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ON THE FLUCTUATIONS OF NUCLEAR MATTER

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It is shown that the production of energetic nuclear fragments in collisions with fast nucleons can be interpreted in terms of collisions of the incoming nucleon with the density fluctuations of the nuclear matter.

1. INTRODUCTION

THE motion of nucleons in nuclei can result in short-lived tight nucleon clusters, in other words, in density fluctuations of nuclear matter. Since such clusters are relatively far removed from the other nucleons of the nucleus, they become atomic nuclei of lower mass in a state of fluctuating compression.

In their study of the scattering of 675-Mev protons by light nuclei, Meshcheriakov and coworkers[1,2] observed recently certain effects which confirm the existence of such fluctuations, at least for the simplest nucleon-pair fluctuations, which lead to the formation of a compressed deuteron.

We recall in this connection reports in earlier works[3,4] that high-energy nucleons can split nuclei into "supra-barrier" fragments, i.e., fragments with an energy much larger than their binding energy and the energy of the Coulomb barrier. However, there was a lack of quantitative experimental data on which to base the theoretical analysis.

Some authors related this curious process, without foundation, to hypothetical long-range nuclear forces. Others tried to connect it with nuclear many-body forces.

The experimental data on the emission of high-energy deuterons from light nuclei give support to the idea that "supra-barrier" fragments are produced also by direct collision of an incoming nucleon with a tight nucleon cluster that results from density fluctuations of the nuclear matter. We offer in the following a quantitative argument in favor of the production of fast deuterons and other "supra-barrier" fragments by such fluctuations.

Concerning the nuclear many-body forces, it should be noted that, according to existing estimates,[5] there is no reason to believe that they are considerably stronger than the two-body forces. At the instant of dense clustering both paired and collective interactions may take place. However, at present there exists no experimental information which would allow an explanation of this interaction, or in particular allow a determination of the relative contributions of the paired and the collective interactions.

2. INTERACTION OF DEUTERONS WITH FAST PROTONS

It was shown experimentally[1,2] that scattering of 675-Mev protons by deuterium produces, in addition to scattered nucleons, a small number of undestroyed deuterons of high energy (up to 660 Mev). This shows that in such collisions the nucleon imparts an appreciable fraction of its momentum to the deuteron as a whole.

According to the fluctuation representation, this collision takes place at a time when the nucleons of the deuteron are at a close distance R and are strongly interacting. Under these conditions the incoming nucleon can transfer its momentum to the tight nucleon pair as a whole. The cross section to be expected for such a special collision will be

$$\sigma = \sigma_d W_d(R), \tag{1}$$

where σ_d is the total cross section for quasi-elastic collisions and $W_d(R)$ is the probability of finding the two nucleons of the deuteron at a distance less than R. The distance R has to be of the order of the range of the strong nuclear interaction, i.e., $(2-3)\hbar/Mc$. Denoting the wave function of the deuteron by $\psi_d(R)$, we have

$$W_d(R) = 4\pi \int_0^R \psi_d^2(r) r^2\, dr = \frac{4\pi}{3}\psi^2(0) R^3. \tag{2}$$

In order to calculate $W_d(R)$ one has to know the deuteron wave function close to $r = 0$. The usual asymptotic wave function $\psi_d = \sqrt{\alpha/2\pi}e^{-\alpha r}/r$ ($1/\alpha = 4.3 \times 10^{-13}$ cm) is completely useless for this purpose, since it approaches infinity for $r = 0$. The deuteron wave function has no singularities. Therefore we can approximate $u = r\psi_d$ by $\sqrt{\alpha/\pi}\, e^{-\alpha r}$ for large r and by $u = \psi(0)[r - \beta r^2 + \ldots]$ for small r and match the functions and derivatives at $r = b = 1/2\beta$. The quantity 2β is the logarithmic derivative $(\psi'/\psi)_0$. This yields $\psi(0) \sim \beta\sqrt{\alpha/2\pi}$.

Using Hulten's function

$$r\psi_d = \sqrt{\alpha/2\pi}\,(e^{-\alpha r} - e^{-\beta r})$$

We obtain the same result. According to known data β is a few times larger than α. Restricting ourselves to an order-of-magnitude estimate, we have

$$\psi_d(0) = \beta\sqrt{\alpha/2\pi}, \tag{3}$$

where the quantity β is to be considered as determining the logarithmic derivative of the deuteron wave function at $r = 0$.

