SATURATION EFFECT IN A SYSTEM WITH THREE ENERGY LEVELS

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The response of a system with three levels (E_1, E_2, and E_3) to an alternating field with harmonics $\omega_{31} = (E_3 - E_1)/\hbar$, $\omega_{21} = (E_2 - E_1)/\hbar$, and $\omega_{32} = (E_3 - E_2)/\hbar$ is examined. An expression which can be used in the theory of quantum oscillators or amplifiers, is derived for the dielectric constant (or magnetic permeability).

MUCH attention has lately been paid in radio spectroscopy to various kinds of quantum-mechanical amplifiers and oscillators (see, for example, Refs. 1 — 10). One proposed system employs three paramagnetic-resonance energy levels.[7] It is therefore interesting to consider the effect of an alternating high-frequency field on a system with three energy levels.

Let the system considered have levels E_1, E_2, and E_3 and let $E_1 < E_2 < E_3$. Furthermore, let the system be under the influence of high-frequency field F, whose spectral expansion includes harmonics with the following frequencies:

$$\omega_{21} = (E_2 - E_1)/\hbar;\ \omega_{31} = (E_3 - E_1)/\hbar;\ \omega_{32} = (E_3 - E_2)/\hbar.$$

It is the aim of this work to investigate the behavior of the system in the presence of such a field.

The system can be described with the aid of an density matrix ρ. The latter obeys the equation

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar}(H\rho - \rho H) + \frac{\delta \rho}{\delta t}, \tag{1}$$

where H is the Hamiltonian of the system in an external field, $\delta\rho/\delta t$ is the change in the density matrix resulting from various kinds of relaxation processes. If the relaxation is due to collisions in a gas, it is possible to show (see, for example, Ref. 11), that

$$\delta\rho/\delta t = -\tau^{-1}(\rho - \rho_0), \tag{2}$$

where τ is the relaxation time and ρ_0 is the density matrix in the equilibrium state. Obviously, a similar equation describes the transition to the equilibrium state in paramagnetic systems (see, for example, Ref. 12). Thus, collecting (1) and (2), we get

$$\partial\rho/\partial t = -(i/\hbar)(H\rho - \rho H) - \tau^{-1}(\rho - \rho_0). \tag{3}$$

This equation can be readily generalized by considering that in radio spectroscopy, generally speaking, two relaxation times τ_1 and τ_2 are used. Here τ_1 corresponds to the relaxation of the system level population, and τ_2 is the relaxation of the average dipole moment of the system, i.e., on the average, within a time τ_1 the diagonal elements of ρ go into ρ_0 in the absence of a field, and within a time τ_2 the average dipole moment of the system becomes equal to zero (i.e., the non-diagonal elements of ρ vanish). Our generalization will then be that the quantity $\tau^{-1}(\rho - \rho_0)$ of Eq. (3) is a matrix of the form

$$[\tau^{-1}(\rho - \rho_0)]_{mn} = \begin{cases} \tau_1^{-1}(\rho - \rho_0)_{mn} & \text{for } m = n \\ \tau_2^{-1}(\rho - \rho_0)_{mn} & \text{for } m \neq n. \end{cases} \tag{4}$$

In the presence of an alternating electrical or magnetic field, the Hamiltonian of our system is

$$H = H_0 - \mu F(t), \tag{5}$$

where μ is the electric or magnetic dipole moment (and accordingly F is either an electric or a magnetic field). In a representation in which the operator H_0 is diagonal and has eigenvalues E_1, E_2 and E_3, Eq. (3) becomes

$$\frac{\partial \rho_{mn}}{\partial t} + i\omega_m \rho_{mn} = \frac{i}{\hbar} F \sum_{l=1}^{3} (\mu_{ml}\rho_{ln} - \rho_{ml}\mu_{ln}) - [\tau^{-1}(\rho - \rho_0)]_{mn}, \tag{6}$$

where m, n, = 1, 2, 3; $\omega_{mn} = (E_m - E_n)/\hbar$.

