $\sigma_3 \ll \sigma_2$ while $\sigma_2 \sim \sigma_1$, i.e.

$$|f-g| \ll |f|, \quad |f| \sim |g|, \tag{4}$$

and all the scattering processes are characterized by a single function f, which may thus be related to the structure of the nucleon.

3. Similar considerations apply to the interaction of π mesons with nucleons. Five basic processes are now possible, and their cross sections can similarly be expressed in terms of two scattering amplitudes for isotopic spins $T = {}^3/_2$ (amplitude f) and $T = {}^1/_2$ (amplitude g):

$$\begin{array}{ll} 1)\ \pi^+ + p \to \pi^+ + p, & \sigma_1 = |f|^2; \\ 2)\ \pi^0 + p \to \pi^0 + p, & \sigma_2 = {}^1/_9 |2f+g|^2; \\ 3)\ \pi^0 + p \to \pi^+ + n, & \sigma_3 = \sigma_5 = {}^2/_9 |f-g|^2; \\ 4)\ \pi^- + p \to \pi^- + p, & \sigma_4 = {}^1/_9 |f+2g|^2; \\ 5)\ \pi^- + p \to \pi^0 + n, & \sigma_5 = \sigma_3 \end{array} \tag{5}$$

(these, of course, are not the same f and g as for the nucleon-nucleon scattering). At small energies, the scattering cross sections for π^+ and π^- mesons are different. At high energies, inelastic processes come into play. In comparison with these, the charge-exchange scattering cross section σ_5 is small, while σ_1, σ_2 and σ_4, which include diffraction scattering, are of the same order of magnitude as σ_{inel}, i.e., $\sigma_1 \sim \sigma_2 \sim \sigma_4 \sim \sigma_0$, $\sigma_3 \sim \sigma_5 \ll \sigma_0$, or

$$|f-g| \ll |f|, \quad |f| \sim |g|, \tag{6}$$

as in the case of nucleon-nucleon collisions. For mesons too, therefore, all the essential processes of elastic scattering are determined at high energies by a single function f. In other words, the scattering does not depend upon the isotopic spin.

4. We now show, on the basis of general considerations, that the imaginary part of the scattering amplitude f is large compared with its real part, so that all the scattering processes can be described at high energy by one real function.

In order to distinguish among the five types of pion-nucleon scattering processes, we introduce the complex scattering amplitudes $f_i(k)$, $i = 1, \dots, 5$, as functions of the meson momentum k. Then $f_1(k) \equiv f(k)$ and $f_4(k) = [f(k) + 2g(k)]/3$, and therefore $g(k) = [3f_4(k) - f_1(k)]/2$. It follows then that

$$f_5(k) = \frac{\sqrt{2}}{3}(f_1 - g) = \frac{1}{\sqrt{2}}(f_1 - f_4).$$

Separating the real and imaginary parts D(k) and A(k) of $f_1(k)$ and $f_4(k)$ respectively, we may write

$$f_1(k) = D_+(k) + iA_+(k), \quad f_4(k) = D_-(k) + iA_-(k). \tag{7}$$

The imaginary part of the forward scattering amplitude is related to the total cross section σ_0 by a formula which, inasmuch as the total cross sections for π^+ and π^- mesons practically coincide at large energies, may be written as

$$A_+(k) \approx A_-(k) = k\sigma_0 / 4\pi. \tag{8}$$

Therefore

$$f_5 = (f_1 - f_4)/\sqrt{2} = (D_+(k) - D_-(k))/\sqrt{2}. \tag{9}$$

One may further apply the relation [see, for example, Ref. (4)] $D_-(k) = D_+(-k)$, which we gave for energies much larger than the rest mass of the meson. Eq. (9) then takes the following form:

$$f_5 = (D_+(k) - D_+(-k)) / \sqrt{2}. \tag{10}$$

It is easily seen from the dispersion relations that link the real part of the forward scattering amplitude with the total scattering cross section,[4] that within given limits, the total cross sections at high energies $D_\pm(k)$ cannot increase faster than the first power of k. Thus, even if $D_+(k) = \alpha k$, where α is a constant, $D_-(k) = D_+(-k) = -\alpha k$, and therefore, $f_5(k) = \sqrt{2\alpha k} = \sqrt{2}\, D_+(k)$. Owing to the fact that at large energies, from Eq. (6), $|f_5| \ll |f_1|$, and $f_1 = D_+ + iA_+$, the following inequality holds:

$$|D_+(k)| \ll |A_+(k)|. \tag{11}$$

Note that according to Eq. (8), $A_+(k)$ increases with energy as the first power of k as long as σ_0 does not vanish, thus if $D_+(k)$ increases more slowly, Eq. (11) is fulfilled all the more.

[1] Eisberg, Fowler, Lea, Shephard, Shutt, Thorndike, and Whittemore, Phys. Rev. **97**, 797, (1955).

[2] S. Z. Belen'kii, J. Exptl. Theoret. Phys. (U.S.S.R.) **30**, 983, (1956), Soviet Phys. JETP **3**, 813 (1956).

[3] L. B. Okun' and I. Ia. Pomerantchuk, J. Exptl. Theoret. Phys. (U.S.S.R.) **30**, 424 (1956), Soviet Phys. JETP **3**, 307 (1956).

[4] Goldberger, Miyazawa, and Oehme, Phys. Rev. **99**, 986 (1955).

Translated by M. A. Melkanoff
252

SOVIET PHYSICS JETP VOLUME 6 (33), NUMBER 5 MAY, 1958

INVERSE DISPERSION RELATIONS

V. Z. BLANK and D. V. SHIRKOV

Joint Institute of Nuclear Research

Submitted to JETP editor May 17, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 1251-1253 (November, 1957)

"Inverse" dispersion relations express the imaginary part of a scattering amplitude as a Cauchy integral of the real part. The integral over the unphysical region is evaluated by making use of ordinary "direct" dispersion relations. The forward scattering amplitudes for charged pions on nucleons are discussed in detail. The inverse dispersion relations for this case are written out explicitly in a form in which no unobservable quantities appear.

STARTING from quantum field theory, various authors[1,2] have derived dispersion relations for the scattering of pions and photons on nucleons and for the photoproduction of pions. These relations express the real part $D(E)$ of the amplitude as a Cauchy integral of the imaginary part $A(E)$, thus

$$D(E) = \frac{1}{\pi} \mathrm{P} \int_{-\infty}^{\infty} \frac{A(E')}{E' - E} dE'. \tag{1}$$

The negative part of the energy range can be eliminated by using symmetry properties, and for forward scattering the part of the integral arising from unphysical positive energies can be computed explicitly. This reflects the fact that in the unphysical region the function $A(E)$ can be represented as a sum of δ-functions. The integral over the unphysical range then depends only on the coupling constants of the relevant interactions.