AGING EQUATION FOR GAMMA QUANTA

B. V. NOVOZHILOV

Institute of Chemical Physics, Academy of Sciences, U.S.S.R.

Submitted to JETP editor June 6, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) 33, 1287-1289 (November, 1957)

An equation is derived for the distribution function of multiply-scattered low energy γ-quanta ($E \ll m_0c^2$), analogous to the aging equation for neutrons. The spatial and wavelength distributions obtained for the quanta are in satisfactory agreement with the numerical solutions of the Boltzmann equation.

In the neutron attenuation theory one frequently employs the so-called aging equation,[1] derived on the assumption that all neutrons of specified initial energy behave identically in the sense of energy loss. This is true when the energy lost by a neutron in one collision is considerably smaller than its initial energy, i.e., when the neutrons are moderated in a medium whose nuclei have a mass much greater than the neutron mass.

An approximation analogous to the aging approximation for neutrons can be devised also for γ quanta moving in a scattering and absorbing medium. In this case the criterion of applicability of the aging equation will be the inequality $E \ll m_0c^2$, where E is the energy of the γ quanta and m_0c^2 is the rest energy of the electron. Actually, the change in the wavelength $\lambda = m_0c^2/E$ of a quantum, in the case of Compton scattering by an angle θ, is

$$\lambda' - \lambda = 1 - \cos\theta, \tag{1}$$

where λ and λ' are the wavelengths before and after scattering. In order for the inequality $\Delta\lambda \ll \lambda$ to be satisfied, as required for the derivation of an aging equation, we must have $\lambda \gg 1$, i.e., $E \ll m_0c^2$. If we denote by $\xi(\lambda)$ the average wavelength change $\overline{\lambda' - \lambda}$ resulting from a single Compton scattering event, we can write

$$\xi(\lambda) = 1 - \overline{\cos\theta}, \tag{2}$$

where the average cosine of the scattering angle is

$$\overline{\cos\theta} = \frac{\pi r_0^2}{\sigma(\lambda)}\lambda\left[6\lambda^2 + 6\lambda - 2 + \frac{2}{(\lambda+2)^2} + (1 - \lambda - 6\lambda^2 - 3\lambda^3)\ln\frac{\lambda+2}{\lambda}\right], \tag{3}$$

[$\sigma(\lambda)$ is the Compton cross section, and r_0 is the classical radius of the electron]. We list below several values of $\xi(\lambda)$.

In the derivation of the aging equation we assume that the source emits quanta of wavelength λ_0, and that the angular distribution of the quanta is spherically symmetrical (for the energies considered here, this is true already after a few collisions). Assuming that the wavelength of the quantum changes continuously, and not jumpwise, we can write

λ	$\xi(\lambda)$	λ	$\xi(\lambda)$
1	0.708	12	0.940
2	0.786	15	0.947
4	0.859	17	0.950
6	0.904	20	0.955
8	0.924	∞	1.000
10	0.935		

$$d\lambda = \mu(\lambda)\xi(\lambda)c\,dt, \tag{4}$$

where $d\lambda$ is the change in the wavelength of the quantum during the time dt, and $\mu(\lambda)$ is the linear coefficient of absorption for the Compton process. An exact connection is thus established between the wavelength of the γ quantum and the time elapsed from the instant of its emission.

Let $n(\mathbf{r}, t)dt$ be the density at the point $\mathbf{r}$, of the quanta whose diffusion time ranges from t to $t + dt$. For the distribution function $n(\mathbf{r}, t)$ one can write a balance equation

$$\frac{\partial n(\mathbf{r}, t)}{\partial t} = -\operatorname{div}\mathbf{j}(\mathbf{r}, t) - \mu_{\mathrm{ph}}(t)\,n(\mathbf{r}, t)\,c. \tag{5}$$