We shall seek a solution of these equations for

$$F = F_{31}\cos\Omega_{31}t + F_{32}\cos\Omega_{32}t + F_{21}\cos\Omega_{21}t, \tag{7}$$

where

$$\Omega_{31} \approx \omega_{31};\ \Omega_{32} \approx \omega_{32};\ \Omega_{21} \approx \omega_{21}. \tag{8}$$

Using condition (8) and introducing

$$D_{mn} = \rho_{mm} - \rho_{nn},\quad D_{0mn} = \rho_{0mm} - \rho_{0nn}, \tag{9}$$

we obtain the approximate equations

$$\frac{\partial \rho_{mn}}{\partial t} + i\omega_{mn}\rho_{mn} + \tau_2^{-1}(\rho_{mn} - \rho_{0mn}) = \frac{i}{\hbar}\mu_{mn} F D_{nm}; \tag{10}$$

$$\frac{\partial D_{12}}{\partial t} + \tau_1^{-1}(D_{12} - D_{012}) = \frac{i}{\hbar} F [2(\mu_{12}\rho_{21} - \rho_{12}\mu_{21}) + (\mu_{13}\rho_{31} - \rho_{13}\mu_{31}) - (\mu_{23}\rho_{32} - \rho_{23}\mu_{32})]; \tag{11}$$

$$\frac{\partial D_{13}}{\partial t} + \tau_1^{-1}(D_{13} - D_{013}) = \frac{i}{\hbar} F [2(\mu_{13}\rho_{31} - \rho_{13}\mu_{31}) + (\mu_{12}\rho_{21} - \rho_{12}\mu_{21}) - (\mu_{32}\rho_{23} - \rho_{32}\mu_{23})]; \tag{12}$$

$$\frac{\partial D_{23}}{\partial t} + \tau_1^{-1}(D_{23} - D_{023}) = \frac{i}{\hbar} F [2(\mu_{23}\rho_{32} - \rho_{23}\mu_{32}) + (\mu_{21}\rho_{12} - \rho_{21}\mu_{12}) - (\mu_{31}\rho_{13} - \rho_{31}\mu_{13})]. \tag{13}$$

We shall seek solutions of (10) — (13) in the form*

$$\rho_{mn} = \rho_{mn}^{(+)} \exp\{i\Omega_{mn}t\} + \rho_{mn}^{(-)} \exp\{-i\Omega_{mn}t\}\ (\Omega_{mn} \equiv \Omega_{nm}),\ m \neq n, \tag{14}$$

$$D_{mn} = \text{const}. \tag{15}$$

We neglect here the higher harmonics and non-resonant terms, which is fully justified if condition (8) is taken into account.

Since the equilibrium matrix ρ_0 is diagonal, we obtain from Eq. (10)

$$\rho_{mn}^{(\pm)} = (i/2\hbar) F_{mn}\mu_{mn}D_{mn} / [i(\omega_{mn} \pm \Omega_{mn}) + \tau_2^{-1}]; \tag{16}$$

Here $F_{mn} = F_{nm}$. From (11) — (13) and (16), using again condition (8), we get

$$\begin{aligned} D_{12}(1 + 2\gamma_{12}) - D_{23}\gamma_{23} + D_{13}\gamma_{13} &= D_{012}; \\ -D_{12}\gamma_{12} + D_{23}(1 + 2\gamma_{23}) + D_{13}\gamma_{13} &= D_{023}; \\ D_{12}\gamma_{12} + D_{23}\gamma_{23} + D_{13}(1 + 2\gamma_{13}) &= D_{013}, \end{aligned} \tag{17}$$

where

$$\gamma_{mn} = F_{mn}^2 |\mu_{mn}|^2 \tau_1\tau_2^{-2} / 2\hbar^1 [(\omega_{mn} - \Omega_{mn})^2 + \tau_2^{-2}]. \tag{18}$$

