ON THE PROBLEM OF THE EXCITED STATES OF NUCLEONS

N. F. NELIPA

Physical Institute of the Academy of Sciences, S.S.S.R.

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It is shown that within the limits of the Markov equation,[1] two different classes of excited states exist for the nucleon: short lived (nuclear times) and long lived ($\sim 10^{-10}$ sec). These states may be identified with hyperons. Taking account of the latter in intermediate states leads to resonance effects. The scattering of π mesons by protons is used as a methodological example. The calculated maxima in the total scattering cross sections are quite close to the experimentally observed values.

1. Markov[1] has proposed a nonlocal equation for the description of excited nucleons. We wish to point out an interesting peculiarity of the Markov equation. It appears that two very different classes of excited states exist within the framework of this equation:* short lived (nuclear times) and relatively long lived ($\sim 10^{-10}$ sec). Generally speaking, the excited states of the nucleon may be identified with heavy particles.[1] Then unstable and relatively stable heavy particles will correspond to these classes of excited states. Taking account of these particles, similar to isobars,[2] may lead to resonance effects in intermediate states. As an example having methodological significance, the scattering of π mesons by protons is considered. If the first excited state of the nucleon is taken[3,4] to be a Λ^0 particle, for example, calculations lead to the presence of maxima in the total π^-—p scattering cross sections at π^--meson energies close to those found experimentally. Thus, in principle, a new possibility emerges for the explanation of experimental facts.

2. The Markov equation is[1]

$$\{\gamma_\nu \partial / \partial X_\nu + a\,[\partial^2 / \partial\zeta_\nu \partial\zeta_\nu + V(\zeta_\nu\zeta_\nu)]^n + m_0 + W(x')\}\,\psi = 0, \tag{1}$$

where $\zeta_\nu = r_\nu/r_0$ is a dimensionless internal degree of freedom, r_0 is a constant with the dimensions of length, X_ν is the coordinate of the "center of mass" of the particle, $W(x')$ is the energy operator of the interaction between the meson and the nucleon fields, and a and m_0 are constants.

It is characteristic of Eq. (1) that the interaction $W(x')$ is considered to be point-like, but displaced with respect to the "center of mass" of the particle (in the simplest case, $x'_\nu = X_\nu + r_\nu$); quanta of the field (for example, mesons) may be emitted and absorbed at any point of space surrounding the nucleon.

The choice of a concrete form of Eq. (1) is quite arbitrary because of the indefiniteness of the expression for $V(\zeta_\nu\zeta_\nu)$ and the value of n. However, to elucidate the possibilities of Eq. 1, we may limit ourselves to the case of $n = 1$ and

$$V(\zeta_\nu\zeta_\nu) = \zeta_\nu\zeta_\nu + 2\,\frac{(k_\nu \partial / \partial\zeta_\nu)^2 - k_\nu\zeta_\nu}{k_\nu k_\nu}. \tag{2}$$

Then the wave function of the unexcited Eq. 1 will have the form

$$\psi = u(X)\,\chi_{n_1 n_2 n_3 n_0}(\zeta_\nu). \tag{3}$$

The function χ describes the internal states of the particle. The set of numbers (n_1, n_2, n_3, n_0) characterize the degree of excitation of the nucleon. For the excited state of a nucleon with nonvanishing momentum **p**,

$$\chi_{n_1 n_2 n_3 n_0} = N H_{n_1}(\zeta_1)\,H_{n_2}(\zeta_2)\,H_{n_3}(\zeta_3)\,H_{n_0}(\zeta_0)\,\exp\left\{-\tfrac{1}{2}\,[\zeta_\nu\zeta_\nu + 2\,(p_\nu\zeta_\nu)^2 / M_0^2]\right\}, \tag{4}$$

where N is a normalization factor, $H_n(\xi)$ is a Chebyshev-Hermite polynomial, and M_0 is the mass of the unexcited nucleon.

*This results from the existence of definite selection rules for the matrix elements over internal variables.

The function u(X) in Eq. (3) describes the motion of the "center of mass," and is a solution of the usual Dirac equation with mass

$$M_n = m_0 + a(n_1 + n_2 + n_3 + n_0 + 2) = M_0 + a(n_1 + n_2 + n_3 + n_0). \tag{5}$$

The constant a characterizes the energy spacing between two excitations of the nucleon and is determined by experiment. The equidistant spectrum is due to the coice of $V(\xi_\nu \xi_\nu)$ in the form of Eq. (2).