The first term in the right half, which takes into account the escape of the quanta, will be replaced by $D\nabla^2 n(\mathbf{r}, t)$, where the diffusion coefficient is

$$D = c/3\,\mu(1-\overline{\cos\theta}). \tag{6}$$

The second term describes the absorption of the quanta as a result of the photoeffect; $\mu_{ph}(t)$ is the linear absorption coefficient for the photoeffect, taken at a wavelength corresponding to the diffusion time t. Changing from the temporal distribution function $n(\mathbf{r}, t)$ with respect to time to the wavelength distribution function $n(\mathbf{r}, \lambda)$ through the use of the relation $n(\mathbf{r}, t) = n(\mathbf{r}, \lambda)\,d\lambda/dt$, and replacing the differentiation with respect to time by differentiation with respect to λ, we get

$$c\mu(\lambda)\xi(\lambda)\frac{\partial[cn(\mathbf{r},\lambda)\mu(\lambda)\xi(\lambda)]}{\partial\lambda} = D\nabla^2[n(\mathbf{r},\lambda)\mu(\lambda)\xi(\lambda)c] - \mu_{ph}(\lambda)\mu(\lambda)c\xi(\lambda)n(\mathbf{r},\lambda). \tag{7}$$

Introducing the symbols

$$q(\mathbf{r},\lambda) = n(\mathbf{r},\lambda)\mu(\lambda)\xi(\lambda)c, \quad \tau = \frac{1}{3}\int_{\lambda_0}^{\lambda} d\lambda\,[\mu(\lambda)\xi(\lambda)]^{-2}, \quad \varkappa = 3\mu_{ph}(\lambda)\mu(\lambda)\xi(\lambda) \tag{8}$$

the aging equation can be written as

$$\partial q(\mathbf{r},\tau)/\partial\tau = \nabla^2 q(\mathbf{r},\tau) - \varkappa(\tau)q(\mathbf{r},\tau). \tag{9}$$

In analogy with neutron attenuation, τ can be called the age of the γ quantum, and $q(\mathbf{r}, \tau)$, the attenuation density.

Let us consider a point source in an infinite medium. If the source is placed at the origin and emits S quanta per unit time, Eq. (9) is augmented by the initial condition

$$q(\mathbf{r}, 0) = S\delta(\mathbf{r}), \tag{10}$$

and the solution of the equation becomes

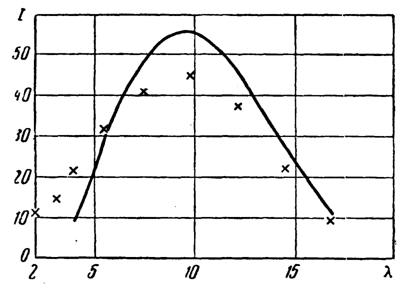
$$q(\mathbf{r},\tau) = S(4\pi\tau)^{-3/2}\exp\left[-\frac{r^2}{4\tau} - \int_0^\tau \varkappa\,d\tau\right]. \tag{11}$$

Let us note that

$$\int_0^\tau \varkappa\,d\tau = \int_{\lambda_0}^{\lambda} d\lambda\,\mu_{ph}(\lambda)/\mu(\lambda)\xi(\lambda).$$

The literature contains no experimental data on the gamma spectrum with which to compare the distribution (11). There is merely a mention[2] of an investigation of the spectrum of γ-quanta from Hg^{203} ($E_0 = 0.275$ Mev) in water.

However, the distribution (11) can be compared with the results of a numerical solution of the kinetic equation for the distribution function of the quanta.[3] The results of these comparisons are shown in the figure. The ordinates represent the energy I transported by the quanta in all directions per unit time, across a unit area, within a unit energy interval. The source energy is $E_0 = 0.25$ Mev ($\lambda = 2$). The quanta propagate in water, and their distance from the source is four mean free paths of quanta of energy E_0. The solid curve is obtained from Eq. (11), while the crosses indicate the results of Goldstein and Wilkins.[3] The discrepancy at small wavelengths is attributed to the fact that the aging equation is not valid at energies close to the source energy. The discrepancy over the remaining interval is due first to errors in Ref. 3, which are estimated by the authors to be 10 — 15%, and second to the rather high initial energy of the quanta ($\lambda_0 = 2$, while the condition for the validity of the approximation is $\lambda \gg 1$).



[1] A. I. Akhiezer and I. Ia. Pomeranchuk. Некоторые вопросы теории ядра (Certain Problems in the Theory of the Nucleus) GTTL, 1950.

[2] M. M. Weiss and W. Bernstein. Phys. Rev. 92, 1264 (1954).

[3] H. Goldstein and J. Wilkins, Calculations of the Penetration of γ Rays. Final report, NYO-3075. 1954.

Translated by J. G. Adashko