From (2) and (3) we find

$$W_d(R) = \frac{2}{3}\alpha\beta^2 R^3. \tag{4}$$

Expressing R in units of $\hbar/Mc = 2 \times 10^{-14}$ cm this becomes

$$W_d(R) = 7 \cdot 10^{-5} R^3 (\beta/\alpha)^2. \tag{4'}$$

The experimental value for this quantity is 7×10^{-3}. Thus $R^3(\beta/\alpha)^2 \sim 10^2$, a fully reasonable value.

We note that neither the pick-up theory nor the impulse approximation are applicable in the present case. In both these theories it is assumed that the incoming nucleon interacts either with one nucleon or with two nucleons, but in an independent fashion. In the present case the momentum transfer is so large that the process is due to very high harmonics of the deuteron wave function, i.e., to such states in which both nucleons are very close together. One therefore cannot consider their collision with a third nucleon as an independent event.

For other light nuclei, an estimate of the function W(R) is still more difficult. For tritium, for example, we find that $W_T(R)$ is approximately on the order of $W_d^2(R)$, with a correction for the fact that the quantity α_T will be larger than α by a factor $\sqrt{m_T\epsilon_T/m_d\epsilon_d}$ (here ϵ_T and ϵ_d are the binding energies of tritium and deuterium respectively; m_T and m_d are reduced masses of tritium and deuterium, respectively, relative to one nucleon removed from the nucleus). This reduction of α_T reflects the fact that the tritium represents a tighter nucleon cluster than the deuteron.

For He, similarly, α_{He} will be larger than α by a factor $\sqrt{m_{He}\epsilon_{He}/m_d\epsilon_d}$.

Taking the above value for $W_d(R)$, we find $W_T \sim 2 \times 10^{-4}$ and $W_{He} \sim 2 \times 10^{-5}$.

These numerical values can be checked experimentally.

3. ESTIMATE OF THE FLUCTUATIONS IN NUCLEI

Let the wave function of a nucleus $A = Z + N$ be

$$\Psi_A = \Psi_A(x_1, x_2, \ldots, x_Z;\ y_1, y_2, \ldots, y_N), \tag{5}$$

where $x_1, x_2, \ldots$ are the coordinates of the protons, and $y_1, y_2, \ldots$ the coordinates of the neutrons. As is well known, the density operator, say of the protons, is given by

$$\rho(x) = \sum_{k=1}^{Z} \delta(x - x_k). \tag{6}$$

Similarly, one can introduce a second-order density operator

$$\rho(x, x') = \sum_{k \neq s}^{Z} \delta(x - x_k)\,\delta(x - x_s). \tag{6'}$$

In general the density operator of order a, involving z protons and n neutrons ($a = n + z$), will be

$$\rho(x, x', \ldots x^{(z)};\ y, y', \ldots y^{(n)}) = \sum_{k \neq s \neq \ldots} \delta(x - x_k)\,\delta(x' - x_s) \ldots,\ \delta(y^{(n)} - y_n). \tag{7}$$

The mean value of this density is

$$\overline{\rho(x, x', \ldots y^{(n)})} = \int \Psi_A^* \rho(x, x', \ldots y^{(n)})\, \Psi_A\, dx_1\, dx_2 \ldots dy_N. \tag{8}$$

In order to obtain the exact value of this integral, one needs to know the wave function Ψ_A. However, one can find an estimate of (8) by observing that the integral should be equal to

$$\overline{\rho(x, x', \ldots y^{(n)})} = MD(x, x' \ldots x^{(z)}, y, y', \ldots y^{(n)}), \tag{9}$$

where D is the probability density for the protons (z) and neutrons (n) to be at the positions ($x, x', \ldots y^{(n)}$), and M is equal to the number of permutations of the protons and the neutrons that would realize this configuration.

We are interested in the case where the relative coordinates of these nucleons, $\xi_1, \xi_2, \ldots \xi_{a-1}$, are within a small volume $\Omega < R^3$. Introducing further the center-of-mass coordinate of the cluster, X, and integrating with respect to $\xi_1, \xi_2 \ldots \xi_{a-1}$ over the volume Ω and with respect to X over the nuclear volume, we find

$$\overline{\rho_a(\Omega)} = MW_a(R), \tag{10}$$

where

$$W_a(R) = \int_{\Omega} D(\xi_1, \xi_2, \ldots \xi_{a-1}\ X)\, d\xi_1 \ldots d\xi_{a-1}\, dX \tag{11}$$

is the probability for such a cluster, compressed into the small volume Ω, to appear.