It is clear that the Markov equation yields an increasing mass spectrum, Eq. (5), and is thus essentially different from other relativistic equations for excited elementary particles (see, for example, Ref. 5).

3. We shall show now that definite selection rules exist for the matrix elements over internal variables, which leads to two classes of excited states of nucleons: short lived (allowed transitions) and relatively long lived (forbidden transitions).

For definiteness, let us consider the matrix element of the transition of a nucleon with momentum $\mathbf{p}_0$ and any (n_1, n_2, n_3, n_0) into a state with momentum $\mathbf{p}'$ and any (n_1', n_2', n_3', n_0'), with absorption of a meson with momentum $\mathbf{k}_0$ (so that $\mathbf{p}' = \mathbf{p}_0 + \mathbf{k}_0$). To simplify the calculations, we transform to the center-of-mass system of the meson and nucleon, (i.e., set $\mathbf{p}_0 + \mathbf{k}_0 = 0$). The momentum of the nucleon in the intermediate state in the c.m.s. is zero in this case, and therefore the form of the function χ is simplified. Moreover, we orient the x-axis along the nucleon momentum in the c.m.s. and limit ourselves to the case of pseudoscalar interaction between the meson and nucleon with pseudoscalar coupling. Then, bearing Eq. (4) in mind, the transition matrix element over the internal variables may be written in the present case in the c.m.s.,

$$I_{nn'} = N \int\limits_{-\infty}^{+\infty} H_{n_1}(\zeta_1) \ldots H_{n_0'}(\zeta_0)\, e^B d\zeta_1 d\zeta_2 d\zeta_3 d\zeta_0, \tag{6}$$

where N is the normalization constand for the Chebyshev-Hermite polynomials,

$$B = -\zeta_1^2 - \zeta_2^2 - \zeta_3^2 - M_0^{-2}(q_1\zeta_1 + E\zeta_0)^2 - i(q_1\zeta_1 - \varepsilon\zeta_0) r_0, \qquad q = |\mathbf{p}_0| = |\mathbf{k}_0|,$$

q_1 is the projection of the vector $\mathbf{q}$ on the x-axis, ϵ is the meson energy, and E is the energy of the nucleon.

We evaluate the integral in Eq. (6) by generalizing somewhat a method which we have used previously.[6] We start with the equalities[7]

$$\exp\{-t_1^2 + 2t_1\zeta_1\} = \sum_{n_1=0}^{\infty} \frac{1}{n_1!} H_{n_1}(\zeta_1)\, t_1^{n_1}, \ldots \tag{7}$$

$$\exp\{-t_1'^2 + 2t_1'\zeta_1\} = \sum_{n_1'=0}^{\infty} \frac{1}{n_1'!} H_{n_1'}(\zeta_1)\, t_1'^{n_1'} \ldots \tag{8}$$

We multiply both sides of Eqs. (7) and (8) term by term and then multiply both sides of the result by e^B. Then we integrate both sides over $\zeta_1, \zeta_2, \zeta_3, \zeta_0$ from $-\infty$ to $+\infty$ and obtain

$$\begin{gathered} N\pi^2 \frac{M_0}{E} \exp\left[-\frac{1}{4} r_0^2 \left(q_1^2 + \varepsilon^2 + 2\frac{\varepsilon}{E} q_1^2\right)\right] [-\alpha(t_1 + t_1') + \beta(t_0 + t_0') - 2\frac{q_1}{E}(t_1 + t_1')(t_0 + t_0') + 2(t_1 t_1' + t_0 t_0')] \\ = \sum_{n_1=0}^{\infty} \ldots \sum_{n_0'=0}^{\infty} I_{nn'} \frac{1}{n_1!} \ldots \frac{1}{n_0'!} t_1^{n_1} \ldots t_0'^{n_0'}, \end{gathered} \tag{9}$$

where

$$\alpha = ir_0(q_1 + q_1\varepsilon / E), \quad \beta = ir_0(q_1^2/E + \varepsilon). \tag{10}$$