From Eqs. (16), (17), and (18) we get

$$D_{12} = \frac{1}{\Delta}[D_{012}(1 + 2\gamma_{23} + 2\gamma_{13} + 3\gamma_{23}\gamma_{13}) - D_{013}\gamma_{13}(1 + 3\gamma_{23}) + D_{023}\gamma_{23}(1 + 3\gamma_{13})], \tag{19}$$

$$D_{23} = \frac{1}{\Delta}[D_{023}(1 + 2\gamma_{12} + 2\gamma_{13} + 3\gamma_{13}\gamma_{12}) - D_{013}\gamma_{13}(1 + 3\gamma_{12}) + D_{012}\gamma_{12}(1 + 3\gamma_{13})], \tag{20}$$

$$D_{13} = \frac{1}{\Delta}[D_{013}(1 + 2\gamma_{12} + 2\gamma_{23} + 3\gamma_{12}\gamma_{23}) - D_{023}\gamma_{23}(1 + 3\gamma_{12}) - D_{012}\gamma_{12}(1 + 3\gamma_{23})], \tag{21}$$

$$\rho_{32}^{(-)} = \frac{F_{23}\mu_{32}}{2\hbar} \frac{\left[D_{023} - \dfrac{D_{013}\gamma_{13}(1 + 3\gamma_{12}) + D_{012}\gamma_{12}(1 + 3\gamma_{13})}{1 + 2\gamma_{12} + 2\gamma_{13} + 3\gamma_{13}\gamma_{12}}\right]\left(\omega_{32} - \Omega_{32} + \dfrac{i}{\tau_2}\right)}{(\omega_{32} - \Omega_{32})^2 + \tau_2^{-2} + \dfrac{F_{23}^2|\mu_{32}|^2\tau_1\tau_2^{-1}}{2\hbar^2}\left(\dfrac{2 + 3\gamma_{12} + 3\gamma_{13}}{1 + 2\gamma_{12} + 2\gamma_{13} + 3\gamma_{13}\gamma_{12}}\right)}, \tag{22}$$

*The transient process will not be considered.

$$\rho_{31}^{(-)} = \frac{F_{13}\mu_{31}}{2\hbar} \frac{\left[D_{013} - \dfrac{D_{023}\gamma_{23}(1+3\gamma_{12}) + D_{012}\gamma_{12}(1+3\gamma_{23})}{1+2\gamma_{12}+2\gamma_{23}+3\gamma_{12}\gamma_{23}}\right]\left(\omega_{31} - \Omega_{31} + \dfrac{i}{\tau_2}\right)}{(\omega_{31}-\Omega_{31})^2 + \tau_2^{-2} + \dfrac{F_{13}^2|\mu_{13}|^2\tau_1\tau_2^{-1}}{2\hbar^2}\left(\dfrac{2+3\gamma_{12}+3\gamma_{23}}{1+2\gamma_{12}+2\gamma_{23}+3\gamma_{12}\gamma_{23}}\right)}. \tag{23}$$

$$\rho_{21}^{(-)} = \frac{F_{12}\mu_{21}}{2\hbar} \frac{\left[D_{012} - \dfrac{D_{013}\gamma_{13}(1+3\gamma_{23}) + D_{023}\gamma_{23}(1+3\gamma_{13})}{1+2\gamma_{13}+2\gamma_{23}+3\gamma_{13}\gamma_{23}}\right]\left(\omega_{21} - \Omega_{21} + \dfrac{i}{\tau_2}\right)}{(\omega_{21}-\Omega_{21})^2 + \tau_2^{-2} + \dfrac{F_{21}^2|\mu_{21}|^2\tau_1\tau_2^{-1}}{2\hbar^2}\left(\dfrac{2+3\gamma_{13}+3\gamma_{23}}{1+2\gamma_{13}+2\gamma_{13}+3\gamma_{12}\gamma_{23}}\right)}, \tag{24}$$

where

$$\Delta = 1 + 2(\gamma_{12} + \gamma_{23} + \gamma_{13}) + 3(\gamma_{12}\gamma_{13} + \gamma_{12}\gamma_{13} + \gamma_{23}\gamma_{13}), \tag{25}$$

$$D_{mn} = -D_{nm}, \quad \rho_{mn}^{(+)} = (\rho_{nm}^{(-)})^*. \tag{26}$$

The last equality follows from the fact that the matrix ρ is Hermitian.