Since only small values of R are of interest, this probability is close to the probability of a similar density fluctuation of a free nucleus of atomic number $a = n + z$. This allows the determination of this probability from studies of collisions of a nucleon with a free nucleus (A).*

For light nuclei, M is the number of states by which one can construct the cluster of interest, i.e., it is roughly proportional to Z^2.

On the other hand, in nuclei where the density of nuclear matter is distributed like in a liquid drop, the number M will be proportional to the number of fragments with $a = z + n$ contained in the nucleus; it thus is proportional to Z. In heavy nuclei one further has to account for the probability, P, that an energetic fragment will leave the nucleus from a certain depth.

Assuming that the energetic fragments appear uniformly with the volume of the nucleus, move in the direction of the momentum of the incoming fast nucleon, and have a mean free path

$$l = 1/n_0 \sigma_a, \tag{12}$$

(where n_0 is density of nucleons in the nucleus, and $\sigma_a \sim \pi r_0^2 a^{2/3}$ is the cross section of a fragment of

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*Leksin and Kumekin[6] have made an attempt to determine this probability for carbon from (p, C) collisions. Within the accuracy of this experiment, they did not observe energetic protons scattered backwards from the carbon as a whole. From the estimates made above for T and He, one can conclude that the probability for an appropriate fluctuation in carbon is completely negligible.

atomic weight a) it is easy to show that the probability that the fragment will emerge from the nucleus is

$$P = \frac{3}{\eta}\left[\frac{1}{2} - \frac{1}{\eta^2} + \left(\frac{1}{\eta} + \frac{1}{\eta^2}\right)e^{-\eta}\right], \tag{13}$$

where $\eta = D/\ell$, and $D = 2r_0A^{1/3}$ is the diameter of the nucleus. In particular, if $P_A \ll 1$ then

$$P = (\pi r_0^2/\sigma_a)A^{-1/3} = \sigma_A/A\sigma_a, \quad \sigma_A = \pi r_0^2 A^{2/3}.$$

Combining all factors, we obtain for the production of a fast fragment of atomic number a from a nucleus of atomic number A the following cross section

$$\Sigma_a = PM\sigma_a W_a(R) \cong (\sigma_A/A)MW_a(R), \tag{14}$$

while the yield of fragments a per collision is given by

$$q_a = PM(\sigma_a/\sigma_A)W_d(R) = (M/A)W_a(R), \tag{15}$$

where σ_A is the total cross section of the target nucleus.

We now consider the special case of the deuteron (a = 2). For light nuclei the number M will be given by the number of ways in which a deuteron can be obtained from the nucleons of the nucleus. In the deuteron, the spins of the neutron and proton are parallel. Therefore $M_d = 2\,(ZN/4) = \frac{1}{2}\,ZN$. We thus have for deuterons produced from light nuclei

$$\Sigma_d' = \frac{1}{2}(ZN/A)\sigma_A W_d(R). \tag{16}$$

For heavy nuclei, owing to saturation of nuclear forces, we have $M_d = Zn$, where n is the number of neutron neighbors of a proton with antiparallel spin. This number is ~ 6. We therefore have for heavy nuclei

$$\Sigma_d'' = (Zn/A)\sigma_A W_d(R). \tag{16'}$$

The yield for this case is almost constant:

$$q_d'' = (Zn/A)W_d(R). \tag{15'}$$

Considering that $\sigma_d = 70 \times 10^{-27}$ cm^2 and $W_d(R) \sim 7 \times 10^{-3}$, we have from (16) the following values for Σ_d in millibarns:

Element	Li	Be	C	O
Σ_d	1.4	2.1	3.6	5.8.

As for heavy elements, we expect, according to (15), a deuteron yield $q_d'' \sim 2\%$.

The obtained results agree essentially with the experimental values.* A more conclusive check could be obtained from a study of deuteron yields from heavier nuclei.

We further note that the given estimate of density fluctuations of tritium, $W_T(R) \sim 2 \times 10^{-4}$, leads to an expected tritium yield of 2 — 3% of the deuteron yield. This is not in disagreement with Ref. 1.

Quantitative calculations for fragments heavier than deuterons lack a sufficient theoretical foundation. It would therefore be of great interest to determine experimentally the probability of large momentum transfers for such nuclei. It would then be possible to calculate the yield of such fragments from heavy nuclei.

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*Comparing the present results with the data of Table 3, Ref. 1, one has to keep in mind that there the cross section is given for all deuterons while we give only the cross section for fast deuterons. According to estimates of Refs. 1, 2 these numbers should differ by a factor of 3. Therefore our table agrees satisfactorily with the data of Ref. 1.

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