Equation (9) is the expansion of the exponent in a Maclaurin series in powers of t and t′, the coefficients of which are the desired integrals. By the general rule for finding the coefficients of the expansion of a function in a Maclaurin series, we obtain

$$I_{nn'} = N\pi^2 \frac{M_0}{E} \exp\left[-\frac{1}{4} r_0^2\left(q_1^2 + \varepsilon^2 + 2\frac{\varepsilon}{E} q_1^2\right)\right] \frac{\partial^{n_1+\ldots+n_0'}}{\partial t_1^{n_1} \ldots \partial t_0'^{n_0'}} \exp\left[-\alpha(t_1 + t_1') + \beta(t_0 + t_0') - 2\frac{q_1}{E}(t_1 + t_1')(t_0 + t_0') + 2(t_1 t_1' + t_0 t_0')\right]_{t_1=t_2 \ldots =t_0'=0}. \tag{11}$$

In the case of an arbitrary orientation of the vector $\mathbf{q}$, we must replace q_1 by $\mathbf{q}$ in Eq. (11), adding the corresponding terms in t. In particular, if $n_1 = n_2 \ldots = n_0' = 0$, then

$$I_{00} = \frac{M_0}{E} \exp\left[-\frac{1}{4} r_0^2\left(q^2 + \varepsilon^2 + 2\frac{\varepsilon}{E} q^2\right)\right]. \tag{12}$$

If $N \equiv n_1 + n_2 + n_3 = 0$, $n_0 = 0$ and $N' = 1$, $n_0' = 0$, then

$$I_{01} = \frac{1}{\sqrt{2}} I_{00} i r_0 \left[1 + \frac{\varepsilon}{E}\right] \mathbf{q}, \tag{13}$$

For $N = n_0 = 0$ and $N' = n_0' = 1$,

$$I_{02} = \frac{1}{2} I_{00}\left[-2\frac{\mathbf{q}}{E} + r_0^2\left(1 + \frac{\varepsilon}{E}\right)\left(\frac{q^2}{E} + \varepsilon\right)\mathbf{q}\right]. \tag{14}$$

If $r_0 a \sim r_0/\lambda \ll 1$, Eq. (13) is considerably smaller than Eq. (14). Using Eq. (11), it is easy to convince ourselves that generally speaking, if

$$N' = n_0', \tag{15}$$

then $I_{nn'}$ will contain terms independent of $r_0 q$, i.e., $I_{nn'}$ will be relatively large; in the other cases, $I_{nn'}$ will be a relatively small quantity, of the order of $r_0 q$. Consequently, condition (15) corresponds to allowed transitions. For the case $N' \neq n_0'$, the transitions will be less probable.*

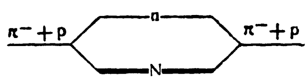
Since t and t′ enter symmetrically into Eq. (11), the selection rules for N and n_0 will be the same as for N′ and n_0'.

4. As an example having methodological significance, we will consider the scattering of π^- mesons on nucleons within the framework of Eq. (1). For simplicity, we will limit ourselves to calculations of the pseudoscalar theory, with pseudoscalar coupling between the meson and the nucleon.

We will first consider the scattering of π^- mesons by protons:

$$\pi^- + p \to \pi^- + p. \tag{I}$$

We use non-covariant perturbation theory for the calculations. The sequences of events shown in the figure correspond to process I in the first nonvanishing approximation. The lower sequence corresponds to all cases in which an excited state of the nucleon is formed in the intermediate state.



In the c.m.s. the transition matrix element corresponding to the first sequence may be written

$$\frac{H_{An}H_{nF}}{E - E'} = \frac{2\pi g^2}{\varepsilon} \frac{(u_0^* \beta\gamma_5 u')(u'^* \beta\gamma_5 u)}{E + \varepsilon - E'} I_0; \tag{16}$$

$$I_0 = \int \chi_{0000}(\zeta_\nu, q_\nu)\, e^{iq_\nu \zeta_\nu r_0} \chi_{0000}(\zeta_\nu, p_\nu')\, d\zeta_\nu \int \chi_{0000}(\zeta_\nu, p')\, e^{-iq_\nu' \zeta_\nu r} \chi_{0000}(\zeta_\nu, q_\nu')\, d\zeta_\nu. \tag{17}$$

Here, the nucleon momentum in the intermediate state $p' = 0$. The transition matrix element

$$\frac{H_{AN}H_{NF}}{E - E_N} = \frac{2\pi g^2}{\varepsilon} \frac{(u_0^* \beta\gamma_5 u_N)(u_N^* \beta\gamma_5 u)}{E + \varepsilon - E_N} I_n. \tag{18}$$

corresponds to the second sequence. We will assume that nucleons in excited states interact with mesons in the same way as protons and neutrons.[3]