In addition to the quantities (22) – (24) and their complex conjugates, there is a whole series of quantities, such as $\rho_{13}^{(-)}$, $\rho_{12}^{(-)}$, etc., which can be neglected in this approximation.

With the solution obtained it is possible to find the average dipole moment of the system using

$$P = \mathrm{Sp}(\rho\mu) \equiv \sum_{m,\, n=1}^{3} \rho_{mn}\mu_{nm} \tag{27}$$

to calculate the dielectric constant (or magnetic permeability) at the frequencies Ω_{31}, Ω_{21}, and Ω_{32}, and to obtain the absorption coefficient at these frequencies.

Let us derive the corresponding formulas for resonance at the frequency Ω_{32}. Analogous formulas are obtained for other frequencies. The average dipole moment per unit volume* (polarization vector) at Ω_{32} has the form

$$P_{32} = \rho_{32}^{(-)}\mu_{23}\exp\{-i\Omega_{32}t\} + \rho_{23}^{(+)}\mu_{32}\exp\{i\Omega_{32}t\} = \mathrm{Re}\,(2\rho_{32}^{(-)}\mu_{23}\exp\{-i\Omega_{32}t\}) \tag{28}$$

(The remaining terms give a negligibly small contribution at the frequency Ω_{32}). On the other hand, the complex dielectric constant † is determined through the complex polarization coefficient κ:

$$\varepsilon = 1 + 4\pi\varkappa, \tag{29}$$

where κ is determined from the relation

$$P_{32} = \mathrm{Re}\,(\varkappa F_{23}\exp\{-i\Omega_{32}t\}). \tag{30}$$

Using Eqs. (22) and (28) – (30) we get

$$\varepsilon = 1 + 4\pi\cdot 2\rho_{32}^{(-)}\mu_{23}/F_{23} = 1 + 4\pi\frac{|\mu_{23}|^2}{\hbar} \frac{\left[D_{023} - \dfrac{D_{013}\gamma_{13}(1+3\gamma_{12}) - D_{012}\gamma_{12}(1+\gamma_{13})}{1+2\gamma_{12}+2\gamma_{13}+3\gamma_{13}\gamma_{12}}\right]\left(\omega_{32} - \Omega_{32} + \dfrac{i}{\tau_2}\right)}{(\omega_{32}-\Omega_{32})^2 + \tau_2^{-2} + \dfrac{F_{23}^2|\mu_{32}|^2\tau_1\tau_2^{-1}}{2\hbar^2}\,\dfrac{2+3\gamma_{12}+3\gamma_{13}}{1+2\gamma_{12}+2\gamma_{13}+3\gamma_{13}\gamma_{12}}}. \tag{31}$$

The absorption coefficient α is determined from (see, for example, Ref. 11)

$$\alpha = (4\pi\Omega_{32}/c)\,\mathrm{Im}\,\varkappa = \frac{4\pi\Omega_{32}}{c}\frac{|\mu_{23}|^2}{\hbar} \frac{\left[D_{023} - \dfrac{D_{013}\gamma_{13}(1+3\gamma_{12}) - D_{012}\gamma_{12}(1+\gamma_{13})}{1+2\gamma_{12}+2\gamma_{13}+3\gamma_{13}\gamma_{12}}\right]\tau_2^{-1}}{(\omega_{32}-\Omega_{32})^2 + \tau_2^{-2} + \dfrac{F_{23}^2|\mu_{32}|^2\tau_1\tau_2^{-1}}{2\hbar^2}\,\dfrac{2+3\gamma_{12}+3\gamma_{13}}{1+2\gamma_{12}+2\gamma_{13}+3\gamma_{13}\gamma_{12}}}. \tag{32}$$

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* We shall assume henceforth that the ρ's are normalized to unit volume.

†To be specific, we shall talk of a dielectric constant, even though all the formulas obtained are equally applicable to the magnetic permeability.

We see immediately from (32) that by proper choice of γ_{13} and γ_{12} (or one of these), i.e., by choice of the fields F_{13} and F_{12}, it is possible to obtain negative absorption at the frequency Ω_{32}. In this case this system can operate as an oscillator or amplifier. It is necessary for this purpose that the following condition be satisfied.

$$\left[D_{023} - \frac{D_{013}\gamma_{13}(1+3\gamma_{12}) - D_{012}\gamma_{12}(1+\gamma_{13})}{1+2\gamma_{12}+2\gamma_{13}+3\gamma_{13}\gamma_{12}}\right] < 0. \tag{33}$$

Let us consider the case when $\gamma_{12} = 0$, i.e., the field $F_{12} = 0$. Then Eqs. (31) — (33) become

$$\varepsilon = 1 + 4\pi \frac{|\mu_{23}|^2}{\hbar} \frac{[D_{023} - D_{013}\gamma_{13}/(1+2\gamma_{13})](\omega_{32} - \Omega_{32} + i/\tau_2)}{(\omega_{32}-\Omega_{32})^2 + \tau_2^{-2} + \frac{F_{23}^2 |\mu_{32}|^2 \tau_1 \tau_2^{-1}}{2\hbar^2} \frac{2+3\gamma_{13}}{1+2\gamma_{13}}}, \tag{31a}$$

$$\alpha = \frac{4\pi\Omega_{32}}{c} \frac{|\mu_{23}|^2}{\hbar} \frac{[D_{023} - D_{013}\gamma_{13}/(1+2\gamma_{13})]\tau_2^{-1}}{(\omega_{32}-\Omega_{32})^2 + \tau_2^{-2} + \frac{F_{23}^2 |\mu_{32}|^2 \tau_1 \tau_2^{-1}}{2\hbar^2} \frac{2+3\gamma_{13}}{1+2\gamma_{13}}}, \tag{32a}$$

$$[D_{023} - D_{013}\gamma_{13}/(1+2\gamma_{13})] < 0. \tag{33a}$$

The last inequality can be satisfied if

$$D_{023} - D_{013}/2 < 0. \tag{33b}$$

Let us note that (33a) changes into (33b) if

$$2\gamma_{13} \gg 1. \tag{34}$$

Taking it into account that $D_{023} = \rho_{022} - \rho_{033}$ and $D_{013} = \rho_{011} - \rho_{033}$, condition (33b) can be written

$$\rho_{022} < {}^1/_2 (\rho_{011} + \rho_{033}). \tag{35}$$

It is now easy to understand the presence of negative absorption at a frequency Ω_{32}. In fact, let the equilibrium level populations ρ_{022}, ρ_{011}, and ρ_{033} satisfy the condition (35).*

A sufficiently strong field F_{13} can then saturate the populations of levels 1 and 3, i.e., in the presence of field F_{13} the populations of levels 1 and 3 become equal to $\sim (\frac{1}{2})(\rho_{011} + \rho_{033})$. But since condition (35) is satisfied, this means that at the upper level 3 the population is greater than at the lower level 2. In that case the field F_{23} induces emission at a frequency $\Omega_{32} \approx \omega_{32}$ instead of absorption. This is precisely negative absorption.

The square brackets in expressions (31), (32) and (31a), (32a) play the role of the differences in the populations of levels 2 and 3 in the presence of the field. These differences are negative under conditions (33) and (33a).