The concrete form of I_n in Eq. (18) is determined by the degree of excitation of the nucleon in the

*The same selection rules were obtained by L. G. Zastavenko in another way, see Ref. 4.

intermediate state, i.e., by the values of the numbers $n_1' \ldots$. If we consider only those excitations in the intermediate state for which $N' \equiv n_1' + n_2' + n_3' = 1$, $n_0' = 1$, then three terms will enter into I_2, corresponding to the triple degeneracy of the sequence with excited nucleons in the intermediate state.*

The differential scattering cross section in the c.m.s. of the meson and nucleon can now be written

$$d\sigma/d\Omega = (g^4/2W^2)\{[(E\varepsilon + q^2)^2 + M_0^2\varepsilon^2 + (W^2 - M_0^2)\, q^2\cos\vartheta]\,(\Phi_0 + \Phi_2)^2 + [E^2 - q^2\cos\vartheta + M_0^2]\, a^2\Phi_2^2 - 2M_0 a\Phi_2(\Phi_0 + \Phi_2)\,[2E\varepsilon + q^2(1+\cos\vartheta)]\}, \tag{19}$$

where ϵ and E are the energies of the meson and nucleon, respectively, $W = E + \epsilon$, and

$$\Phi_0 = I_0/(M^2 - W_0^2),\ \Phi_2 = I_2/(W^2 - M_2^2),\ I_0 = M_0^2/E^2,\ I_2 = (M_0^2 q^2/E^4)\cos\vartheta.$$

To obtain the expressions for I_0 and I_2, we assumed that the excited state of the nucleon is a particle with a lifetime of the order of 10^{-10} sec (for example, Λ^0, etc.). Then[4] $r_0 \sim 10^{-19}$ cm and the exponential factor entering into I_0 and I_2 may be replaced by unity for practically all possible momenta. It is clear that in this case the magnitude of r_0 does not play an essential role.

Integrating Eq. (10) over the angles, we obtain the total cross section of process (I):

$$\sigma^- = (2\pi g^4/W^2)\,\Phi_0^2\,[(E\varepsilon + q^2)^2 + M_0^2\varepsilon^2] + (4\pi g^4/3W^2)\,F_2\Phi_0\,[(W^2 - M_0^2)\,q^2 - 2M_0 a q^2] + (2\pi g^4/3W^2)\,F_2^2[(E\varepsilon + q^2)^2 + M_0^2\varepsilon^2 + a^2(E^2 + M_0^2) - 2M_0 a\,(2E\varepsilon + q^2)], \tag{20}$$

where $F_2 = M_0^2 q^2/E^4(W^2 - M_2^2)$.

The most interesting thing in this expression is the presence of maxima in the cross section at $W^2 = M_2^2$, or more generally, at $W^2 = M_n^2$. The value of M_n is determined by the magnitude of a, i.e., the energy spacing between two excitations of the nucleon. If we assume that the excited state of the nucleon is, for example, a Λ^0-particle, then a $\sim$ 180 Mev, and the cross section will have maxima at kinetic energies of the scattered π^- mesons of $\sim$ 300, 910 and 1570 Mev in the laboratory system for M_2, M_4 and M_6, respectively. This discloses, in principle, a new possibility of explaining the experimentally-observed maxima in the scattering of π^- mesons by nucleons, occurring, as is well known,[8] at meson energies $\sim$ 200 and 900 Mev. To draw more definite conclusions, it is necessary to study the same problem in the theory of damping, and also to keep in mind that our considerations are based on the assumption that $V(\zeta_\nu \zeta_\nu)$ has the form of Eq. (2), and that $n = 1$.

5. Evidently, the presence of maxima in the scattering cross section of π^- mesons by nucleons is due to a term of the type $1/(W^2 - M_n^2)$. It is easy to see that for the process

$$\pi^+ + p \to \pi^+ + p \tag{II}$$

this factor appears only if it is admitted that an excited doubly charged particle appears in the intermediate state.† Therefore, process II requires additional study.

In conclusion, I wish to thank Prof. M. A. Markov for suggesting this topic, and for helpful advice during the work.

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*The less interesting case $N' = 1$, $n_0' = 0$ was considered by a Filiukov (Dissertation, Moscow State University, 1955).

†A similar situation occurs in the theory of isobaric states (see Ref. 2).