Let us note that with the aid of formula (31) it is possible to obtain directly the amplitude and frequency of the steady-state oscillations of the generator, as well as the gain of the amplifier. For this purpose it is necessary to substitute the real and imaginary parts of (31) into the corresponding formulas of the theory of the molecular generator and amplifier (see, for example, Ref. 1).

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ON THE FLUCTUATIONS OF NUCLEAR MATTER

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It is shown that the production of energetic nuclear fragments in collisions with fast nucleons can be interpreted in terms of collisions of the incoming nucleon with the density fluctuations of the nuclear matter.

1. INTRODUCTION

THE motion of nucleons in nuclei can result in short-lived tight nucleon clusters, in other words, in density fluctuations of nuclear matter. Since such clusters are relatively far removed from the other nucleons of the nucleus, they become atomic nuclei of lower mass in a state of fluctuating compression.

In their study of the scattering of 675-Mev protons by light nuclei, Meshcheriakov and coworkers[1,2] observed recently certain effects which confirm the existence of such fluctuations, at least for the simplest nucleon-pair fluctuations, which lead to the formation of a compressed deuteron.

We recall in this connection reports in earlier works[3,4] that high-energy nucleons can split nuclei into "supra-barrier" fragments, i.e., fragments with an energy much larger than their binding energy and the energy of the Coulomb barrier. However, there was a lack of quantitative experimental data on which to base the theoretical analysis.

Some authors related this curious process, without foundation, to hypothetical long-range nuclear forces. Others tried to connect it with nuclear many-body forces.

The experimental data on the emission of high-energy deuterons from light nuclei give support to the idea that "supra-barrier" fragments are produced also by direct collision of an incoming nucleon with a tight nucleon cluster that results from density fluctuations of the nuclear matter. We offer in the following a quantitative argument in favor of the production of fast deuterons and other "supra-barrier" fragments by such fluctuations.

Concerning the nuclear many-body forces, it should be noted that, according to existing estimates,[5] there is no reason to believe that they are considerably stronger than the two-body forces. At the instant of dense clustering both paired and collective interactions may take place. However, at present there exists no experimental information which would allow an explanation of this interaction, or in particular allow a determination of the relative contributions of the paired and the collective interactions.

2. INTERACTION OF DEUTERONS WITH FAST PROTONS

It was shown experimentally[1,2] that scattering of 675-Mev protons by deuterium produces, in addition to scattered nucleons, a small number of undestroyed deuterons of high energy (up to 660 Mev). This shows that in such collisions the nucleon imparts an appreciable fraction of its momentum to the deuteron as a whole.

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ERRATA TO VOLUME 6

Page	Line	Reads	Should Read
643	16 from bottom	where $\kappa = \pi a^2 \Omega - \ldots$	where $\kappa = \pi a^2 \Omega \varphi - \ldots$
690	8 from bottom	$\ldots \sin [- \ldots$	$\ldots \sin \vartheta [- \ldots$
	5 from bottom	$\ldots \sin 2\vartheta \sqrt{\frac{1}{3}} \ldots$	$\ldots \sin 2\vartheta \left[\sqrt{\frac{1}{3}} \ldots \right.$
809	9 from top	$\ldots \left(\frac{1}{2 \sinh u} + \ldots \right.$	$\ldots \left(\frac{1}{\sinh u} + \ldots \right.$
973	unnumbered equation	$\ldots C^{S'-\mu';\, S\mu}_{n\mu-\mu'} T^{(n)}_{\mu'-\mu'}.$	$\ldots C^{S'-\mu';\, S\mu}_{n\mu-\mu'} < S' \Vert T^{(n)} \Vert S^{-1} > \times T^{(n)}_{\mu'-\mu}.$
975	5 from bottom	...of a particle by a nucleus ...	...of a particle in state a by a nucleus ...
992	Eq. (18)	$\ldots \tau_1 \tau_2^{-2} / 2\hbar^1 \ldots$	$\ldots \tau_1 \tau_2^{-1} / 2\hbar^2 \ldots$